

For TS(1) we have to calculate, for each n : t_n (1 flop) x'_n (3 flops—see equation (3) from the previous exercise) and $x_{n+1} = x_n + hx'_n$ (2 flops) giving a grand total of 6 flops.

For TS(2) we have the additional cost of computing x''_n (3 flops— $(1 - 2t)$ was evaluated for x' and does not need to be recalculated) and adding $\frac{1}{2}h^2x''_n$ (only 2 flops since the number $\frac{1}{2}h^2$ need only be calculated once, at the start of the exercise, and not on each step). Thus TS(2) costs 5 flops more than TS(1).

By a similar argument, TS(3) costs 5 flops more than TS(2).

	TS(1)	TS(2)	TS(3)
Cost	6	11	16

For the data in Table 3.1: assuming that the GE for TS(1) is proportional to h : at $h = .15$ the GE is $-.1148$, so the constant of proportionality is $-.1148/.15 \approx 0.77$. Thus, $GE \approx .77h$ and this will achieve the target GE of 0.01 with $h = .01/.77 \approx 0.013$. Thus, to integrate to $t = 1.2$ will require $1.2/.013 \approx 93$ steps. Each step costs 6 flops, so the cost of this calculation is about $6 \times 93 = 558$ flops.

The final column in Table 3.1 suggests that the GE for TS(2) $\approx 0.138h^2$. This will achieve the target GE of 0.01 with $h = \sqrt{0.01/0.138} \approx 0.27$. Thus, to integrate to $t = 1.2$ will require $1.2/.27 \approx 5$ steps. Each step costs 11 flops, so the cost of this calculation is about 55 flops.

Thus TS(1) requires about 10 times as much computational effort as TS(2) in order to achieve the required accuracy.