Solutions for homework 2

6.2. Runge-Kutta Methods

Exercise 5, page 259
Compute by hand the first five iterations using the second-order Runge- Kutta method with step-size $h = 0.1$.
Arrange the results in a tabular form: $y' = t - 2y, y(0) = 1$.

time	Eulersolution	RK2solution	RK4solution	exact
0.0	1	1	1	1
0.1	0.8	0.825	0.823416666666667	1.1103418361513
0.2	0.65	0.6905	0.687905338888889	1.24280551632034
0.3	0.54	0.58921	0.586021031126296	1.39971761515201
0.4	0.462	0.51515	0.511668285550803	1.58364939528254
0.5	0.4096	0.46342	0.459856547656627	1.79744254140026

Compute the first two stages of RK2:

$$s_1 = f(t_0, y_0) = t_0 - 2y_0 = 0 - 2 \times 1 = -2;$$
$$s_2 = f(t_1, y_0 + h \cdot s_1) = f(0.1, 1 + 0.1 \times (-2)) = f(0.1, 0.8) = 0.1 - 2 \times 0.8 = -1.5$$

and then

$$y_1 = y_0 + \frac{h}{2}(s_1 + s_2) = 1 + \frac{0.1}{2} * (-2 - 1.5) = 1 - 0.175 = 0.825.$$

For each initial value problem presented in Exercises 20-23, compute by hand the first three iterations using the fourth-order Runge-Kutta method with step size $h = 0.1$.
Arrange your results in a tabular form similar to that presented in Exercise 1.

Exercise 23.

$z' = x - 2z, \quad z(0) = 1.$

n	x_n	z_n	s_1	s_2	s_3	s_4	0.1	$\frac{\Delta x}{6}(s_1 + 2s_2 + 2s_3 + s_4)$
0	0.0	1.	$-2.$	-1.7500	-1.7750	-1.5450	0.1	-0.1766
1	0.1	0.8234	-1.5468	-1.3422	-1.3626	-1.1743	0.1	-0.1355
2	0.2	0.6879	-1.1758	-1.0082	-1.0250	-0.8708	0.1	-0.1019
3	0.3	0.5860						

(You can check your results with RK4Exercise23at6_2.)
We note that the exact solution is

$$y(t) = \frac{5}{4}e^{-2t} + \frac{1}{4}(2t - 1).$$

Remark the plots of exact solution versus the Euler, RK2 and RK4 approximations.