3102 Homework set IV (due March 12)

You need to work out one of the three problems to get full credits. You should choose the suitably challenging ones for your own sake. You are of course encouraged to work out as much as you can.

Level I Problems:

(1) Consider the leptonic decay of a charged pion

$$\pi^-(k) \to \mu^-(p_1)\bar{\nu}_\mu(p_2). \tag{1}$$

The amplitude can be written as

$$i\mathcal{M} = G_F H^\mu \ \bar{u}(p_1)\gamma_\mu P_L v(p_1), \tag{2}$$

where the chirality projection operator is $P_L = (1-\gamma_5)/2$, and H^μ is the hadronic current which can be parameterized by the poin decay constant f_π

$$H^\mu = V_{ud} f_\pi k^\mu. \tag{3}$$

• Calculate the differential decay rate $d\sigma/d\cos\theta$, where θ is the angle of the muon momentum with respect to the pion momentum in the pion rest frame.
• What should be changed to obtain the leptonic decay $\pi^- \to e^-\nu_e$?
• What should be changed to obtain the leptonic decays of K^-, D^- and B^-? [If you wish to go further, figure out their partial decay widths from the PDG ($\Gamma_i = BR/\tau$, with BR is the branching fraction and τ the life-time) and check the consistency with your numbers (orders of magnitude).]

(2) Consider pair production of charge scalars in e^+e^- collisions via a virtual photon exchange

$$e^-(p_a)\ e^+(p_b) \to \gamma^* \to H^-(p_1)\ H^+(p_2), \tag{4}$$

where the scalar QED coupling is given by $ie(p_2-p_1)_\mu$.
• Calculate the scattering amplitudes for massless electrons, and comment on the threshold behavior and partial wave composition.
• Calculate the unpolarized total cross section σ and plot the numerical results for $m_H = 120$ GeV versus the c.m. energy $\sqrt{s} \approx 240 - 1000$ GeV.
• Calculate the unpolarized differential cross section $d\sigma/d\cos\theta$ and plot the numerical results for $\sqrt{s} = 500$ GeV.
[Choose the adequate units and appropriate scales to make the plots nice-looking.]

Level II Problems:

(1) The same as (2) in Level I.

(2) Consider the associated production of ZH^0 in e^+e^- collisions via a virtual Z exchange

$$e^-(p_a)\ e^+(p_b) \to Z^* \to Z(p_1)\ H^0(p_2), \tag{5}$$

• Calculate the scattering amplitude for massless electrons, but for a polarized Z boson of both transverse polarization $\epsilon^\mu_\pm(p_1)$ as well as longitudinal polarization $\epsilon^\mu_L(p_1)$.
Comment on the threshold behavior and partial wave decomposition.
• Calculate the total cross sections for $m_z = 90$ GeV and $m_H = 120$ GeV versus the c.m. energy $\sqrt{s} \approx 210 - 1000$ GeV.
• Calculate the corresponding differential cross sections $d\sigma_{\pm,L}/d\cos\theta$ and plot the numerical results for $\sqrt{s} = 500$ GeV.

Level III Problems:

(1) Same as (2) in Level II.

(2) The most general heavy fermion 2-body decay:
Consider a top quark decay

$$t(\lambda_t, p_t) \to b(\lambda_b, p_b) + W(\lambda_w, p_w), \tag{6}$$

where λ's are the physical helicities $\lambda_t, \lambda_b = \pm 1/2$ (not the chiralities!); $\lambda_w = \pm 1, 0$ for the transverse and longitudinal polarizations. Assume the most general coupling to be

$$i\frac{g}{\sqrt{2}}\gamma^\mu(g_L P_L + g_R P_R), \tag{7}$$

where $P_{L\ (R)}$ are the left (right) projection operators with coupling constants g_L, g_R.
• Calculate the (12) helicity amplitudes $i\mathcal{M}(\lambda_t, \lambda_b, \lambda_w, \cos\theta_b)$, where $\cos\theta_b$ is the angle of the b momentum with respect to the top momentum in the top rest frame. Given them in terms of the d-functions $d^J_{\delta\lambda_i,\delta\lambda_j}$.
• Take the limit $m_b \to 0$ and comment on the meaning of the amplitude simplication.
• In the Standard Model, $g_L = 1,\ g_R = 0$, corresponding to the pure $V-A$. Calculate the ratio of the partial widths for the longitudinally polarized $W(\lambda = 0)$ and transversely polarized $W(\lambda = \pm 1)$ (Take the limit $m_b \to 0$).