3102 Homework set III (due Feb. 26, makeup day)

You need to work out one of the three problems to get full credits. You should choose the suitably challenging one for your own sake. You are of course encouraged to work out as much as you can.

Level I Problems:
(1) For a π^0, μ^-, τ^-, D^+ and B^+, calculate their decay lengths respectively for the energy $E = 10$ GeV. Based on your results, comment on how those particles may show up in a collider detector. [You may use the Review of Particle Properties from PDG.]

(2) An e^+e^- collider was designed to have the lab-frame energy $E_{e^+} = 3.1$ GeV and $E_{e^-} = 9$ GeV. What is its c.m. energy? It was designed to produce a particle in resonance. Find out what it is. What is the advantage for this asymmetric production?

(3) The amplitude for the electroweak Goldstone boson scattering is calculated to be

$$i\mathcal{M}(w^+w^+ \to w^+w^+) \approx \frac{m_H^2}{v^2}\left(\frac{t}{t-m_H^2} + \frac{u}{u-m_H^2}\right), \quad v = 246 \text{ GeV},$$

where $t, u = -\frac{s}{2}(1 \mp \cos\theta)$. Assuming the theory to be valid at very high energies $s \gg m_H^2$, using the partial wave unitarity, find the Higgs mass bound.

Level II Problems:
(1) Consider a $2 \to 2$ scattering process $p_a + p_b \to p_1 + p_2$. Assume that $m_a = m_1$ and $m_b = m_2$. Show that two of the Mandelstam variables can be written as

$$\begin{aligned} t &= -2p_{cm}^2(1-\cos\theta_{a1}^*), \\ u &= -2p_{cm}^2(1+\cos\theta_{a1}^*) + \frac{(m_1^2-m_2^2)^2}{s}, \end{aligned}$$

p_{cm} and θ^* are the momentum magnitude and the scattering polar angle, respectively, in the c.m. frame.
Note: t is negative dfinite; $t \to 0$ in the collinear limit.

(2) Properly treating the integration variables for the 2-body phase space, verify that the $2 \to 2$ cross section can be written as

$$\frac{d\sigma_2}{d\Omega} = \frac{d\sigma_2}{2\pi\ d\cos\theta} = \frac{1}{64\pi^2 s} \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{\lambda^{1/2}(s, m_a^2, m_b^2)}\ |\mathcal{M}|^2,$$

$$\frac{d\sigma_2}{dt} = \frac{1}{16\pi} \frac{1}{\lambda(s, m_a^2, m_b^2)}\ |\mathcal{M}|^2,$$

where the angles are in the c.m. frame.

(3) A particle of mass M decays to two equal-mass (m) particles isotropically in its rest frame. What does the momentum distribution look like in a frame in which the particle M is moving with a speed β_z? Compare the result with your expectation for the shape change of a basket ball. What is the reason for the difference?

Level III Problems:

(1) Same as (3) in Level I.

(2) Same as (3) in Level II.

(3) A particle of mass M decays to 3 particles $M \to a + b + c$. Properly treating the 3-body phase space element, show that the decay width can be expressed as

$$d\Gamma_3 = \frac{M}{2^8 \pi^3}\ |\mathcal{M}|^2\ dx_a dx_b.$$

$$x_i = \frac{2E_i}{M}, \quad r_i = \frac{m_i}{M}, \quad (i = a, b, c,\ \sum_i x_i = 2).$$

where the integration limits for (x_a, x_b) are

$$2r_a \le x_a \le 1 + r_a^2 - r_b^2 - r_c^2 - 2r_b r_c, \quad x_- \le x_b \le x_+,$$

$$x_\pm = \frac{(2 - x_a)(1 + r_a^2 + r_b^2 - r_c^2 - x_a) \pm (x_a^2 - 4r_a^2)^{1/2} \lambda^{1/2}(1 + r_a^2 - x_a, r_b^2, r_c^2)}{2(1 - x_a + r_a^2)}.$$