

3102 Homework set I (due Jan. 18)

You need to work out one of the three problems to get full credits. You should choose the suitably challenging one for your own sake. You are of course encouraged to work out as much as you can.

Level I Problem:

(1) Ancient Chinese philosopher Chuang Tzu stated: *“Take a foot-long stick, cut it in half every day, you’d never come to the end”*. Comment on this statement based on your modern physics knowledge and be quantitative. Go as far as you can if assuming we have the time and powerful tools to continue the job.

(2) Chairman Mao, when meeting the Japanese physicist S. Sakata (who was one of the pioneers for the quark model), stated: *“Elementary particles can’t be elementary”*. Sheldon Glashow appreciated this philosophy and hence proposed that we should name the substructural particles under quarks “Maon”. Do you think that “Maons” must exist or not, based on physics arguments rather than your belief?

(3) In 1928, P. Jordan and E.P. Wigner proposed the “second quantization” for electron wave functions by introducing the “anticommutation relations”

$$\{b_r, b_{r'}^\dagger\} = \delta_{rr'}, \quad \{b_r, b_{r'}\} = \{b_r^\dagger, b_{r'}^\dagger\} = 0, \quad (1)$$

where b_r^\dagger and b_r are the creation and annihilation operators:

$$b_r^\dagger|0\rangle = |1_r\rangle \quad \text{and} \quad b_r|0\rangle = 0, \quad (2)$$

with r denoting the relevant quantum numbers for a given particle. Show that this formulation satisfies the Pauli exclusion principle. You can introduce the occupation number operator $N_r = b_r^\dagger b_r$, if desirable.

Level II Problem:

(1) The electromagnetic, weak charged, and weak neutral currents are defined, respectively, as

$$\begin{aligned} J_{em}^\mu &= Q_f \bar{\psi}_f(x) \gamma^\mu \psi_f(x), \\ J_C^\mu &= \bar{\psi}_f(x) \gamma^\mu \frac{(1 - \gamma_5)}{2} \psi_f(x), \\ J_N^\mu &= \bar{\psi}_f(x) \gamma^\mu (v + a\gamma_5) \psi_f(x). \end{aligned}$$

Calculate the divergences of the currents $\partial_\mu J^\mu$ (you may use the Dirac equation), and understand your results.

(2) Derive the current-current interactions for the weak charged current and the electromagnetic current respectively (with the help of the SM Lagrangian). Show that the weak currents lead to 4-fermion interactions at low energies responsible for the β decay (a simple step); and the electromagnetic currents lead to the Coulomb's law in the non-relativistic limit (a bit tedious).

Level III Problem:

(1) Go through the model construction based on the $SO(3)$ gauge group in the reference Phys. Rev. Lett. 28 (1972) p. 1494, by H. Georgi and S. Glashow, as detailed as you can.

(2) Compare this model with the SM for the particle spectra and their interactions, and comment on how to test them experimentally.