Problem 1 Find the sum of the following series.

\[ a) \sum_{n=2}^{\infty} \frac{(-2)^n - 2 \cdot 3^n}{4^{n-1}} \]

\[ b) \sum_{n=2}^{\infty} \left[ \cos \left( \frac{\pi}{n} \right) - \cos \left( \frac{\pi}{n+2} \right) \right] \]

Problem 2 Solve the initial value problem.

\[ y'' + 9y = 0, \quad y(0) = 1, \quad y'(0) = 2 \]

Problem 3

a) Write down a formula for the n-th term \( a_n \) of the following sequence.

\[ 3, \frac{5}{2}, \frac{7}{3}, \frac{9}{4}, \frac{11}{5}, \ldots \]

b) Does this sequence converge? If it does, what does it converge to?

Problem 4 Solve the differential equation.

\[ 4y'' - 5y' + y = e^{2x} \]

Problem 5 Determine whether the following series converges or diverges. Full explanation is required. Name all the tests and theorems used and indicate why they are applicable.

\[ \sum \frac{n + \sin n}{\sqrt{2n^4 + 3n \cdot \ln n}} \]

Problem 6

a) Convert the polar equation \( r(\theta) = 4 \sin(\theta) \) to \((x, y)\)–coordinates and identify the curve.

b) Find parametric equations of the line tangent to \( r(\theta) = \cos(\theta) - 1 \) at the point where \( \theta = -\frac{\pi}{3} \).
Problem 7 Solve the differential equation.

\[ xy' + 2y = \sin(2x + 3) \]

Problem 8 Determine whether the following series converges absolutely, converges conditionally or diverges. Full explanation is required. Name all the tests and theorems used and indicate why they are applicable.

\[ \sum \frac{(-3)^n \cdot (n!)^2}{(2n + 1)!} \]

Problem 9 Determine whether or not the sequence \( \{a_n\} \) converges. If it does, find its limit.

a) \( a_n = \ln(2n + 1) - \ln(3n + 1) \)

b) \( a_n = \frac{(-1)^n}{\sqrt{n}} \)

Problem 10 a) Write down the logistic differential equation with the carrying capacity 100 and the constant \( k = 2 \).

b) For a solution \( y(x) \) to the above differential equation with \( y(0) = 20 \), find \( y(4) \).