Theorems you should be able to PROVE

1. The Fundamental Theorem of Invertible Matrices (you may be asked to prove the equivalence of any 2 of the 15 statements.)
2. Matrix transformation is linear.
3. Any linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation.
4. Formulate and prove 2 properties of linear transformations.
5. Kernel of a linear transformation $T: V \to W$ is a subspace of $V$.
6. Image of a linear transformation $T: V \to W$ is a subspace of $W$.
7. A linear transformation $T$ is one-to-one if and only if $\text{Ker}(T) = \{0_V\}$.
8. A one-to-one linear transformation preserves linear independence of vectors.
10. Isomorphic finite-dimensional vector spaces have the same dimension.
11. An $n-$dimensional vector space is isomorphic to $\mathbb{R}^n$.
12. A linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ is one-to-one if and only if $T$ is onto.
13. The composition of the linear transformations is linear.
14. If $A_T$ is the matrix of a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$, and $A_S$ is the matrix of a linear transformation $S: \mathbb{R}^m \to \mathbb{R}^l$, then the matrix $A_{S \circ T}$ of $S \circ T: \mathbb{R}^n \to \mathbb{R}^l$, is $A_{S \circ T} = A_S \cdot A_T$.
15. The inverse of the invertible linear transformation is unique.
16. A linear transformation $T: V \to W$ is invertible if and only if it is one-to-one onto.
17. A linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ is invertible if and only if $\text{rk}(A_T) = n$, where $A_T$ is the matrix of $T$.
18. If $A_T$ is the matrix of the invertible transformation $T: \mathbb{R}^n \to \mathbb{R}^n$, then the matrix of the inverse transformation $T^{-1}$ is $A_T^{-1}$.
19. If the coordinates of the vector $x$ in the old basis $\mathcal{B}$ are $[x]_\mathcal{B}$, its in the new basis $\mathcal{B}'$ are $[x]_\mathcal{B}'$, and $P$ is the change of basis matrix, then $[x]_\mathcal{B} = P \cdot [x]_\mathcal{B}'$.
20. If $A_T$ is the matrix of a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$, in the basis $\mathcal{B}$, and $P$ is the change of basis matrix from the old basis $\mathcal{B}$ to the new basis $\mathcal{B}'$, then the matrix of $T$ in the basis $\mathcal{B}'$ is $P^{-1}A_T P$.
21. For any permutations $\sigma, \tau \in S_n$, $\text{sgn}(\tau \circ \sigma) = \text{sgn}(\tau) \cdot \text{sgn}(\sigma)$.
22. A transposition is an odd permutation.
23. Formulate and prove 7 properties of the determinants (you may be asked to prove any one of the properties).

24. For any \( n \times n \) matrices \( A \) and \( B \), \( \det(AB) = \det(A) \cdot \det(B) \).

25. If a matrix \( A \) is invertible, then \( \det(A^{-1}) = \frac{1}{\det(A)} \).


27. Formulate and prove the Cramer’s Rule.

28. State and prove the formula for the inverse of the matrix using the adjoint.

29. Eigenspace corresponding to an eigenvalue \( \lambda \) is a subspace of \( \mathbb{R}^n \).

30. If \( \lambda_1, \ldots, \lambda_k \) are distinct eigenvalues of \( A \), and \( v_1, \ldots, v_k \), are the corresponding eigenvectors, then \( v_1, \ldots, v_k \) are linearly independent.

31. An \( n \times n \) matrix \( A \) is diagonalizable if and only if \( A \) has \( n \) linearly independent eigenvectors.

32. If an \( n \times n \) matrix \( A \) has \( n \) distinct eigenvalues, then \( A \) is diagonalizable.