

1)

$$\begin{aligned} \sigma_B^2 &= \langle (B - \langle B \rangle)^2 \rangle = \int \psi^* (B - \langle B \rangle)^2 \psi dx \\ &= \int \psi^* (\hat{B}^2 + \langle B \rangle^2 - 2\hat{B}\langle B \rangle) \psi dx \\ &= \int \psi^* \hat{B}^2 \psi dx + \langle B \rangle^2 \int \psi^* \psi dx - 2\langle B \rangle \int \psi^* \hat{B} \psi dx \\ &= \langle B^2 \rangle + \langle B \rangle^2 - 2\langle B \rangle \cdot \langle B \rangle \\ &= \langle B^2 \rangle + \langle B \rangle^2 - 2\langle B \rangle^2 \\ &= \langle B^2 \rangle - \langle B \rangle^2 \end{aligned}$$

6)

$$\begin{aligned} E_0 &= \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \\ &= \frac{1}{2} \cdot \frac{6.626 \times 10^{-34} \text{ Js}}{2 \cdot \pi} \sqrt{\frac{5.16 \text{ Nm}^{-1}}{1.008 \times 34.969 \times 1.66 \times 10^{-27}}} \\ &= 2.97 \times 10^{-20} \text{ J} \end{aligned}$$

IF this was converted into translational energy then

$$2.97 \times 10^{-20} = \frac{1}{2} m v^2$$

$$\Rightarrow v = \sqrt{\frac{2 \times 2.97 \times 10^{-20}}{m}} = \sqrt{\frac{2 \times 2.97 \times 10^{-20}}{(1.008 + 34.969) \cdot 1.66 \times 10^{-27}}} = 456 \text{ ms}^{-1}$$

$$|v_{rms}| = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.381 \times 10^{-23} \cdot 300}{(1.008 + 34.969) \cdot 1.66 \times 10^{-27}}} = 456 \text{ ms}^{-1}$$

$$\frac{v}{|v_{rms}|} = \frac{997}{456} = 2.19$$

$$2) \quad \psi_1 = \left(\frac{4\alpha^3}{\pi} \right)^{1/4} x e^{-\alpha x^2/2}$$

$$\langle x \rangle = \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} x e^{-\alpha x^2/2} \cdot x \cdot x e^{-\alpha x^2/2} dx$$

$$= \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} x^3 e^{-\alpha x^2/2} dx$$

odd func. \times

$$= 0$$

$$\langle x^2 \rangle = \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} x e^{-\alpha x^2/2} \cdot x^2 \cdot x e^{-\alpha x^2/2} dx$$

$$= \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} x^4 e^{-\alpha x^2/2} dx$$

$$= \left(\frac{4\alpha^3}{\pi} \right)^{1/2} 2 \int_0^{\infty} x^4 e^{-\alpha x^2} dx$$

$$\int_0^{\infty} x^{2n} e^{-\alpha x^2} dx = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^{n+1} \alpha^n} \sqrt{\frac{\pi}{\alpha}}$$

Now

$$\Rightarrow \langle x^2 \rangle = \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \cdot 2 \cdot \frac{1 \cdot 3}{2^3 \cdot \alpha^2} \sqrt{\frac{\pi}{\alpha}}$$

$$= \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \cdot 2 \cdot \frac{3}{8\alpha^2} \sqrt{\frac{\pi}{\alpha}}$$

$$= \frac{3}{2\alpha}$$

$$\Rightarrow \sigma_x = \sqrt{\frac{3}{2\alpha} - 0} = \sqrt{\frac{3}{2\alpha}}$$

$$\langle P_x^2 \rangle = \frac{3}{2} \hbar^2 \alpha$$

$$\begin{aligned} \text{Thus } \sigma_{P_x} &= \sqrt{\langle P_x^2 \rangle - \langle P_x \rangle^2} \\ &= \sqrt{\frac{3}{2} \hbar^2 \alpha} \end{aligned}$$

$$\begin{aligned} \Rightarrow \sigma_x \sigma_{P_x} &= \sqrt{\frac{3}{2\alpha}} \cdot \sqrt{\frac{3}{2} \hbar^2 \alpha} \\ &= \sqrt{\frac{9}{4} \hbar^2} \\ &= \frac{3}{2} \hbar \geq \frac{\hbar}{2} \end{aligned}$$

$$\langle P_x \rangle = \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} x e^{-\alpha x^2/2} \left(-i\hbar \frac{\partial}{\partial x} x e^{-\alpha x^2/2} \right) dx$$

$$= \left(\frac{4\alpha^3}{\pi} \right)^{1/2} (-i\hbar) \int_{-\infty}^{\infty} x e^{-\alpha x^2/2} \left[e^{-\alpha x^2/2} - \alpha x \cdot x e^{-\alpha x^2/2} \right] dx$$

$$= -i\hbar \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \left[\int_{-\infty}^{\infty} x e^{-\alpha x^2} dx - \alpha \int_{-\infty}^{\infty} x^3 e^{-\alpha x^2/2} dx \right]$$

Both terms are 0 \Rightarrow odd func. $f(x)$

$$= 0$$

[think of this answer is true]

$$\langle P_x^2 \rangle = \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} x e^{-\alpha x^2/2} \left(-i\hbar \frac{\partial}{\partial x} \right) \left(-i\hbar \frac{\partial}{\partial x} x e^{-\alpha x^2/2} \right) dx$$

$$= - \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \hbar^2 \int_{-\infty}^{\infty} x e^{-\alpha x^2} \frac{\partial}{\partial x} \left[e^{-\alpha x^2/2} - \alpha x^2 e^{-\alpha x^2/2} \right] dx$$

$$= - \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \hbar^2 \int_{-\infty}^{\infty} x e^{-\alpha x^2} \left[-\alpha x e^{-\alpha x^2/2} - 2\alpha x e^{-\alpha x^2/2} + \alpha x^2 (\alpha x) e^{-\alpha x^2/2} \right] dx$$

$$= - \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \hbar^2 \left[\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx + 2\alpha^2 \int_{-\infty}^{\infty} x^4 e^{-\alpha x^2} dx \right]$$

$$= - \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \hbar^2 \left[-6\alpha \int_0^{\infty} x^2 e^{-\alpha x^2} dx + 2\alpha^2 \int_0^{\infty} x^4 e^{-\alpha x^2} dx \right]$$

$$= - \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \hbar^2 \left[-6\alpha \cdot \frac{1}{2^2 \alpha} \sqrt{\frac{\pi}{\alpha}} + 2\alpha^2 \cdot \frac{1 \cdot 3}{2^3 \alpha^2} \sqrt{\frac{\pi}{\alpha}} \right]$$

$$= - \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \hbar^2 \left[-\frac{6}{4} \sqrt{\frac{\pi}{\alpha}} + \frac{6}{8} \sqrt{\frac{\pi}{\alpha}} \right] = + \left(\frac{4\alpha^3}{\pi} \right)^{1/2} \hbar^2 \frac{3}{4} \sqrt{\frac{\pi}{\alpha}}$$

$$4) \quad \psi_2(x) = \left(\frac{\alpha}{4\pi}\right)^{1/4} (2\alpha x^2 - 1) e^{-\alpha x^2/2}$$

$$\hat{H}\psi = -\frac{\hbar^2}{2\mu} \frac{\partial}{\partial x} \cdot \frac{\partial}{\partial x} \left(\frac{\alpha}{4\pi}\right)^{1/4} (2\alpha x^2 - 1) e^{-\alpha x^2/2} + \frac{1}{2} kx^2 \psi_2$$

$$= -\frac{\hbar^2}{2\mu} \frac{\partial}{\partial x} \left[\left(\frac{\alpha}{4\pi}\right)^{1/4} \cdot \left\{ (4\alpha x) e^{-\alpha x^2/2} + (2\alpha x^2 - 1)(-\alpha x) e^{-\alpha x^2/2} \right\} \right] + \frac{1}{2} kx^2 \psi_2$$

$$= -\frac{\hbar^2}{2\mu} \frac{\partial}{\partial x} \left[\left(\frac{\alpha}{4\pi}\right)^{1/4} \left\{ 4\alpha x e^{-\alpha x^2/2} + (\alpha x - 2\alpha^2 x^3) e^{-\alpha x^2/2} \right\} \right] + \frac{1}{2} kx^2 \psi_2$$

$$= -\frac{\hbar^2}{2\mu} \left[\left(\frac{\alpha}{4\pi}\right)^{1/4} \left\{ 4\alpha e^{-\alpha x^2/2} - (\alpha x) 4\alpha x e^{-\alpha x^2/2} - \alpha x (\alpha x - 2\alpha^2 x^3) e^{-\alpha x^2/2} + (\alpha - 6\alpha^2 x^2) e^{-\alpha x^2/2} \right\} \right] + \frac{1}{2} kx^2 \psi_2$$

$$= -\frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/4} \left[4\alpha - 4\alpha^2 x^2 - \alpha^2 x^2 + 2\alpha^3 x^4 + \alpha - 6\alpha^2 x^2 \right] e^{-\alpha x^2/2} + \frac{1}{2} kx^2 \psi_2$$

$$= -\frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/4} \left[5\alpha - 11\alpha^2 x^2 + 2\alpha^3 x^4 \right] e^{-\alpha x^2/2} + \frac{1}{2} kx^2 \psi_2$$

$$= -\frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/4} 5\alpha e^{-\alpha x^2/2} - \frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/4} e^{-\alpha x^2/2} (2\alpha x^2 - 11)\alpha^2 x^2 + \frac{1}{2} \frac{\alpha^2 \hbar^2}{\mu} x^2 (2\alpha x^2 - 1) \left(\frac{\alpha}{4\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

$$= -\frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/4} 5\alpha e^{-\alpha x^2/2} - \frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/2} e^{-\alpha x^2/2} \alpha^2 x^2 (2\alpha x^2 - 11 - 2\alpha x^2 + 1)$$

$$= -\frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/4} 5\alpha e^{-\alpha x^2/2} + \frac{\hbar^2}{2\mu} \left(\frac{\alpha}{4\pi}\right)^{1/2} e^{-\alpha x^2/2} \alpha^2 x^2 (10)$$

$$= \frac{\hbar^2}{2\mu} \cdot 5\alpha \psi_2 = \frac{\hbar^2}{2\mu} \cdot 5 \sqrt{\frac{\mu k}{\hbar^2}} \psi_2$$

$$= \frac{5}{2} \hbar \sqrt{\frac{k}{\mu}} \psi_2 \Rightarrow \boxed{E_2 = \frac{5}{2} \hbar \sqrt{\frac{k}{\mu}}}$$

7)

7.21

$$\frac{-\hbar^2}{2\mu r_0} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial Y(\theta, \phi)}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 Y(\theta, \phi)}{\partial\phi^2} \right] = E Y(\theta, \phi)$$

$$\text{L.H.S.} = \frac{-\hbar^2}{2\mu r^2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\frac{5}{16\pi} \right)^{1/2} (-6\cos\theta \sin\theta) \right]$$

$$= \frac{-\hbar^2}{2\mu r^2} \left[\frac{1}{\sin\theta} \left(\frac{5}{16\pi} \right)^{1/2} \frac{\partial}{\partial\theta} (-6\cos\theta \sin^2\theta) \right]$$

$$= \frac{-\hbar^2}{2\mu r^2} \left[\frac{1}{\sin\theta} \left(\frac{5}{16\pi} \right)^{1/2} \{6\sin^3\theta - 12\cos^2\theta \sin\theta\} \right]$$

$$= \frac{-\hbar^2}{2\mu r^2} \left(\frac{5}{16\pi} \right)^{1/2} (6\sin^2\theta - 12\cos^2\theta)$$

$$= \frac{-\hbar^2}{2\mu r^2} \left(\frac{5}{16\pi} \right)^{1/2} (6 - 6\cos^2\theta - 12\cos^2\theta)$$

$$= \frac{+\hbar^2}{2\mu r^2} 6 (3\cos^2\theta - 1) \left(\frac{5}{16\pi} \right)^{1/2}$$

$$= \frac{3\hbar^2}{\mu r^2} Y(\theta, \phi)$$

$$\Rightarrow E_e = \frac{3\hbar^2}{\mu r^2}$$

$$\left\{ \begin{array}{l} \text{Nak } E_l = \frac{\hbar^2}{2\mu r^2} l(l+1) \\ \Rightarrow l=2 \end{array} \right.$$

8) 7:22

$$\int_0^{2\pi} \psi_n^* \psi_m d\phi = \frac{1}{2\pi} \int_0^{2\pi} e^{in_e\phi} e^{-im_e\phi} d\phi = \frac{1}{2\pi} \int_0^{2\pi} e^{i(n_e - m_e)\phi} d\phi$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \{ \cos(n_e - m_e)\phi + i \sin(n_e - m_e)\phi \} d\phi$$

$$= \frac{1}{2\pi} \left[\frac{1}{(n_e - m_e)} \left\{ \sin(n_e - m_e)\phi \Big|_0^{2\pi} \right\} - \frac{i}{(n_e - m_e)} \left\{ \cos(n_e - m_e)\phi \Big|_0^{2\pi} \right\} \right]$$

$$= \frac{1}{2\pi} \left[\frac{1}{(n_e - m_e)} \{ \sin 2\pi(n_e - m_e) - \sin 0 \} - \frac{i}{(n_e - m_e)} \{ \cos 2\pi(n_e - m_e) - \cos 0 \} \right]$$

$$= \frac{1}{2\pi} \left[0 - \frac{i}{(n_e - m_e)} (1 - 1) \right] = 0$$

9) 7:29

$$\frac{n_j}{n_{j=0}} = \frac{2j+1}{1} e^{-\frac{h^2 j(j+1)}{2IkT}}$$

$$I = \mu r^2 = \frac{1.008 \times 34.969}{1.008 + 34.969} \times 1.66 \times 10^{-27} \times (1.27 \times 10^{-10})^2$$

$$= 2.733 \times 10^{-47}$$

$$\Rightarrow \frac{n_1}{n_0} = 3 e^{-\frac{(1.055 \times 10^{-34})^2 (1+1)}{(2.733 \times 10^{-47} \times 1.381 \times 10^{-23} \times 300)}}$$

$$= 2.708$$

$$\frac{n_5}{n_0} = 11 e^{-\frac{(1.055 \times 10^{-34})^2 5(6)}{(2.733 \times 10^{-47} \times 1.381 \times 10^{-23} \times 300)}}$$

$$= 2.369$$