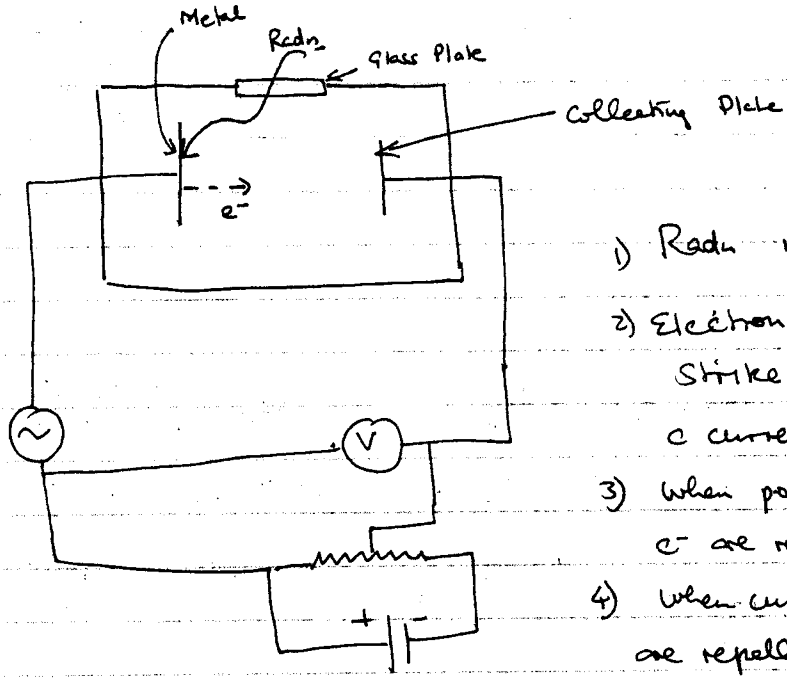
HW # 1

(b)

(1)

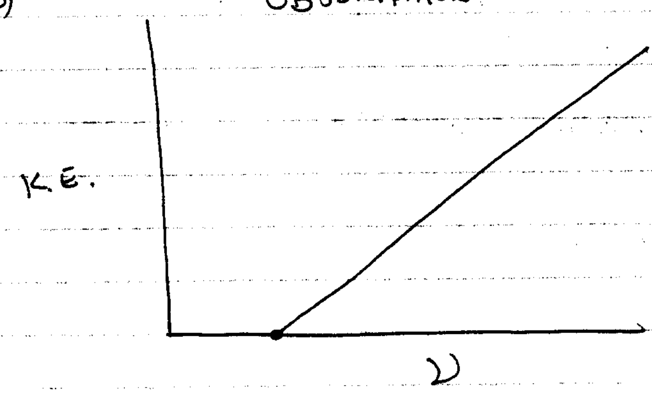
(a)



1) Radn impinges on metal.
2) Electron are released and Strike collecting plate to yield a current
3) When potential is made negative e^- are repelled & current drops
4) When current is zero ~~all~~ even fastest electrons are repelled the potential is measured

$K.E. = eV_{stop}$
↳ Potn when current is zero

b) OBSERVATION



① ~~K.E.~~ ~~as~~ electrons are ejected only till a certain frequency ν reached
② K.E. $\propto \nu$
③ K.E. independent of intensity of light.

Implications. 1) Energy of light must be $E = \beta \nu$
↳ constant

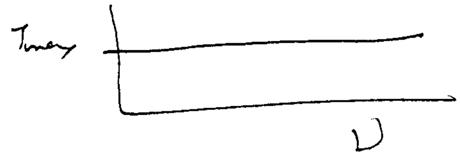
2) light can act as a particle, it can give all its energy to an electron.

3) ~~tog~~

(c) If light were a wave then

i) $E_{rad^n} \propto$ Intensity of wave $\Rightarrow$ $T_{max} \propto$ Intensity

ii) If Intensity is sufficient to overcome work function, then electrons would be emitted @ all freq.



(d)

$$T_{max} = h\nu - \Phi = \frac{hc}{\lambda} - \Phi \quad \text{— (1)}$$

Plot T_{max} vs $1/\lambda$

[Remember UNITS; T_{max} are 4.49×10^{-19} J etc. [$T_{max} \times 10^{19}$ is provided] λ should be converted into meters]

Best fit yields $K.E. = -3.97362 \times 10^{-19} + 2.11171 \times 10^{-25} \frac{1}{\lambda}$

Slope $= hc = 2.11171 \times 10^{-25}$

$$h = \frac{2.11171 \times 10^{-25}}{2.998 \times 10^{8}\ ms^{-1}} \approx 7 \times 10^{-34}\ Js$$

Intercept: From EQ 1. $\Rightarrow$ when $T_{max} = 0$, {ie. @ ν_{th}}

$$\Phi = \frac{hc}{\lambda_{th}}$$

We have $T_{max} = -3.97362 \times 10^{-19} + \frac{2.11171 \times 10^{-25}}{\lambda}$

$T_{max} = 0$ when $\lambda_{th} = \frac{2.11171 \times 10^{-25}\ Jm}{3.97362 \times 10^{-19}\ J}$

$= 5.3 \times 10^{-7}$ m

$$\Rightarrow \Phi = \frac{hc}{\lambda_{th}} = 4 \times 10^{-19}\ J$$

(2)

$$\frac{N_2}{N_1} = e^{-\Delta E/kT}$$

$$kT = 1.381\times10^{-23}\times 298 = 4.1\times10^{-21}\ J$$

(a) $\Delta E = 5\times10^{9}\ s^{-1} \times 6.6266\times10^{-34}\ Js = 3.313\times10^{-24}$

$$\Rightarrow \frac{N_2}{N_1} = e^{-3.313\times10^{-24}/4.1\times10^{-21}} = e^{-8\times10^{-4}} = 0.9992$$

(b) $\Delta E = 500\ cm^{-1} \times 6.626\times10^{-34}\ Js \times 3\times10^{10}\ cms^{-1}$

$$= 9.939\times10^{-21}\ J$$

$$\frac{N_2}{N_1} = e^{-9.94\times10^{-21}/4.1\times10^{-21}} = e^{-2.42}$$

$$= 0.088$$

(c) $\Delta E = 2eV \times 1.6\times10^{-19}\ J/eV$

$$= 3.204\times10^{-19}\ J$$

$$\frac{N_2}{N_1} = e^{-3.204\times10^{-19}J/4.1\times10^{-21}}$$

$$= e^{-78} \simeq 1\times10^{-34}$$, No population in state 2.

(d) $\Delta E = 500\times10^{6}\ s^{-1} \times 6.626\times10^{-34}\ Js$

$$= 3.31\times10^{-25}$$

$$\frac{N_2}{N_1} = e^{-3.31\times10^{-25}/4.1\times10^{-21}}$$

$$= e^{-8\times10^{-5}} = .99992$$

Note the trends $\Rightarrow$ they will be important.

3) Require $\hat{O} f(x) = a f(x)$, with a a constant

a) $$\frac{d^3}{dx^3} x^3 = \frac{d^2}{dx^2} 3x^2 = \frac{d}{dx} 6x = 6$$

Not an eigenfunction

b) $$x \frac{\partial}{\partial x} xy + y \frac{\partial xy}{\partial y}$$

$$= xy + yx = 2xy$$

xy is an eigenfnc. of $x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$ with an eigenvalue of 2

c) $$\frac{\partial^2}{\partial \theta^2} \sin\theta \cos\theta = \frac{1}{2} \frac{\partial^2}{\partial \theta^2} \sin 2\theta$$

$$= \frac{1}{2} \frac{\partial 2\cos 2\theta}{\partial \theta} = \frac{\partial}{\partial \theta} \cos 2\theta$$

$$= -2 \sin 2\theta$$

$$= -2 \cdot 2 \sin\theta \cos\theta$$

$$= -4 \sin\theta \cos\theta$$

$\sin\theta \cos\theta$ is an eigenfn of $\frac{\partial^2}{\partial \theta^2}$ with an eigenvalue of -4.

d) $$\frac{\partial^2}{\partial x^2} e^{-kx} = -k \frac{\partial}{\partial x} e^{-kx} = +k^2 e^{-kx}$$

e^{-kx} is an eigenfnc with an eigenvalue of k^2

4) ~~P2~~

$$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} + \frac{2}{r}\right]Ae^{-br}$$

$$= \frac{1}{r^2}\frac{d}{dr}r^2\left(-Abe^{-br}\right) + \frac{2A}{r}e^{-br}$$

$$= \frac{1}{r^2}\left[2r\left(-Abe^{-br}\right) + r^2\left(+Ab^2e^{-br}\right) + \frac{2A}{r}e^{-br}\right]$$

$$= \frac{1}{r^2}\left[-2Abre^{-br} + r^2Ab^2e^{-br}\right] + \left[\frac{2}{r}Ae^{-br}\right]$$

$$= ~~2Ae^{-br}(~~ \left[-\frac{2Ab}{r}e^{-br} + Ab^2e^{-br}\right] + \left[\frac{2Ae^{-br}}{r}\right]$$

$$= Ae^{-br}\left[\frac{-2b}{r} + b^2 + \frac{2}{r}\right]$$

$$= Ae^{-br}\left[\frac{2-2b}{r}\right] + b^2Ae^{-br}$$

This term must vanish for Ae^{-br} to be an eigenfunction of $\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} + \frac{2}{r}$

This requires that $b = 1$

When $b = 1$

$$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} + \frac{2}{r}\right]Ae^{-br} = Ae^{-br}\left[\frac{2-2}{r}\right] + b^2Ae^{-br}$$

$$= b^2Ae^{-br}$$

5.) FOR a set of functions,

$$\psi_1, \psi_2, \psi_3, \ldots, \psi_i \ldots \psi_j, \ldots$$

ORTHOGONALITY implies that $\int \psi_i^* \psi_j \, dx = 0$ if $i \neq j$

$i = 1, 2, 3 \ldots$
$j = 1, 2, 3 \ldots$

NORMALIZED implies that $\int \psi_i^* \psi_i \, dx = 1$ for all i

Completeness implies that another function, f, may be written as:

$$f(x) = \sum_i b_i \psi_i^*$$

)

Normalization requires that $\quad$ for PIB $0 \le x \le a$

$$\int_{-\infty}^{\infty} \psi_n^* \psi_n dx = \int_0^a \psi_n^* \psi_n dx = 1$$

$$\Rightarrow N^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

Let $y = \frac{n\pi x}{a}$ when $x = 0$, $y = 0$; $x = a$; $y = n\pi$

$dy = \frac{n\pi}{a} dx \Rightarrow dx = \frac{a}{n\pi} dy$

$$\Rightarrow \frac{a}{n\pi} N^2 \int_0^{n\pi} \sin^2 y \, dy = \frac{a}{n\pi} N^2 \left| \frac{y}{2} - \frac{\sin 2y}{4} \right|_0^{n\pi} = 1$$

$$\Rightarrow \frac{a}{n\pi} \cdot N^2 \cdot \frac{n\pi}{2} = 1$$

$$\Rightarrow N^2 = \frac{2}{a} \quad \text{or} \quad N = \sqrt{\frac{2}{a}}$$

Are the functions orthogonal: Determine $\int \psi_n^* \psi_m \, dx$ $\quad m \neq n$

~~$\frac{2}{a} \int \psi_1$~~

$$\Rightarrow \frac{2}{a} \int_0^a \sin\frac{n\pi x}{a} \sin\frac{m\pi x}{a} = \frac{2}{a}\left[\int_0^a \cos(m-n)\frac{\pi x}{a} dx - \int_0^a \cos(m+n)\frac{\pi x}{a} dx\right]$$

$$= \frac{2}{a}\left[\frac{a \sin(m-n)\frac{\pi x}{a}}{(m-n)\pi}\Bigg|_0^a - \frac{a}{(m+n)\pi} \sin(m+n)\frac{\pi x}{a}\Bigg|_0^a\right] = 0$$

functions are orthogonal.