Solution

PRACTICE FINAL EXAM

This is a closed book-exam. You may use a calculator and one two-sided sheet of notes. Mark all answers clearly and be sure to include work in cases where partial credit may be awarded. Necessary tables are attached.

Section 1: Multiple Choice: For questions 1-16, circle the best response to each question.

1.) Which of the following is a matched-pairs design?
   a. Measure level of right hand gripping strength for one random sample, and measure the left hand gripping strength of a different random sample.
   b. Measure levels of right hand gripping strength and left hand gripping strength of the individuals in a single random sample.

2.) Which of the following is a design that involves two independent samples?
   a. Take an SRS from Pennsylvania and calculate the percentage who approve of President Bush's policies in Iraq. Take an SRS from Ohio and calculate the percentage who approve of President Bush's policies in Iraq.
   b. Take an SRS of residents from California and calculate the percentage who approve of George Bush's policies in Iraq and the percentage of those who approve of George Bush's policies in The United States.

Use the following to answer questions 3-5:

An SRS of 100 laborers who use the services of a national temporary employment agency found that in the past year the average number of days worked by these laborers was \( \bar{x} = 107 \) days, with standard deviation \( s = 45 \) days. Assume the distribution of the number of days worked in the population of laborers using this employment agency is approximately normal, with mean \( \mu \). Are these data evidence that \( \mu \) has lowered from the value of 120 days of 5 years ago? To determine this, we test the hypotheses

\[ H_0 : \mu = 120 \]
\[ H_a : \mu < 120 \]
3.) The appropriate degrees of freedom for this test are

a. 45.
b. 99.
c. 100.
d. 120.

4.) Based on the data, the value of the one-sample t statistic is

a. 3.41.
b. 2.89.
c. 2.67.
d. 2.38.

5.) The P-value for the one-sample t test is

a. larger than 0.10.
b. between 0.10 and 0.05.
c. between 0.05 and 0.01.
d. below 0.01.

6.) Does the mean cost of text books per semester differ for math students and students in the liberal arts? A sample of six math students and a sample of six liberal arts students were asked how much their text books had cost the previous semester. The results (in dollars) are given below.

<table>
<thead>
<tr>
<th>Math students</th>
<th>420</th>
<th>395</th>
<th>200</th>
<th>350</th>
<th>410</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal Arts students</td>
<td>330</td>
<td>470</td>
<td>300</td>
<td>210</td>
<td>500</td>
<td>200</td>
</tr>
</tbody>
</table>

To analyze this data we should use

a. the matched-pairs t test.
b. the one-sample t test
c. the two-sample t test
d. Any of the above are valid.
7.) Which of the following statements is true?

a. Two-sample $t$ procedures are less robust than the one-sample $t$ methods.
b. In planning a two-sample study, it is best to choose equal sample sizes.
c. In planning a two-sample study, if the two population distributions have different shapes, then you can use samples of size 5.
d. None of the above is true.

8.) We look at a random sample of statistics students’ Verbal SAT scores to see if in general they average more than 530. What is the most appropriate statistical test?

a. $t$ test about a regression coefficient
b. $t$ test about a mean with a two-sided alternative hypothesis
c. $t$ test about a mean with a one-sided alternative hypothesis
d. Chi-Square ($\chi^2$) test

9.) Four runners are asked to run a 10-kilometer race on each of two consecutive weeks. In one of the races the runners wear one brand of shoe and in the other a second brand. All runners are timed and are asked to run their best in each race. The results (in minutes) are given below.

<table>
<thead>
<tr>
<th>Runner</th>
<th>Brand1</th>
<th>Brand2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.23</td>
<td>32.02</td>
</tr>
<tr>
<td>2</td>
<td>29.33</td>
<td>28.98</td>
</tr>
<tr>
<td>3</td>
<td>30.50</td>
<td>30.63</td>
</tr>
<tr>
<td>4</td>
<td>32.20</td>
<td>32.67</td>
</tr>
</tbody>
</table>

To determine if there is evidence that the mean time using brand 1 is less than the mean time using brand 2, we would use

a. the one-sample $z$ test.
b. the matched pairs $t$ test.
c. the two-sample $t$ test.
d. any of the above are valid. It is at the experimenter's discretion.
10.) We want to test for a relationship between gender and whether or not a student smokes. What is the most appropriate statistical test?

a. inference for regression  
b. t test using two independent samples  
c. Chi-Square test  
d. t test for a matched pairs setting

Use the following to answer questions 11-12:

When a police officer responds to a call for help in a case of spousal abuse, what should the officer do? A randomized controlled experiment in Charlotte, North Carolina studied three police responses to spousal abuse: advise and possibly separate the couple, issue a citation to the offender, and arrest the offender. The effectiveness of the three responses was determined by re-arrest rates. The following table shows these rates.

<table>
<thead>
<tr>
<th>Number of subsequent arrests</th>
<th>Assigned Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Arrest</td>
</tr>
<tr>
<td>0</td>
<td>175</td>
</tr>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

11.) Suppose we wish to test the null hypothesis that the proportion of subsequent arrests is the same regardless of the treatment assigned. Under the null hypothesis, the expected number of times no subsequent arrest would occur for the treatment “Advise/Separate” is

a. 177  
b. 181  
c. 187  
d. 543
12.) Suppose we wish to test the null hypothesis that the proportion of subsequent arrests is the same regardless of the treatment assigned. Which of the following statements is true?

a. We cannot test this hypothesis because the police officers did not record the expected counts.

b. The test of the null hypothesis will have a very small $P$-value (below .0001) because the counts in each row are not identical.

c. We cannot test this hypothesis because the expected cell counts are less than five in too many of the cells.

d. The test of the null hypothesis will have a very small $P$-value (below .0001) because there were so few cases where there was more than one re-arrest.

Use the following to answer questions 13-15:

An old saying in golf is "You drive for show and you putt for dough." The point is that good putting is more important than long driving for shooting low scores and hence winning money. To see if this is the case, data on the top 69 money winners on the PGA tour in 1993 are examined. The average number of putts per hole for each player is used to predict their total winnings using the simple linear regression model

$$1993\text{winnings}_j = \alpha + \beta (\text{average number of putts per hole})_j + \varepsilon_j$$

where the deviations $\varepsilon_j$ are assumed to be independent and normally distributed with mean 0 and standard deviation $\sigma$. This model was fit to the data using the method of least squares. The following results were obtained from statistical software.

$$R^2 = 0.081$$
$$s = 281,777$$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Std. Error of Par. Est.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>7,897,179</td>
<td>3,023,782</td>
</tr>
<tr>
<td>Avg. Putts</td>
<td>-4,139,198</td>
<td>1,698,371</td>
</tr>
</tbody>
</table>
13.) Suppose the researchers test the hypotheses

H₀: β₁ = 0  
Hₐ: β₁ < 0

The value of the t statistic for this test is

a. 2.61.  
b. 2.44.  
c. 0.081.  
d. -2.44.

14.) A 95% confidence interval for the slope β₁ in the simple linear regression model is (approximately)

a. 7,897,179 ± 3,023,782.  
b. 7,897,179 ± 6,047,564.  
c. -4,139,198 ± 1,698,371.  
d. -4,139,198 ± 3,396,742.

15.) Which of the following conclusions seems most justified?

a. There is no evidence of a relationship between the average number of putts per round and the 1993 winnings of PGA tour pros.  
b. There is distinct evidence (P-value less than 0.05) that there is a positive correlation between 1993 winnings and average number of putts per round.  
c. There is relatively strong evidence that PGA tour pros who averaged fewer putts per round had higher winnings in 1993.  
d. The presence of strongly influential observations in these data makes it impossible to draw any conclusions about the relationship between 1993 winnings and average number of putts per round.

16.) You have conducted a test of significance for the slope coefficient in a linear regression setting using level of significance α = 0.05. The P-value of your test result was 0.026. You should conclude that

a. there is a linear relationship between the two quantitative variables  
b. the two categorical variables are independent  
c. all of the categorical variables are independent  
d. the null hypothesis is true
Section 2: Workout Problems: For questions 17-19, be sure to show all work in order to receive partial credit.

17.) Researchers at the UCLA Research Center want to compare intensities of earthquakes at San Francisco Bay with earthquakes at the Los Angeles Basin. The data, assumed to be independent samples from the two areas, represent intensity on the Richter scale of tremors felt during several months in 1994.

Two-sample T for SF vs LA

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF</td>
<td>14</td>
<td>2.59</td>
<td>1.02</td>
<td>0.27</td>
</tr>
<tr>
<td>LA</td>
<td>16</td>
<td>2.13</td>
<td>1.09</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Difference = μ₁ - μ₂
Estimate for difference: 0.454
95% CI for difference: (-0.339, 1.248)
T-Test of difference = 0 (vs not =): T-Value = 1.18

a. If researchers wish to test whether mean intensities differ between the two regions, state the appropriate null and alternative hypothesis.

\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 \neq \mu_2 \]

b. What shape is required of the data to ensure robustness of the procedures?

Symmetry and the shape of distribution should be similar.

c. Use the two-sample t-statistic from the output to report a range for the P-value.

\[ t = 1.18 \sim t(df = 13) \]
\[ 1.079 < t = 1.18 < 1.35 \]
\[ 0.2 < P-value < 0.3 \]

d. Is there statistically significant evidence of a difference?

No, because P-value is greater than 0.05
18.) The slope and intercept of a least-squares regression line are calculated and yield \( \hat{y} = 3.4 + .17x \). Our sample size was \( n = 12 \), and we determine the standard error of the slope is \( SE_\beta = .012 \).

a. Give a 90% confidence interval for the true slope of the regression line \( \beta \).

\[
b \pm t_{0.95,10}SE_b = 0.17 \pm 1.812 \times 0.012 = [0.148, 0.192]
\]

b. An experimenter believes \( \beta > 0 \). Write down the appropriate hypotheses.

\[
H_0: \beta = 0 \\
H_a: \beta > 0
\]

c. Determine the value of the test statistic.

\[
t = \frac{b}{SE_b} = \frac{0.17}{0.012} = 14.17 \sim t(df = 10)
\]

d. Determine an interval which must contain the P-value associated with the test statistic in part c.

\[
t = 14.17 > 4.58, \quad P-value < 0.0005
\]

e. What is your conclusion at the .05 level of significance?

Since P-value is less than \( \alpha = 0.05 \), we reject the null hypothesis. That is, there is enough evidence to say that the true slope \( \beta \) is significantly greater than zero.
19.) A random sample of Pennsylvania residents was asked how much the recent increase in gasoline prices has impacted the amount they drive their vehicles. A summary of the data is conveyed in the following table:

<table>
<thead>
<tr>
<th>Impact on # of miles driven/week</th>
<th>Some Impact</th>
<th>No Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban/Suburban</td>
<td>12</td>
<td>39</td>
</tr>
<tr>
<td>Rural</td>
<td>21</td>
<td>28</td>
</tr>
</tbody>
</table>

**a.** State the appropriate hypotheses that test whether these categorical variables are related.

\[ H_0: \text{there is no relationship between location and whether impacted} \]

\[ H_a: \text{there are some relationship between location and whether impacted} \]

**b.** What is the value of your test statistic? What distribution does it follow under the null hypothesis?

\[ \chi^2 = \frac{(12 - 16.83)^2}{16.83} + \frac{(39 - 34.17)^2}{34.17} + \frac{(21 - 16.17)^2}{16.17} + \frac{(28 - 32.83)^2}{32.83} = 4.22 \sim \chi^2_1 \]

**c.** What is the P-value associated with your test statistic in part b?

Since 4.22 > 3.84, \( P-value < 0.05 \)

**d.** Using level of significance \( \alpha = 0.05 \), what do you conclude?

Since P-value is less than 0.05, we reject the null hypothesis.