Sovereign Debt and Default

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Sovereign Debt and Default

- Eaton and Gersovitz (1981) - Debt with Potential Repudiation: Theoretical and Empirical Analysis
- Bulow and Rogoff (1989) - Sovereign Debt: Is to Forgive to Forget?
- Atkeson (1991) - International Lending with Moral Hazard and Risk of Repudiation
- Arellano (2008) - Default Risk and Income Fluctuations in Emerging Economies
- Hatchondo and Martinez (2009) - Long Duration Bonds and Sovereign Defaults
- Chatterjee and Eyigungor (2012) - Maturity, Indebtedness, and Default Risk

Atkeson (1992)

- Constrained optimal pattern of capital flows between a lender and a borrower
- Two impediments to forming contracts
  - lenders cannot observe whether borrowers invest or consume borrowed funds (moral hazard)
  - borrower, as a sovereign nation, may choose to repudiate his debts
- In the optimal contract, the borrowing country experiences a capital outflow when the worst realizations of national output occur.

Model

- Two agents: infinitely-lived risk-averse borrower, and a sequence of short-lived, risk-neutral lenders
- Borrower can borrow b_t, invest I_t which yields stochastic output Y_{t+1}, with a debt repayment schedule $d_{t+1}(Y_{t+1})$
- Lenders have endowment M in each period they are alive
- An allocation is *feasible* if $c_t - b_t + I_t \leq Y_t - d_t(Y_t)$
- Higher investment makes higher output more likely; each output realization has positive probability for any level of investment (lenders cannot infer investment based on output)

Model

- Borrower preferences: $U_t^B = (1-\delta)E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_t)$
- Lender preferences: $U_t^L = -b_t + \delta E_t d_{t+1}(Y_{t+1})$
- An allocation cannot be supported by contracts in equilibrium unless it provides each agent with expected utility at least as great as his reservation utility in every round of contracting

  i.e. an allocation is *individually rational* if $U_t^B \geq U_{aut}^B$ and $U_t^L \geq 0$ for all $t \geq 0$

Model

- An allocation σ is *immune from the threat of repudiation* if for all $t \geq 0$, $Y_{t+1} \in Y$, the continuation allocation $\sigma | Y_{T+1}$ satisfies $U_t^B(\sigma | Y_{t+1}) \geq U_{aut}^B(Y_{t+1})$. (Note that this is different from *individual rationality* which requires that the allocation in expectation have higher utility than autarky)
- An allocation σ is *incentive compatible* if the borrower finds it optimal to carry out the consumption and investment plan specified in the allocation when he takes the lending and repayment plans specified as given: $U^B(\sigma) \geq U^B(\sigma')$ where $\sigma' = (\sigma_c', \sigma_I', \sigma_b, \sigma_d)$

Constrained Pareto Optimum

- An allocation σ is *constrained Pareto optimal* if it maximizes the borrower's payoff subject to the constraints of (1) feasibility, (2) individual rationality, (3) immunity from the threat of repudiation, and (4) incentive compatibility.
- If positive investment is specified in an equilibrium allocation, then full insurance is not incentive compatible: the borrower will invest nothing because his payoff does not depend upon the output from his investment opportunity and thus does not depend upon his level of investment.

Recursive Representation

- Write the problem in recursive representation:

$$
\begin{aligned}
V(Q) = \max_{c,I,b,d'} \quad & \{(1-\delta)u(c) + \delta EV(Q')\} \\
\text{s.t.} \quad & c + I - b \leq Q,\ b, -d' \leq M,\ c, I \geq 0 \\
& b \leq \delta E d'(Y') \\
& V(Q') \geq U^B_{aut}(Y')\ \forall Y' \\
& I \in \arg \max_{\tilde{I} \in [0, Q+b]} (1-\delta)u(Q + b - \tilde{I}) + \delta EV(Q')
\end{aligned}
$$

where $Q = Y - d$

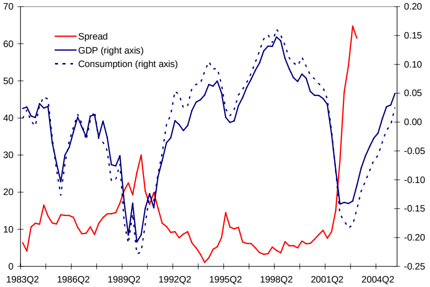
- The constraints are recursive analogues of feasibility, individual rationality, immunity from the threat of repudiation, and incentive compatibility

Characterization of Optimal Contract

- When the immunity from the threat of repudiation constraint binds for some $Y' \in Y$, then the borrower's value is equated to the autarky value for that realization.
- Then the new loan b' will be necessarily lower than $d'(Y')$, resulting in a capital outflow, and a fall in consumption and investment.
- As in Kehoe and Perri (2002), there is no "default" in equilibrium, although there are "debt crises"
- We will now study Arellano (2008) where countries default in equilibrium

Arellano (2008)

- Emerging economies have volatile business cycles and experience economic crises more frequently than developed economies.
- They also face volatile and highly countercyclical interest rates, usually attributed to countercyclical default risk.
- Consider Argentina's default in December 2001



Model

- Small open economy
- Risk neutral competitive foreign lenders
- Representative households have preferences $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$
- Household receives a stochastic stream of income y, which follows Markov process with transition function $f(y', y)$
- Government (benevolent planner) can default on debt, can be temporarily excluded from intertemporal trading and incur direct output costs
- The price of each bond $q(B', y)$ reflects the likelihood of default, such that the creditors break even in expected value

Model

- If government repays its debts,

$$c = y + B - q(B', y)B'$$

- If government defaults,

$$c = y^{def}$$

where $y^{def} = h(y) \leq y$ and $h(y)$ is increasing

- In equilibrium, bond prices must satisfy

$$q(B', y) = \frac{1 - \delta(B', y)}{1 + r}$$

where the probability of default δ is endogenous to the model, and r is the risk free rate.

Option to Default

- Given the option to default,

$$v^o(B, y) = \max\left\{v^c(B, y), v^d(y)\right\}$$

where v^c is the value of not defaulting and v^d is the value of defaulting

Value of Default

- When the government defaults, the economy is in temporary financial autarky,

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^o(0, y') + (1 - \theta) v^d(y')\right] f(y', y) dy'$$

where θ is the probability of regaining access to international credit markets

Value of Not Defaulting

- The value conditional on not defaulting is given by

$$v^c(B, y) = \max_{B'} \left\{ u(y - q(B', y)B' + B) + \beta \int_{y'} v^o(B', y')f(y', y)dy' \right\}$$

Default Policy

- Repayment set

$$A(B) = \left\{ y \in Y \mid v^c(B, y) \geq v^d(y) \right\}$$

- Default set

$$D(B) = \left\{ y \in Y \mid v^c(B, y) < v^d(y) \right\}$$

Recursive Equilibrium

Definition

The recursive equilibrium is a set of policy functions for consumption $c(B, y)$, government asset holdings $B'(B, y)$, repayment sets $A(B)$, and default sets $D(B)$, and price functions $q(B', y)$ such that

1. Taking as given the government policies, households' consumption $c(s)$ satisfies the resource constraint.
2. Taking as given the bond price function $q(B', y)$, the government's policy functions $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$ satisfy the government optimization problem.
3. Bonds prices $q(B', y)$ reflect default probabilities and are consistent with creditors' expected zero profits.

Characterization

- In equilibrium

$$\delta(B', y) = \int_{D(B')} f(y', y) dy'$$

- Default sets are shrinking in assets (Proposition 1)

$$\forall B^1 \leq B^2, \;\; D(B^2) \subseteq D(B^1)$$

intuition: value of not defaulting is increasing in B while that of default is independent of B

Case of i.i.d. Shocks

- Assume endowment shocks are i.i.d., $h(y) = y$ (no output loss), and $\theta = 0$ (permanent exclusion after default)
- Default incentives are stronger the lower the endowment (Proposition 3)

$$\forall y_1 \leq y_2, \quad y_2 \in D(B) \Rightarrow y_1 \in D(B)$$

  intuition: net repayment is more costly when income is low, due to concavity, making default more likely
- Note that this is in contrast with enforcement constraint models with complete markets. In those models, default incentives are higher in times of good shocks

Quantitative Analysis

Table 1. Business Cycle Statistics for Argentina

	Default episode			
	x: Q1–2002	$std(x)$	$corr(x, y)$	$corr(x, r^c)$
Interest rates spread	28.60	5.58	-0.88	
Trade balance	9.90	1.75	-0.64	0.70
Consumption	-16.01	8.59	0.98	-0.89
Output	-14.21	7.78		-0.88

- Consumption and Output (linear trend filtered, 1980.1-2001.4) are negatively correlated with interest rate spreads (EMBI index, 1983.3 - 2001.4) - stronger in default episode
- Consumption more volatile than output
- Trade balance (1993.1-2001.4) is countercyclical
- Interest rate spreads are high and volatile (mean spread 10.25%)

Quantitative Analysis

Table 2. Business Cycle Statistics for Other Defaulters

Ecuador	Default episode			
	x: Q3–1999	$std(x)$	$corr(x,y)$	$corr(x,r^c)$
Interest rate spread	47.58	5.44	-0.63	
Trade balance	10.96	4.47	-0.39	0.05
Consumption	-7.14	2.78	0.92	-0.53
Output	-6.46	2.53		-0.63

Russia	Default episode			
	x: Q4–1999	$std(x)$	$corr(x,y)$	$corr(x,r^c)$
Interest rate spread	30.43	17.5	-0.70	
Trade balance	12.4	5.4	-0.17	0.86
Consumption	-17.2	7.08	0.79	-0.80
Output	-12.6	11.8		-0.70

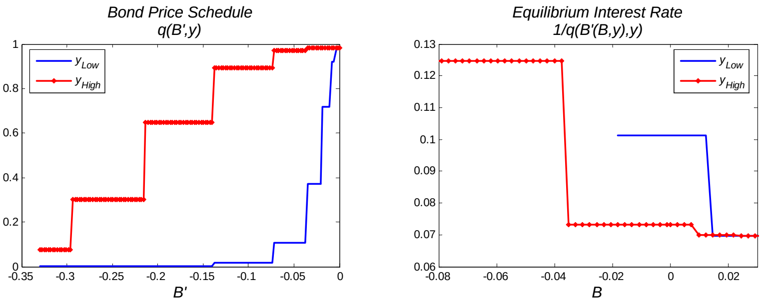
High volatility of interest rates and countercylicality of interest rate and trade balance seem to be common in emerging economies

Calibration

Table 3. Parameters

Risk free interest rate	$r = 1.7\%$	U.S. 5 year bond quarterly yield
Risk aversion	$\sigma = 2$	
Stochastic structure	$\rho = 0.945$, $\eta = 0.025$	Argentina's GDP
Calibration		
	Values	Target Statistics
Discount factor	$\beta = 0.953$	3% default probability
Probability of re-entry	$\theta = 0.282$	Trade balance volatility 1.75
Output costs	$\widehat{y} = 0.969E(y)$	5.53% debt service to GDP

Simulated Results



- Income shocks are 5 percent above and below trend
- Interest rate charged at every loan size is lower in booms
- If incoming debt is below 2 percent of output, then the borrower chooses relatively higher debt in recession. If above, the borrower defaults in recessions and borrows risky in booms.

Capital Outflows in Recession

- Larger capital outflows $(y - c)$ can occur in recessions because interest rates are high and borrowing is constrained
- This result is similar to Atkeson (1991)

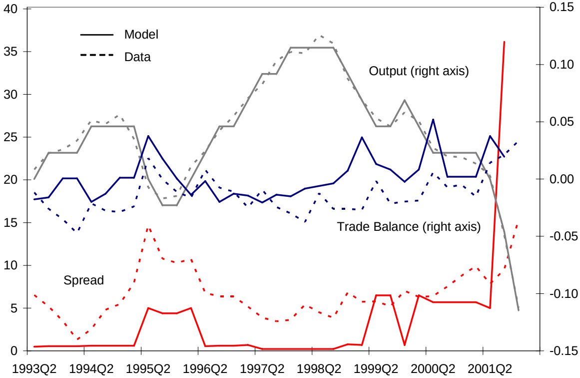
Simulated Results

Table 4.
Business Cycle Statistics in the Benchmark Model

	Default Episodes	$std(x)$	$corr(x,y)$	$corr(x,r^c)$
Interest rates spread	24.32	6.36	-0.29	
Trade balance	-0.01	1.50	-0.25	0.43
Consumption	-9.47	6.38	0.97	-0.36
Output	-9.60	5.81		-0.29
Other Statistics				
Mean Debt (% output)	5.95	Mean Spread		3.58
Default Probability	3.00	Output Deviation in Default		-8.13

- Higher volatility of consumption relative to output, countercylical interest rates, and countercylical trade balance
- High volatility of interest rate spreads - however the mean spread (3.68%) is well below that of Argentina (10.25%)
- Significant collapses in consumption and output, and high spreads - however, the model misses the trade balance reversal

Simulated Results



- Model predicts default in Argentina, feeding in Argentina's GDP starting in 1993
- Overall, matches spread dynamics, but less so with trade balance

Hatchondo and Martinez (2009)

- Extends the baseline sovereign default framework with long duration bonds in a tractable way
- Model generates spreads that are higher and more volatile, closer to the data

Model

- Small open economy
- Risk neutral competitive foreign lenders
- Representative households have preferences $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$
- Household receives a stochastic stream of income y, which follows Markov process with transition function $f(y', y)$
- Government (benevolent planner) can default on debt which is accompanied by direct output costs
- The price of each bond $q(B', y)$ reflects the likelihood of future default, such that the creditors break even in expected value

Long-Duration Bonds

- A bond issued in period t promises an infinite stream of coupons, which decreases at δ
- Law of motion of debt

$$B' = B(1-\delta)(1-d) - i$$

where $d = 1$ in default and i is the current issuance

- Face value of debt is given by

$$\frac{B'}{\delta + r}$$

Model

- If government honors its debt commitments,

$$c = y + B - q(B', y)\,(B' - B(1 - \delta))$$

- If government defaults,

$$c = y - \phi(y) - q(B', y)B'$$

Option to Default

- Given the option to default,

$$v^o(B,y) = \max\left\{v^c(B,y), v^d(y)\right\} \tag{1}$$

- The value of not defaulting is given by

$$\begin{aligned} v^c(B,y) = \quad & \max_{B' \leq 0} \quad \left\{u(c) + \beta \int_{y'} v^o(B',y')f(y',y)dy'\right\} \quad (2) \\ & \text{s.t.} \qquad c = y + B - q(B',y)\left(B' - B(1-\delta)\right) \end{aligned}$$

- The value of defaulting is given by

$$\begin{aligned} v^d(y) = \quad & \max_{B' \leq 0} \quad u(c) + \beta \int_{y'} v^o(B',y')f(y',y)dy' \quad (3) \\ & \text{s.t.} \qquad c = y - \phi(y) - q(B',y)B' \end{aligned}$$

Bond price schedule

- The bond price that satisfies the lender's zero profit condition:

$$\begin{aligned} q(B', y) &= \frac{1}{1+r}\int (1 - d(B', y'))\, dF(y'|y) \\ &+ \frac{1}{1+r}\int [(1 - d(B', y'))(1-\delta)\, q(B^*(B', y'), y')]\, dF(y'|y) \end{aligned} \tag{4}$$

  where $d(B', y')$ and $B^*(B', y')$ denote the default policy and asset policy
- first term: expected value of next-period coupon payment
- second term: expected value of all future coupon payments, summarized by the expected price of the bond next period

Markov Perfect Equilibrium

Definition

A Markov Perfect Equilibrium is a set of policy functions for default $d(B, y)$, borrowing $B'(B, y)$, and price function $q(B', y)$ such that

1. Taking as given the bond price function $q(B', y)$, the policy functions $B'(s)$ and $d(B, y)$ solve the government optimization problem in (1)-(3)
2. Bonds prices $q(B', y)$ reflect government policies and are consistent with creditors' expected zero profits implicit in (4).

Parameters

Parameter	Value	Target
risk aversion σ	2	
interest rate r	0.01	
endowment process ρ, σ	0.9, 0.027	Argentina output (1993.4-2001.3)
output loss λ	0.1,0.2,0.5	$\phi(y) = \lambda y$
discount factor β	0.95	
duration δ	1, 0.045	average duration of Argentine bonds: 4 years

Business Cycle Statistics

- With long term debt, spreads are higher and more volatile, closer to the data.

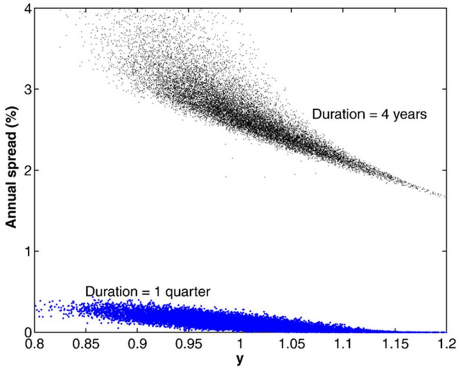
	Data	$\lambda=10\%$		$\lambda=20\%$		$\lambda=50\%$	
		$\delta=1$	$\delta=0.045$	$\delta=1$	$\delta=0.045$	$\delta=1$	$\delta=0.045$
Average duration	4.13	0.25	4.07	0.25	4.08	0.25	4.12
$E(R_s)$	7.44	0.12	3.01	0.11	2.93	0.12	2.73
$\sigma(R_s)$	2.51	0.03	0.27	0.04	0.29	0.06	0.33
$\sigma(y)$	3.17	3.12	3.07	3.05	3.06	3.15	3.07
$\sigma(c)$	2.98	3.21	3.13	3.27	3.23	3.66	3.45
$\sigma(TB/Y)$	1.35	0.20	0.12	0.38	0.26	0.85	0.56
$\rho(c, y)$	0.97	1.00	1.00	0.99	1.00	0.98	0.99
$\rho(TB/Y, y)$	−0.69	−0.46	−0.58	−0.48	−0.60	−0.50	−0.64
$\rho(R_s, y)$	−0.65	−0.93	−0.86	−0.86	−0.86	−0.77	−0.86
$\rho(R_s, TB/Y)$	0.56	0.76	0.83	0.86	0.85	0.93	0.88
Debt/output		0.09	0.10	0.18	0.21	0.44	0.51
Defaults per 100 years		0.12	3.02	0.11	2.92	0.12	2.72

The second column is computed using data from Argentina from 1993 to 2001. Other columns report the mean of the value of each moment in 500 simulation samples.

where $r^* = \dfrac{1}{q(B', y)} - \delta$ and $R_s \equiv \left(\dfrac{1+r^*}{1+r}\right)^4 - 1$

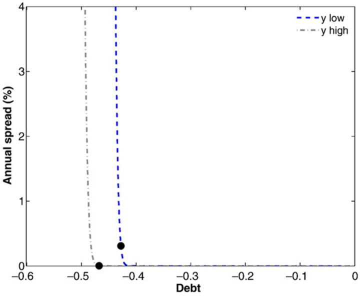
Equilibrium spread levels

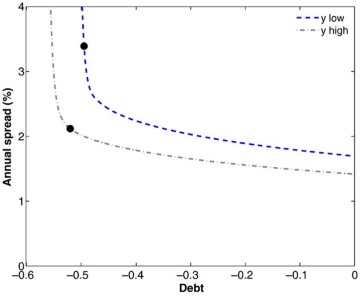
- With one-period bonds, low debt levels command close to a zero spread, since the probability of default next period is small
- With long-term debt, even low debt levels are associated with a significant probability of default in future periods



Bond price schedule

- With long-term debt, the bond price schedule is more sensitive to current endowments. (low shock today implies higher default probabilities in future periods, not just next period)
- Also, since per-period issuance levels are lower, the government may be more willing to choose debt levels that are on the steep section of the schedule (dots illustrate the optimal decision of government with average level of debt)





Chatterjee and Eyigungor (2012)

- Sovereign default model with long-term debt
- Theoretical contribution: provides modification to ensure existence and convergence
- Quantitative contribution: provides solution method to convergence issues

Model

- Small open economy
- Risk neutral competitive foreign lenders
- Representative households have preferences $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$
- Household receives a stochastic stream of income $x = y + m$
  - y follows Markov process with transition function $f(y', y)$
  - m is a transitory iid shock with mean 0.
- Government (benevolent planner) can default on debt, leading to a temporary exclusion from capital markets and direct output costs
- The price of each bond $q(B', y)$ reflects the likelihood of future default, such that the creditors break even in expected value

Long-Duration Bonds

- Long-term debt contracts mature with probability δ
- If the bond does not mature it pays a coupon of z
- Law of motion of debt

$$B' = B(1-\delta)(1-d) - i$$

where $d = 1$ in default and i is the current issuance

Model

- If government honors its debt commitments,

$$c = y + m + [\delta + (1 - \delta)\, z]\, B - q(B', y)\, (B' - B(1 - \delta))$$

- If government defaults,

$$c = y + m - \phi(y)$$

Option to Default

- Given the option to default,

$$v^o(B, y, m) = \max\left\{v^c(B, y, m), v^d(y, -\bar{m})\right\} \tag{5}$$

- The value of not defaulting is given by

$$\begin{aligned} v^c(B, y, m) = \quad & \max_{B' \leq 0} \quad \left\{u(c) + \beta E_{(y',m')|y} v^o(B', y', m')\right\} \\ & \text{s.t.} \qquad c = y + m + [\delta + (1-\delta)\, z]\, B \\ & \qquad\qquad -q(B', y)\,(B' - B(1-\delta)) \end{aligned}$$

- The value of defaulting is given by

$$\begin{aligned} v^d(y, m) = \quad & \max_{B' \leq 0} \quad u(c) + \beta E_{(y',m')|y}\left[\theta v^o(0, y', m') + (1-\theta) v^d(y', m')\right] \\ & \text{s.t.} \qquad c = y + m - \phi(y) \end{aligned}$$

Bond price schedule

- The bond price that satisfies the lender's zero profit condition:

$$\begin{aligned} q(B', y) &= \frac{\delta}{1+r} E_{(y',m')|y} \left[(1 - d(B', y', m')) \right] + \\ &\quad \frac{1-\delta}{1+r} E_{(y',m')|y} \left[(1 - d(B', y', m'))\left(z + q(B^*(B', y', m'), y')\right) \right] \end{aligned} \tag{6}$$

  where $d(B, y, m)$ and $B^*(B, y, m)$ denote the default policy and asset policy
- first term: expected value of next-period maturing debt
- second term: expected value of coupon payments and expected price of the bond next period

Characterizations

1. $d(B, y, m)$ is decreasing in B
2. If $q(B', y)$ is increasing in B', then $B^*(B, y, m)$ is increasing in B
3. There exists an equilibrium price function $q^*(B', y)$ that is increasing in B'
4. $B^*(B, y, m)$ is increasing in m and $d(B, y, m)$ is decreasing in m

Proof of proposition 3 requires the presence of the random variable m with a continuous CDF.

Functional Forms

- Endowments

$$\begin{aligned} \log(y_t) &= \rho \log(y_{t-1}) + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \sigma_\epsilon^2) \\ m_t &\in [-\bar{m}, \bar{m}], \text{ truncated } N(0, \sigma_m^2) \end{aligned}$$

- Utility function

$$u(c) = c^{1-\sigma}/(1-\sigma)$$

- Output loss

$$\phi(y) = \max\{0, d_0 y + d_1 y^2\}$$

  - $d_0 > 0,\ d_1 = 0$: loss proportional to output
  - $d_0 < 0,\ d_1 > 0$: similar to Arellano (2008)

Parameters

Parameter	Value	Target
risk aversion σ	2	
interest rate r	0.01	
y process ρ, σ_ϵ	0.949, 0.027	Argentina output (1980.1-2001.4)
m process $\bar{m}, \sigma_m$	0.006,0.003	
output loss d_0, d_1	-0.189, 0.246	mean and variance of spread (1993.1-2001.4)
discount factor β	0.954	70% of Argentina external debt/output (1993.1-2001.4)
duration δ	0.05	average bonds duration: 5 years
credit access θ	0.0385	average exclusion: 6.5 years
coupon z	0.03	annual coupon 12 percent

Business Cycle Statistics

- Targeted statistics

	Data	Baseline	Arellano
avg. spread	0.0815	0.0815	0.0358
st dev of spread	0.0443	0.0443	0.0636
debt-output	1.00	1.00	0.06

Business Cycle Statistics

Variable	Data	Baseline	1-period debt	Arellano
$\sigma(c)/\sigma(y)$	1.09	1.11	1.59	1.10
$\sigma(NX/y)/\sigma(y)$	0.17	0.20	1.06	0.26
$corr(c, y)$	0.98	0.99	0.73	0.97
$corr(NX/y, y)$	-0.88	-0.44	-0.16	-0.25
$corr(r^* - r, y)$	-0.79	-0.65	-0.55	-0.29
debt service	0.053	0.055	0.699	0.056
default freq	0.125	0.068	0.073	0.030

- The interest rate is defined implicitly by

$$q(B', y) = \underbrace{\left[\delta + (1-\delta)\left(z + q(B', y)\right)\right] / \left[1 + r^*(B', y)\right]}_{\text{present discounted value of promised future payments}}$$

Other Quantitative Properties

- Default frequency increases with maturity because sovereign is more willing to borrow into high-risk regions
  - with short-term debt, increasing price applies B'
  - with long-term debt, increasing price applies to $B' - (1 - \delta)B$
- Welfare is decreasing in maturity
  - effect of higher borrowing costs dominates effect of lower volatility of disposable income
- In presence of rollover crises, long-term debt is superior
  - with long-term debt, less debt needs to be rolled over each period