Lecture 9

• Income disparity among countries
• Endogenous growth: a model of human capital accumulation
We’ve said that the Solow growth model was a good model to explain growth as it was able to replicate the regularities we see in real-world data …
1. There is sustained growth over time:
2. There is a positive correlation between the rate of investment and output per worker across countries:
• In the end, we said that increases in savings were not sufficient to justify the continuous growth over time.
• We concluded that **total factor productivity** (technological progress) was the one that allowed us to grow over time!
• **Growth accounting** allowed us to do this.
• Now let’s talk about convergence.
• Is there a tendency for poor countries to catch up with the rich? The Solow growth model says “yes”!
• As always, we need some assumptions:
  1. We have two identical countries (same TFP, labor force growth rate, and savings rate).
  2. The “rich” country initially has a higher level of capital per worker relative to the “poor” country. (Consequently, it also has a higher output per worker.)
Yet, the model predicts that the poor country will catch up!

In terms of capital per worker:

\[
\frac{szf(k)}{1 + n} + \frac{(1 - d)k}{1 + n}
\]
Yet, the model predicts that the poor country will catch up!

In terms of output per worker:

\[ y = \text{Income Per Worker} \]

\[ y_1^* \]

Time

Rich

Poor
Yet, the model predicts that the poor country will catch up!

In terms of the growth rate of GDP:
Well, the bad news is that there is no convergence among all countries:

(We would like to see a negative correlation so that poor countries are growing faster than rich countries, thus catching up!)
While there is some sort of convergence among rich countries ...
... there is clearly **no convergence** among poor countries!
Why could this happen?

The most obvious reason is to claim that countries do not have access to the same technology, so our assumptions do not hold.

But, (again) why could this happen?

1. Think of government intervention. For example, laws that give unions bargaining power and thus could make firms unable (or even fearful!) of introducing new technologies which are not “approved” by unions.

2. Think again of government intervention. For example, barriers to international trade to “protect” the domestic industry … which also won’t allow the domestic industry to get technologies from abroad.
If this is the case, then TFP for different countries is not the same, so we have multiple steady states across a sample of countries:
So, are poor countries doomed?
Well, the answer is “not exactly.” There are some things that could be done, but it depends on the government to get things right. The government could …

1. Promote **greater competition among firms**: Firms must innovate to remain competitive

2. Promote **free trade**.

3. Do **selective privatization**: Eliminate government ownership when there is no clear benefit of having a public enterprise vs. a private one. (But be really selective!)
The Solow Growth Model is an exogenous growth model.

Long run growth can be accounted for in the Solow Growth Model through the growth of TFP.

But where is this TFP growth coming from? It is exogenous.

Perhaps models of endogenous growth can explain…
Endogenous Growth: A Model of Human Capital Accumulation

In this model, the growth rate of per capita income is determined by the efficiency with which human capital is accumulated, and the fraction of available time devoted to human capital accumulation.
Let’s first look at the representative consumer
Consumer’s Budget Constraint

Consumption is equal to total wage income.

\[ C = wH^S u \]

\( u \): units of time used working

\( H \): human capital
Technology for accumulating human capital

Future human capital depends on current human capital and time devoted to training and education.

\[ H^s' = b(1 - u)H^s \]

u: units of time used working
b: parameter that captures efficiency of human capital accumulating technology (quality of education)
Now let’s consider the representative firm.
Production Function

Firms use labor efficiency units (uH) to produce output:

\[ Y = zuH^d \]

Y: current output
z: marginal product of efficiency units of labor
Firm’s Profits

\[
\pi = zuH^d - wuH^d \\
= (z - w)uH^d
\]

If \(z<w\), firm hires no labor

If \(z>w\), firms desires to hire infinite amount of labor

Can you guess what the equilibrium wage is going to be?
Determination of the Equilibrium Real Wage
Now let’s characterize the competitive equilibrium
Equilibrium Characterization

Consumption is given by:

\[ C = wuH^s \]

\[ C = zuH \]

Human capital evolves in equilibrium according to:

\[ H^s' = b(1 - u)H^s \]

\[ H' = b(1 - u)H \]
Human Capital Accumulation

- The colored line shows future human capital as a function of current human capital.
- $H' > H$ for any $H$, so human capital increases forever.
Growth rate of Human Capital

Equilibrium growth rate of human capital:

\[ \frac{H'}{H} - 1 = b(1 - u) - 1 \]

Note: A decrease in \( u \), i.e. less time working, implies lower initial consumption, but higher human capital growth and thus higher consumption growth.
Growth rate of Consumption

Equilibrium growth rate of consumption:

\[
\frac{C'}{C} - 1 = \frac{zuH'}{zuH} - 1 = \frac{H'}{H} - 1 = b(1 - u) - 1
\]

Human capital, consumption, and output all grow at the same rate in equilibrium.
Growth rate of Consumption

Equilibrium growth rate of consumption:

\[ \frac{C'}{C} - 1 = \frac{zuH'}{zuH} - 1 = \]

\[ \frac{H'}{H} - 1 = b(1 - u) - 1 \]

**Discussion**: So what happens when \( u \) decreases because of, for example, education subsidies?
Effect of a Decrease in $u$ on the Consumption Path
Discussion: what is the implication of this model for convergence? Two otherwise identical countries that differ only in additional incomes...
No Convergence in the Endogenous Growth Model

- Two otherwise identical countries that differ only according to their initial incomes never converge
• How about increasing $b$? This would also lead to higher growth rates...