Lecture 8

- Solow growth model
- Growth accounting

Solow Growth Model

- This is a key model which is the basis for the modern theory of economic growth.
- A key prediction is that technological progress is necessary for sustained increases in standards of living.

Population growth

- In the Solow growth model, population is assumed to grow at a constant rate *n*.

$$N' = (1 + n)N$$

- Discussion: How is this different from the Malthusian view of population growth? What’s the significance?

Representative Consumer

- Consumers are assumed to save a constant fraction *s* of their income, consuming the rest.

$$C = (1 - s)Y$$

- The consumer has one unit of time available, inelastically supplying one unit of time as labor

Representative Firm

The firm produces using as inputs capital and labor:

$$Y = zF(K, N)$$

Representative Firm

Constant returns to scale implies:

$$Y = zF(K, N)$$

$$\Rightarrow \quad \frac{Y}{N} = \frac{zF(K,N)}{N}$$

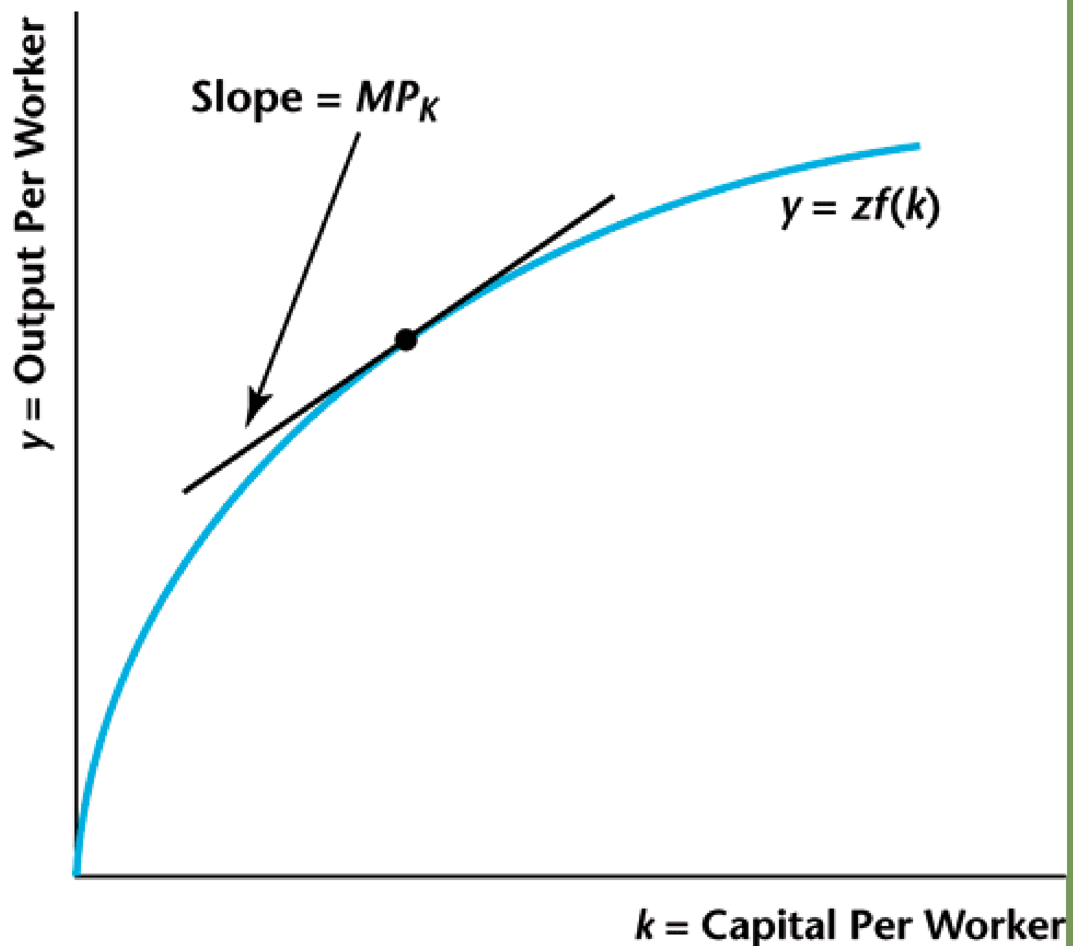
$$\Rightarrow \quad \frac{Y}{N} = zF\left(\frac{K}{N}, 1\right)$$

→ Output per worker depends on capital per worker!

The Per-Worker Production Function

$$\frac{Y}{N} = zF\left(\frac{K}{N}, 1\right)$$

$$y = zf(k)$$



Evolution of the capital stock

Future capital equals the capital remaining after depreciation, plus current investment.

$$K' = (1 - d)K + I$$

Income-Expenditure Identity

The income expenditure identity holds as an equilibrium condition.

$$Y = C + I$$

$$\Rightarrow \quad Y = (1-s)Y + K' \ ? \ (1-d)K$$

$$\Rightarrow \quad K' = sY + (1-d)K$$

Future capital equals **total savings** plus what **remains of current *K***.

Equilibrium capital

Substitute for output from the production function.

$$K' = sY + (1 - d)K$$

$$\Rightarrow \quad K' = szF(K, N) + (1 - d)K$$

Capital per worker

Rewrite in per-worker form.

$$K' = szF(K, N) + (1 - d)K$$

$$\Rightarrow \quad \frac{K'}{N} = szF\left(\frac{K}{N}, 1\right) + (1 - d)\frac{K}{N}$$

$$\Rightarrow \quad \frac{K'}{N'}\frac{N'}{N} = szF\left(\frac{K}{N}, 1\right) + (1 - d)\frac{K}{N}$$

$$\Rightarrow \quad k'(1 + n) = szf(k) + (1 - d)k$$

Capital per worker (cont'd)

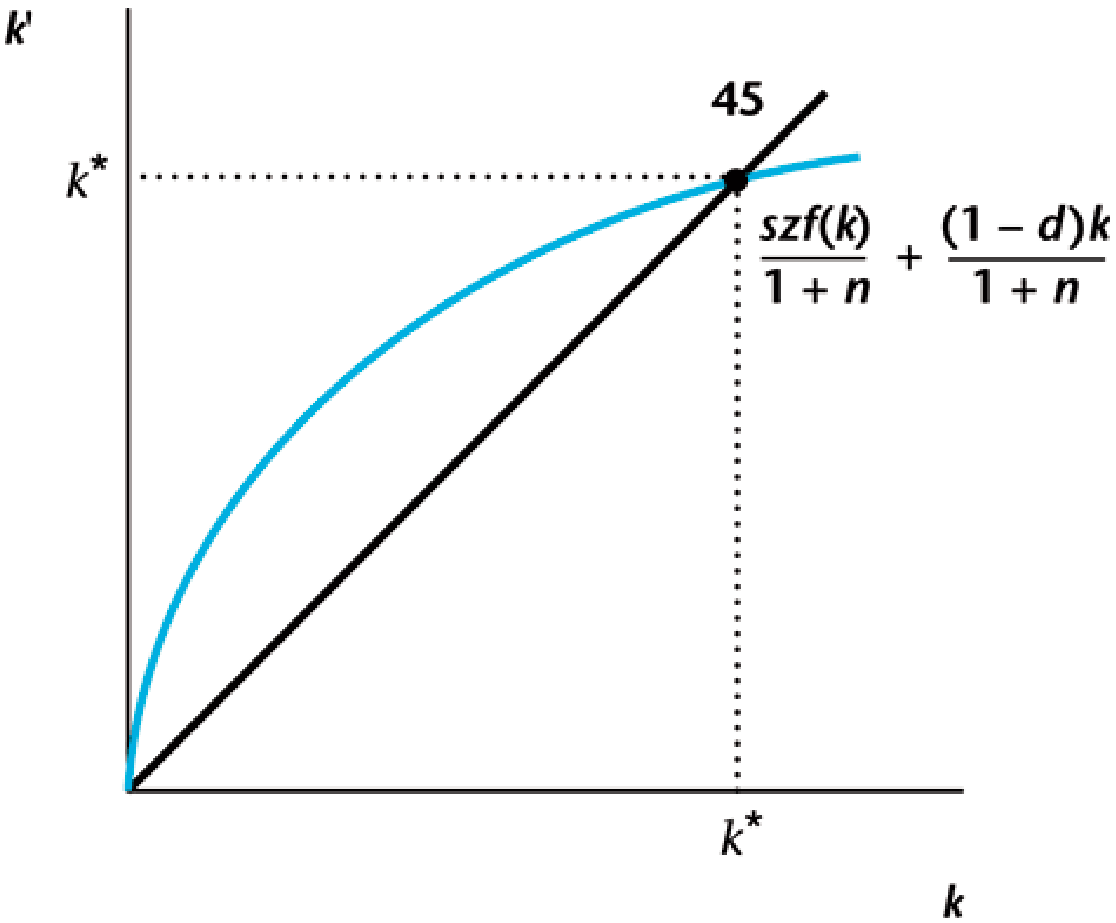
Re-arrange, to get:

$$k'(1+n) = szf(k) + (1-d)k$$

$$\Rightarrow \quad k' = \frac{szf(k)}{1+n} + \frac{(1-d)k}{1+n}$$

We can now use this condition to determine the *steady state* of the model, where k'=k=k*

Steady State Capital per Worker



- k* is the steady state population, determined by the intersection of the curve and the 45 degree line.

Now here's an interesting question: What is the growth rate of k^*?

The answer: zero!

Why? Since it's a **steady state,** it won't move from there!

Another question: What is the growth rate of y^*?

The answer: zero!

Why? Since $k = k^*$ in the long run, output per worker is constant at $y^* = zf(k^*)$.

So … there's no growth in here? Are we forgetting something?

There is growth in this economy!

In the long run, when $k = k^*$, all real aggregate quantities grow at a rate n. Why?

1. The aggregate quantity of capital is $K = k^*N$. Since k^* is constant and N grows at a rate n, K should grow at a rate n.

2. Aggregate real output is $Y = y^*N = zf(k^*)N$, hence Y also grows at a rate n.

3. Consumption and investment follow the same logic once we note that

$$I = sY = szf(k^*)N,$$

$$C = (1 - s)Y = (1 - s)zf(k^*)N.$$

In this way, the Solow growth model is an exogenous growth model!

Now let’s explore how the **steady state** changes with changes in the **savings rate**

Steady state analysis

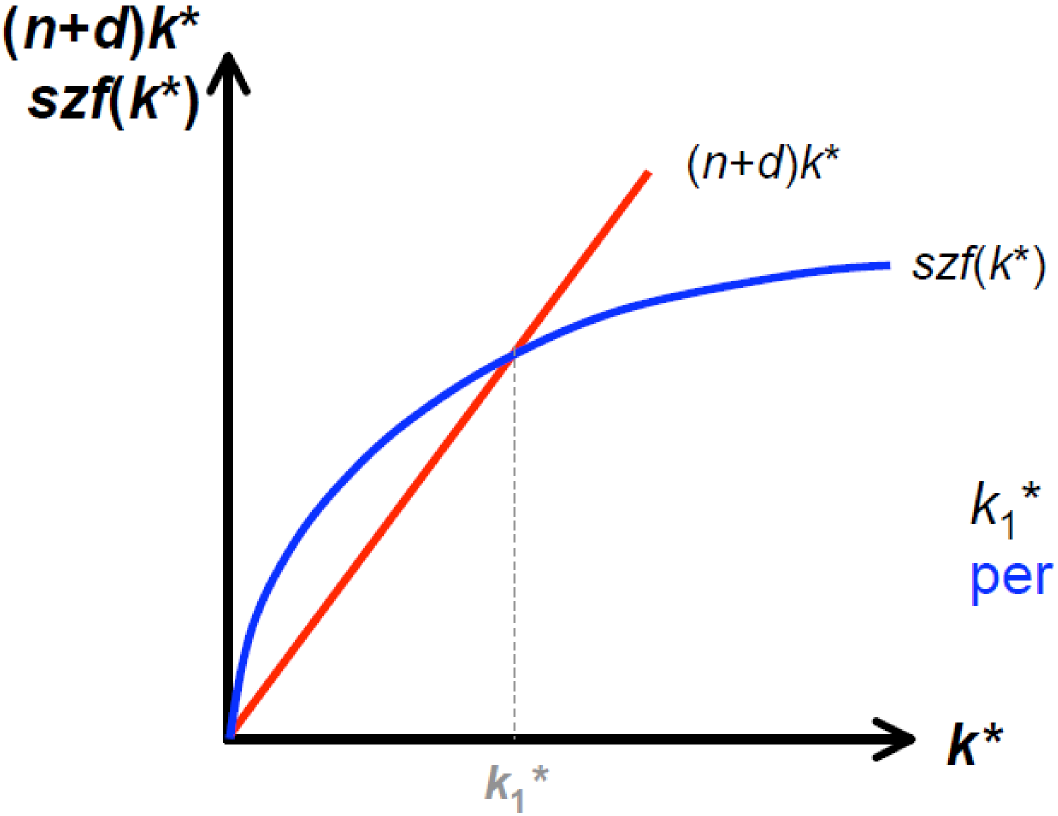
Equation determining the steady state quantity of capital per worker, k^*:

$$k'(1+n) = szf(k) + (1-d)k$$

set k’ = k = k*

$$\Rightarrow \quad szf(k^*) = (n+d)k^*$$

Graphically, to determine k^* we match the left and right hand sides of equation [12]:



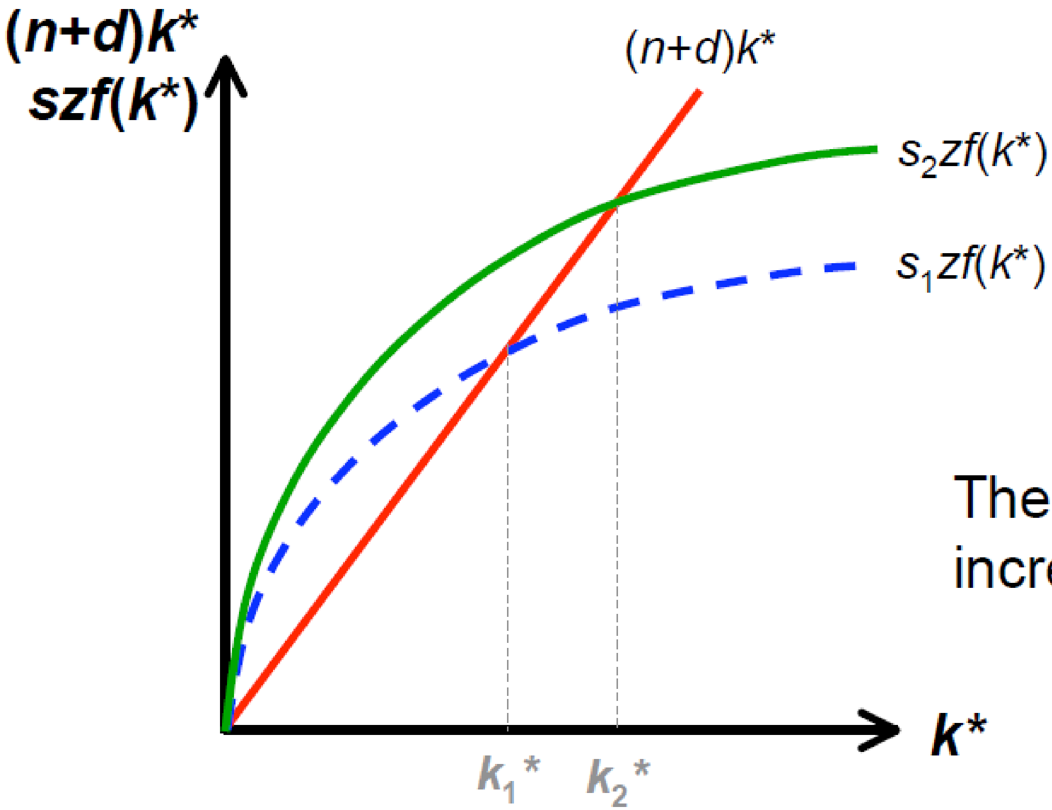
k_1^* is the steady state capital per worker!

We know what happens in the steady state. But now, let's see what happens when we **change the savings rate, *s*.**

Suppose that at some time t_0 the savings rate increases from s_1 to s_2. (This could be due to a change in preferences. What if it was due to a subsidy to savings offered by the government?)

Now that we know how the model operates, it's easy to see (graphically) how this looks.

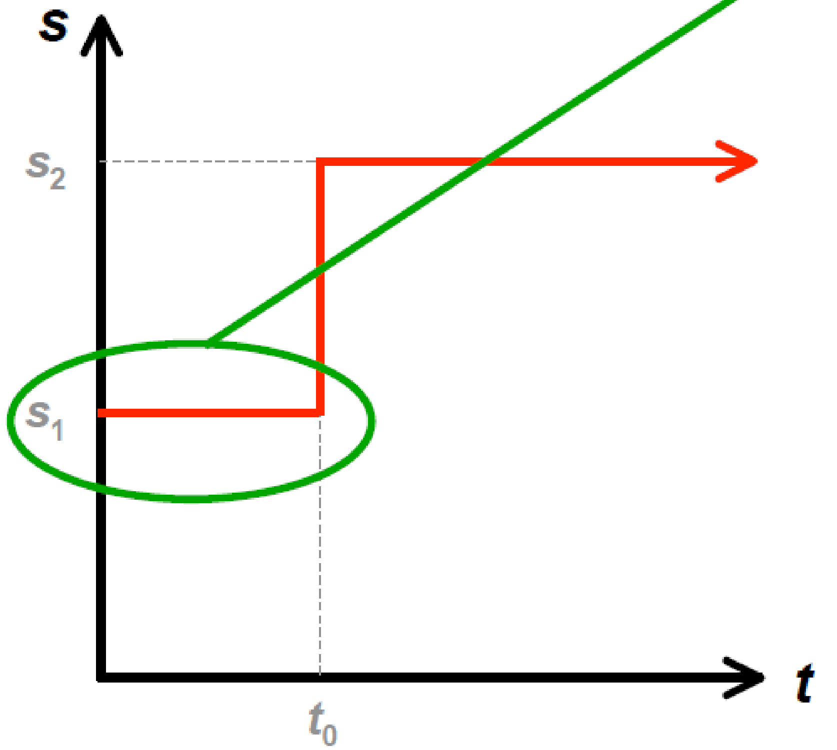
A change in the savings rate looks like this; for $s_2 > s_1$



The steady state capital level increases!

Let’s study the transition the to new steady state as the savings rate increases

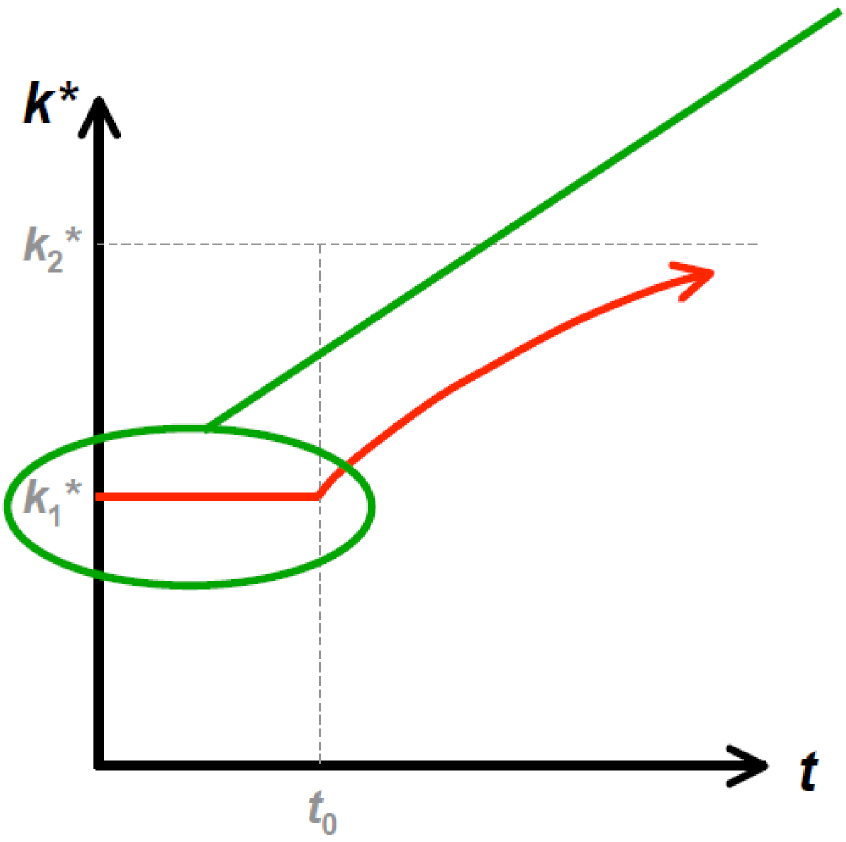
In time: The savings rate



The savings rate starts at a level s_1 …

… then jumps to the new level of savings s_2.

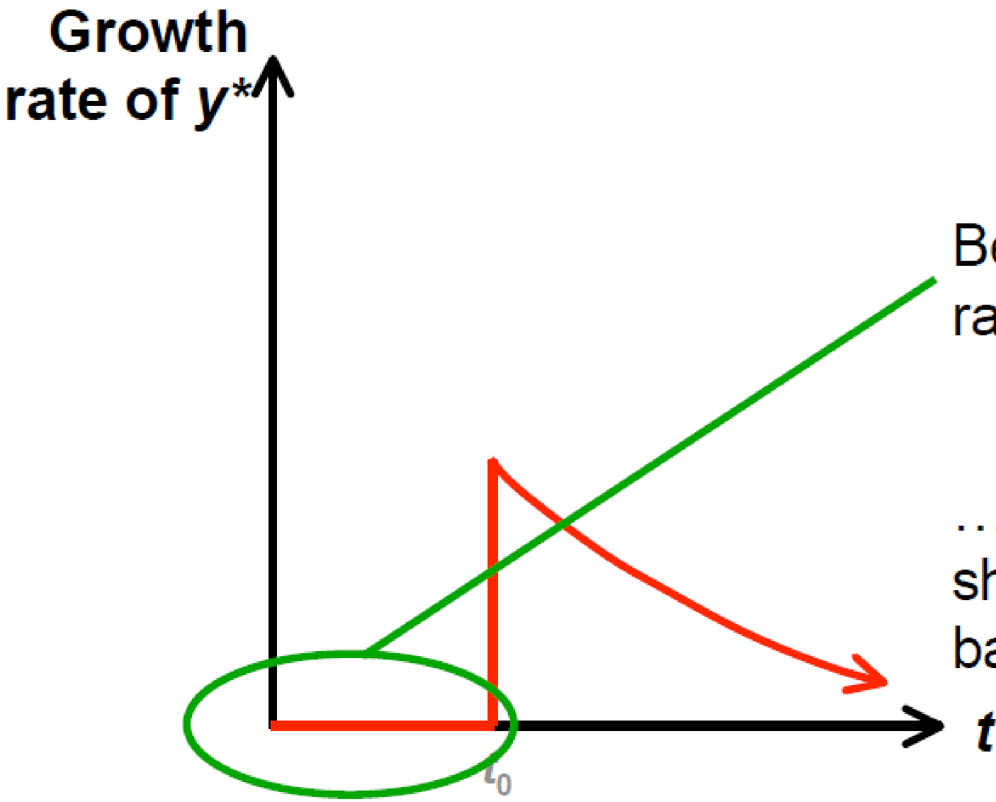
In time: Capital per worker



The capital per worker starts at a level k_1^* …

… then steadily increases towards its new level k_2^*.

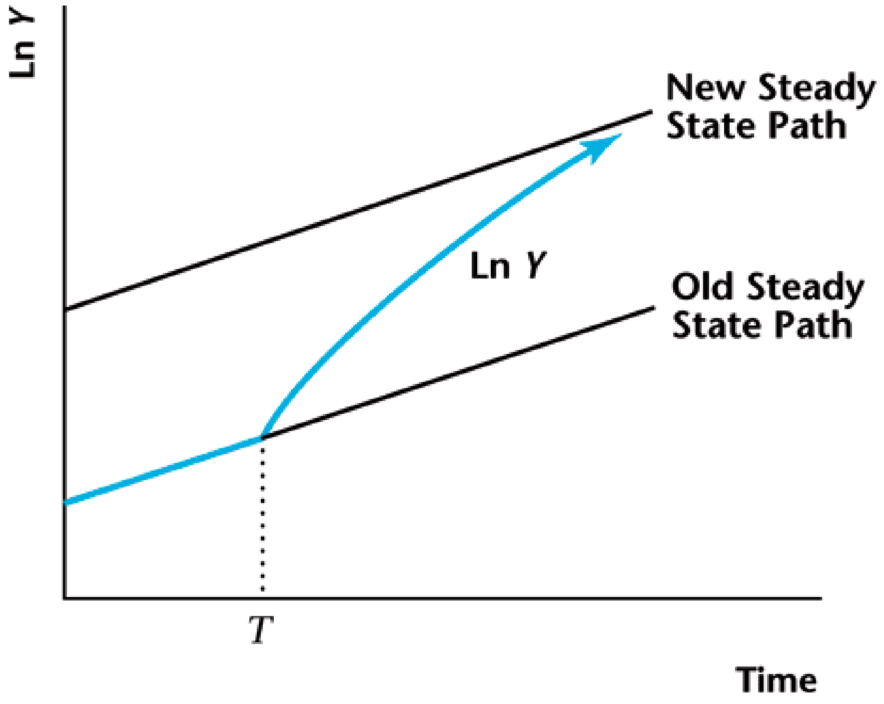
In time: Growth rate of *y**



Before the shock, the growth rate of *y** is zero …

… it jumps at the time of the shock, yet it steadily decreases back to zero again.

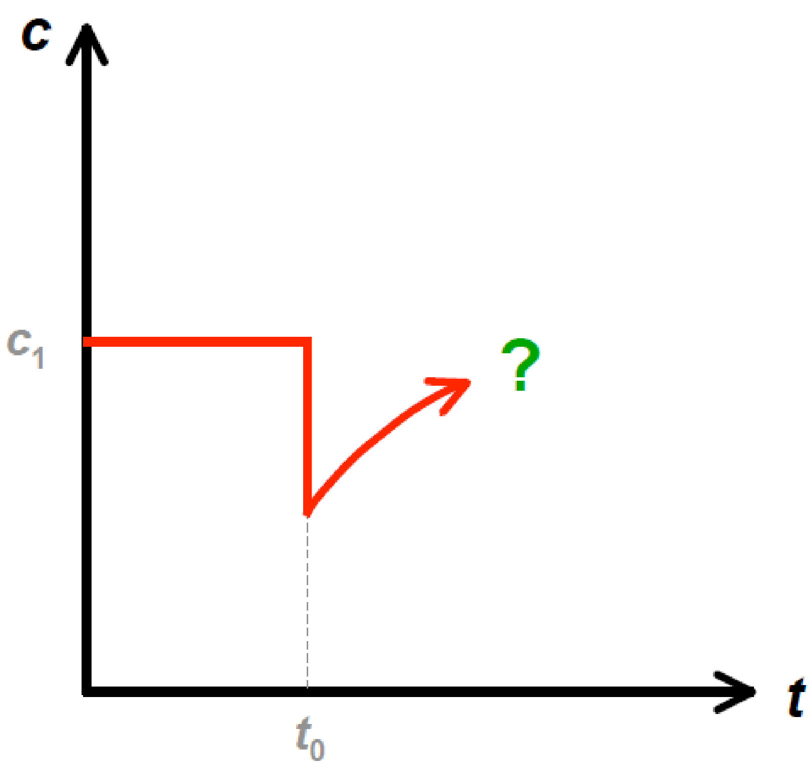
Effect of an Increase in the Savings Rate at Time *T*



What about consumption?

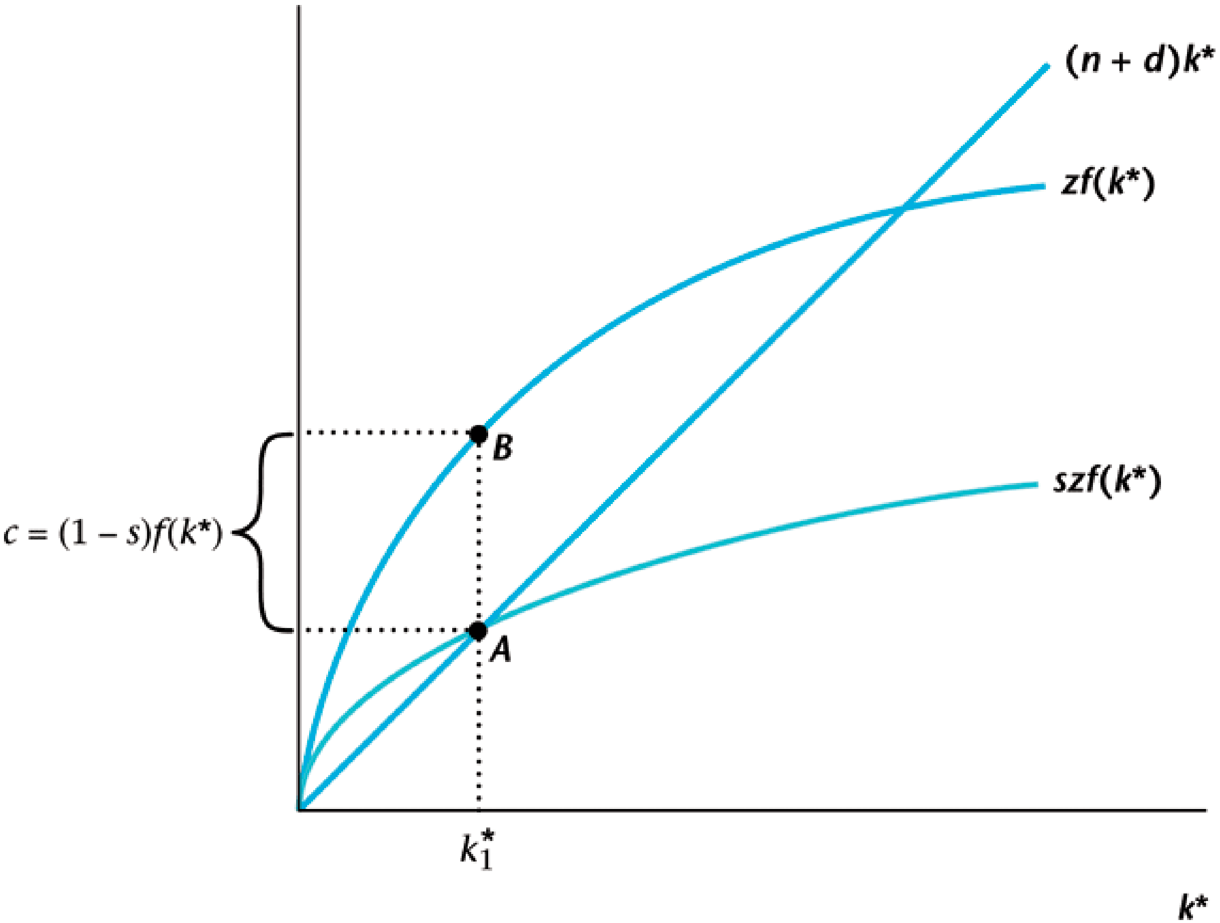
We know consumption per capita equals $(1 - s)zf(k^*)$. Since s changes discontinuously at t_0 and k^* adjusts gradually, it is the case that initially c falls but reverses over time.

Whether per capita consumption is higher or lower at the end is not immediately clear!



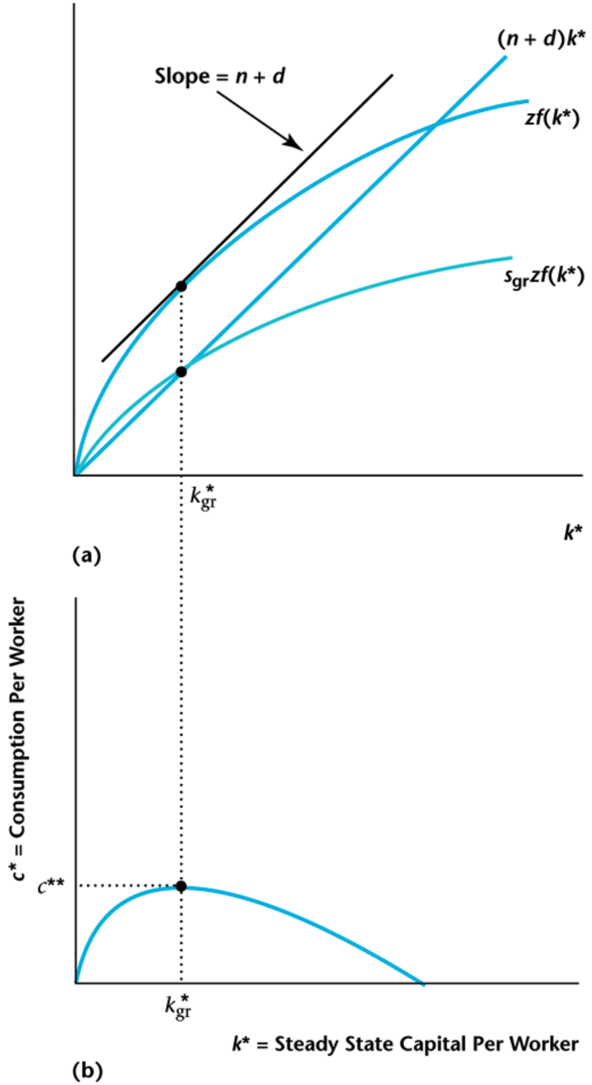
… But we can say a bit about this!

Steady State Consumption per Worker



- Consumption per worker in the steady state is AB (output minus savings), given capital per worker k*

The Golden Rule Quantity of Capital per Worker



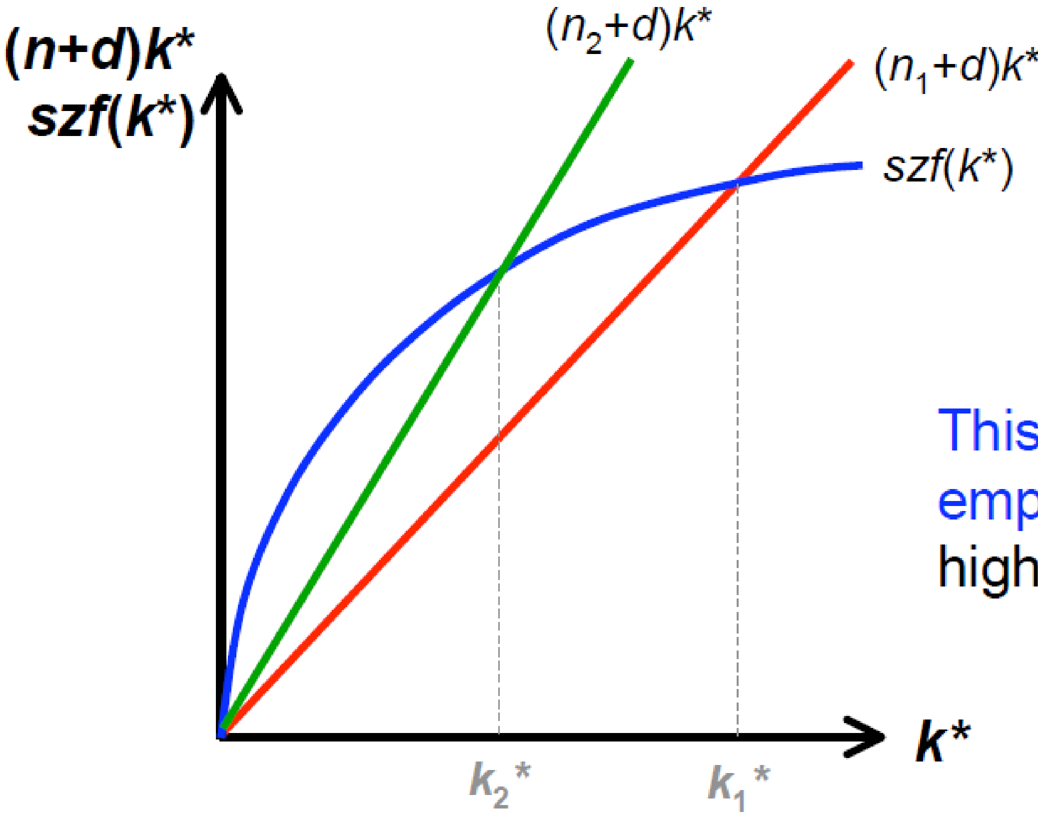
- The golden rule savings rate s_{gr} is where

$$zf'(k^*) = n + d$$

- Note that at the golden rule allocation, consumption is maximized

What happens in the steady state when we **change the population growth rate, *n*?**

This is straightforward:



This shows the second empirical relationship … a higher *n* yields a lower *y*!

So we’ve analyzed how things change when we move the savings rate and the population growth rate.

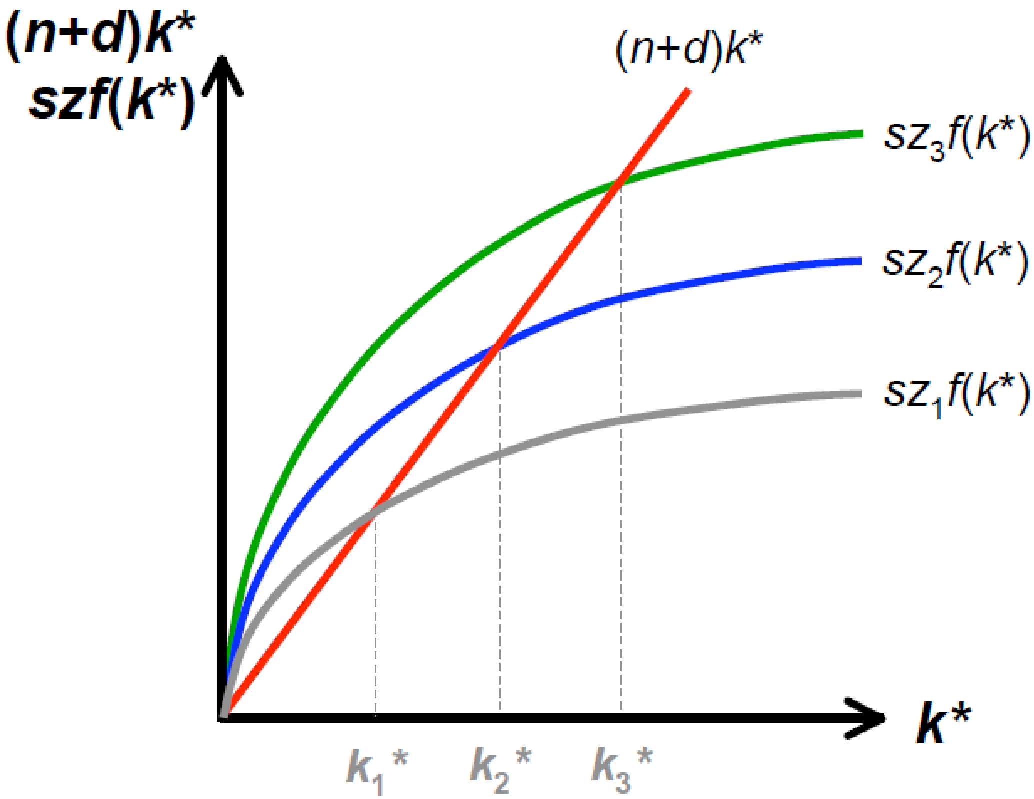
If we want to grow … save more! If we want to grow … enact in population control policies! (But who’s going to implement this? We don’t have a government!)

But … we cannot increase *s* above 1. And there is a “natural limit” on how much we can reduce *n*.

Are we stuck?

(You know the answer is “no, we’re not stuck”.)

Let’s see what happens when there is a positive technological shock; i.e., **an increase in z.** For $z_1 < z_2 < z_3$:



Hence, increases in z translate into long-run growth!

And we've just proved

Improvements in a country's standard of living are brought about in the long run by technological progress.