

# The Lost Generation of the Great Recession

Sewon Hur

University of Pittsburgh

June 16, 2017

# Introduction

- What are the distributional consequences of the Great Recession?
- 2 dimensions that affect generations differently
  - large decline in labor income - hurts young
  - even larger decline in asset prices - hurts old, young can potentially gain from cheaper assets
- Long-term consequences of these channels  $\Rightarrow$  need model to evaluate **lifetime welfare consequences** of recession

## This paper

- Heterogeneous agent life-cycle model with
  - portfolio over risky and risk-free assets
  - household **borrowing constraints**
  - **heterogeneity** in income and wealth
- Feed in features of Great Recession
  - decrease in labor income
  - decrease in risky asset price
- Use model allocations to compute lifetime welfare consequences

## Model findings

- Welfare changes (one-period consumption equivalents) are diverse
- Households with risky assets before Recession suffered the most
  - Young households (21–29) suffered the most (–37 percent)
  - Old households (84–92) suffered the least (–16 percent)
- Households without risky assets fared better
  - Young households (21–29) suffered the most (–9 percent)
  - Pre-retirement households (57–65) gained the most (15 percent)
- On average, middle-aged households (39–47) suffer the largest welfare loss (–24 percent)

## Related Literature

- **Asset prices and generations:** Kiyotaki et al. (2010), Li and Yao (2007)
- **Welfare over the Great Recession:** Bell and Blanchflower (2011), Elsby et al. (2010), Glover et al. (2014), Krueger et al. (2016). Menno and Oliviero (2014), Peterman and Sommer (2014)
- **Life-cycle heterogenous agent models:** Conesa et al. (2008), Del Negro et al. (2010), Heathcote et al. (2010), Huggett (1996)
- **with borrowing constraints:** Chambers et al. (2009), Favilukis et al. (2013), Fernandez-Villaverde and Krueger (2010), Iacoviello and Pavan (2013), Storesletten et al. (2004)

# Model

## Preview of the model

- Households become economically active at age 21
- Choose a portfolio over risky and risk-free assets
- Heterogeneity generated by initial wealth differences and idiosyncratic shocks to labor endowment and risky asset returns
- Young are net buyers, old are net sellers of risky assets
- Young households typically borrow to finance risky assets
- Low wealth, young households more leveraged than others

# Environment

- Continuum of finitely lived households
- Partial Equilibrium
  - exogenous asset prices



## Demographics

- Households indexed by  $i$ , age denoted by  $j \in J \equiv \{1, 2, \dots, \bar{J}\}$
- $\psi_j$ : survival probability from age  $j$  to  $j + 1$ , (let  $\Psi_j = \prod_{k=1}^{j-1} \psi_k$ )
- Retirement at  $j = R$
- Newborns endowed with  $\{\omega_i\}$  wealth

## Household preferences

- Preferences are given by

$$\mathbf{E} \left[ \sum_{j \in J} \beta^{j-1} \Psi_j u_j(c_{ij}) \right]$$

- $c$ : consumption (numeraire)
- $\beta$ : time discount factor
- $u_j(c_{ij}) = u \left( \frac{c_{ij}}{e_j} \right)$ 
  - $e_j$ : number of adult equivalents
  - captures consumption needs of changes in households size

## Household labor income

- Each period, working-age households receive idiosyncratic shocks  $z \in Z$ , which follows a Markov process, with transition matrix  $\Gamma$
- Household of age  $j$  with shock  $z$  and past earnings  $k$  earns:

$$y_j(z, k) = \begin{cases} e^z \eta_j & \text{if } j < R \\ s(k) & \text{otherwise} \end{cases}$$

- $\eta_j$ : age-profile of endowments
- $s(k)$ : retirement income depends on past earnings  $k$
- Let  $\Gamma^j = \Gamma$  for  $j < R - 1$  and  $\Gamma^j = I$  for  $j \geq R - 1$

## Retirement income

- Retirement income is proportional to previous earnings, summarized by  $k$
- Consistent with the U.S. social security system and following Huggett and Parra (2010), marginal benefit rates are

$$\kappa_1 \text{ for } 0 < k \leq k_1$$

$$\kappa_2 \text{ for } k_1 < k \leq k_2$$

$$\kappa_3 \text{ for } k_2 < k \leq \bar{y}$$

- $\bar{k}$ : maximum earnings that count toward retirement income

## Law of motion for past earnings

- Following Kitao (2012) and Peterman and Sommer (2016), previous earnings are summarized by

$$k'(j, z, k) = \begin{cases} \frac{\min\{g(e^z \eta_j), \bar{k}\} + (j-1)k}{j} & \text{if } j \leq j^* \\ \max\left\{k, \frac{\min\{g(e^z \eta_j), \bar{k}\} + (j-1)k}{j}\right\} & \text{if } j^* < j < R \\ k & \text{if } j \geq R \end{cases} \quad (1)$$

- $g$  transforms disposable to gross labor income
- $j^*$ :  $k'$  does not fall below average of first 35 working years

## Assets

- Risky asset  $x \geq 0$ 
  - return subject to idiosyncratic shock  $e^\xi$ , with  $\xi \in \Xi$  and probability  $\pi(\xi)$
  - pay fixed cost  $f$  if  $x > 0$
- Risk-free asset  $b$  with *marginal* interest rate
  - $r_s$  if  $b' \geq 0$
  - $r_b$  if  $b' \in [-\lambda p_x x', 0)$
  - $r_h/\psi_j > r_b$  on portion of debt that exceeds  $\lambda p_x x'$
  - let  $r(b', x', j)$  denote *average* interest rate
- Beginning-of-period wealth:  $a' = b'(1 + r(b', x', j)) + p_x x' e^{\xi'}$

## Household problem

- Household of age  $j$ , wealth  $a$ , endowment shock  $z$ , and prior earnings  $k$  chooses consumption  $c$ , risk-free assets  $b'$ , and risky assets  $x'$  to solve:

$$v(j, a, z, k) = \max_{c, b', x'} u_j(c) + \beta \psi_j \sum_{z' \in Z} \sum_{\xi' \in \Xi} \Gamma_{z, z'}^j \pi(\xi') v(j+1, a', z', k') \quad (2)$$

$$\text{s.t. } c + p_x x' + b' \leq y_j(z, k) - \mathbb{1}_{x' > 0} f + a$$

$$a' = b'(1 + r(b', x', j)) + p_x x' e^{\xi'}$$

$$k' \text{ follows (1)}$$

$$c \geq 0, x' \geq 0$$

- $v_{J+1} = 0$
- Normalize  $p_x = 1$

## Transition Function

- Define the state space  $S \equiv J \times A \times Z \times K$  with Borel  $\sigma$ -algebra  $\mathcal{B}$
- For any set  $\mathcal{S} \in \mathcal{B}$ ,  $\mu(\mathcal{S})$  is a measure of agents in  $\mathcal{S}$
- Define  $Q((j, a, z, k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K})$  as the probability that an agent with state  $(j, a, z, k)$  transits to  $\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}$  next period
- Formally,  $Q : S \times \mathcal{B} \rightarrow [0, 1]$  and

$$Q((j, a, z, k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) = \tag{3}$$

$$\psi_j \mathbb{1}_{\{k'(j, z, k) \in \mathcal{K}\}} \mathbb{1}_{\{j+1 \in \mathcal{J}\}} \sum_{z' \in \mathcal{Z}} \sum_{\xi \in \Xi} \mathbb{1}_{\{a'(j, a, z, k; \xi) \in \mathcal{A}\}} \pi(\xi) \Gamma_{z, z'}^j$$

where  $\mathbb{1}$  is the indicator function,

$a'(j, a, z, k; \xi) = b'(j, a, z, k)(1 + r(b', x', j)) + x'(j, a, z, k)e^\xi$ , and

$b'(j, a, z, k)$  and  $x'(j, a, z, k)$  are the optimal policy functions



## Equilibrium

A *stationary equilibrium* is a value function  $v : S \rightarrow \mathbb{R}$ , policy functions for the households  $c : S \rightarrow \mathbb{R}_{++}$ ,  $b' : S \rightarrow \mathbb{R}$ , and  $x' : S \rightarrow \mathbb{R}_+$  and a stationary measure  $\mu : S \rightarrow \mathbb{R}_+$  such that:

1. the policy functions solve the households' problem in (2) and  $v$  is the associated value function.
2. for all  $(\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) \in \mathcal{B}$ , the invariant measure  $\mu$  satisfies

$$\mu(\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) = \int_{\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}} Q((j, a, z, k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) d\mu(j, a, z, k)$$

where  $Q$  is the transition function defined in (3) and the distribution for newborns is given.

## Rest of talk

- Calibrate model to 2007
  - Important moments: household leverage and risky asset participation
- Show that the model matches data along important dimensions
- Shock the model with recession and present welfare analysis

## Functional forms

- Preferences

$$u_j(c) = \frac{\left(\frac{c}{e_j}\right)^{1-\sigma} - 1}{1-\sigma}, \quad \sigma = 3$$

► Sensitivity

- Period in model: 3 years

## Estimation of endowment process

- Compute residual disposable labor earnings
  - PSID 1970–1997 core sample (ages 21–65)
  - disposable labor earnings using TAXSIM
  - obtain residuals from model with age and year fixed effects

- Model specification

$$\log(y_t) = z_t + \nu_t, \quad \nu_t \sim N(0, \sigma_\nu^2),$$

$$z_t = \rho z_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

where  $y_t$  is residual disposable earnings

- GMM estimation:  $\rho = 0.938$ ,  $\sigma_\varepsilon = 0.215$ ,  $\sigma_\nu = 0.171$

## Endowment process

- Treat transitory shock as measurement error:  $\hat{\sigma}_\nu = 0$
- Convert to 3-year persistence:  $\hat{\rho} = \rho^3 = 0.825$
- Convert variance so that cross-sectional variance is consistent:

$$\hat{\sigma}_\varepsilon = \sigma_\varepsilon \sqrt{\frac{1 - \hat{\rho}^2}{1 - \rho^2}} = 0.351$$

- Discretize using Rouwenhorst method described in Kopecky and Suen (2010) with 11 grid points

## Endowment in retirement

- Normalize average working age disposable-earnings to 1
- Retirement endowments depend on previous earnings  $k$
- Marginal replacement rates are:

$$\begin{cases} 0.90 & \text{if } k \in (0, 0.27] \\ 0.32 & \text{if } k \in (0.27, 1.65] \\ 0.15 & \text{if } k \in (1.65, 3.10] \end{cases}$$

- Consistent with social security replacement rates (Huggett and Parra 2010), adjusting for units

## Main parameters

- 5 parameters jointly calibrated to match 5 data targets

Variables	Value	Targets
Discount factor $\beta^{\frac{1}{3}}$	0.990	average leverage*: 0.23
Risky asset variance $\sigma_{\xi}^2$	0.082	90/50 wealth ratio: 7.43
Participation cost $f$	0.060	risky asset participation: 0.81
Risky return $E(\xi)^{\frac{1}{3}} - 1$	7.9%	risky assets/21–65 disp. lab. inc.: 8.78
Interest rate $(1 + r_s)^{\frac{1}{3}} - 1$	2.5%	safe assets/21–65 disp. lab. inc.: 0.40

- Risky assets: stocks (direct and indirect), real estate, and non-corporate business assets
- Safe assets: all other assets
- \* : Leverage =  $\left[ \frac{-\text{safe}}{\max\{\epsilon, \text{risky}\}} \right]_0^1$

## Other parameters

Variable	Value	Target/Source
Number of cohorts $J$	25	ages 21–95 (3-year intervals)
Retirement $R$	16	ages 66–68
Wealth endowments $\{\omega_i\}$	<a href="#">▶ graph</a>	wealth, ages 18–23 (SCF 2007)
adult equivalents $\{e_j\}$	<a href="#">▶ graph</a>	SCF 2007
Endowment profile $\{\eta_j\}$	<a href="#">▶ graph</a>	disposable labor income (SCF 2007)
Survival prob. $\{\psi_j\}$	<a href="#">▶ graph</a>	2007 US Life Tables
Borrowing spread $r_b - r_s$	1.3%	15-year mortgage rate* (2001–2007)
High spread $r_h - r_s$	8.7%	credit card rate* (2001–2007)
Collateral constraint $\lambda$	0.8	discussion below <a href="#">▶ sensitivity</a>

\* : less 10-year treasury rate

- Residential mortgages: rates and fees higher if loan-to-value higher than 80%
- Commercial loans: typically require 65–80 percent collateral
- Stock based lines of credit loans: typically require 50–65 percent collateral

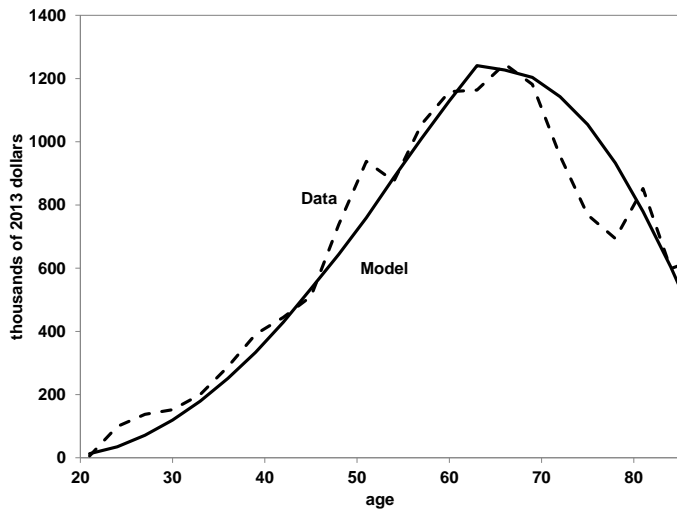


## Steady state

- Model generates
  - wealth profile over age
  - risky asset profile over age
  - risky asset participation over age
  - wealth distribution
  - household leverage

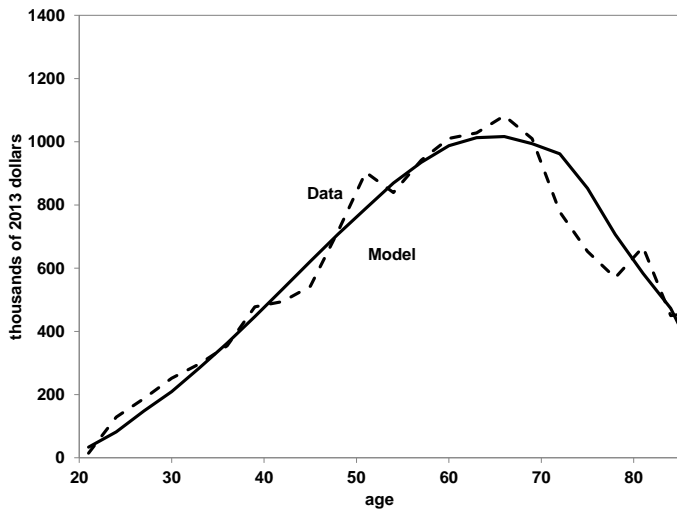
consistent with US 2007 household data

## Household net wealth



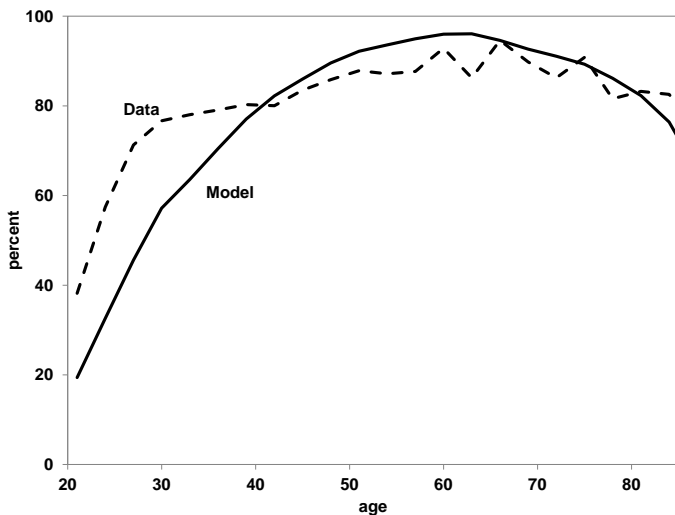
data source: SCF 2007

## Household risky assets



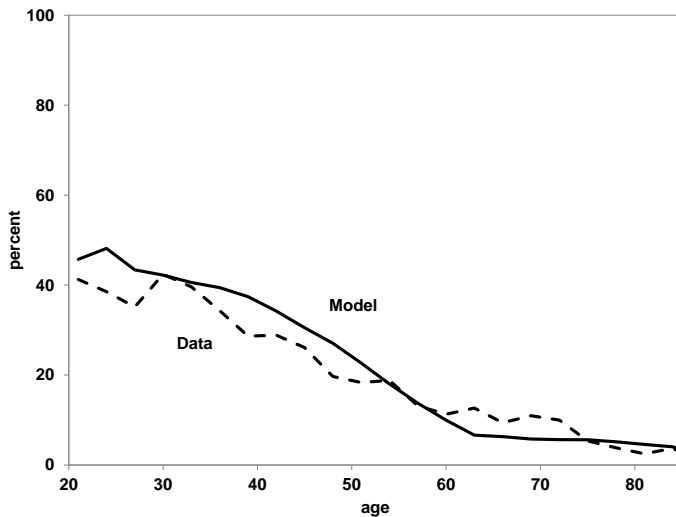
data source: SCF 2007

## Risky asset participation



data source: SCF 2007

# Leverage



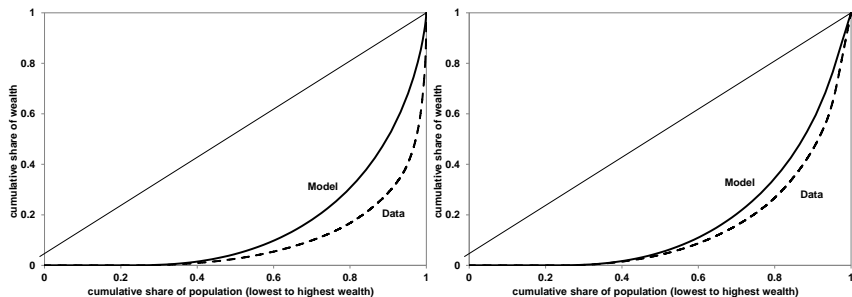
data source: SCF 2007

## Distribution of leverage

pctile age	25		50		75	
	model	data	model	data	model	data
21–29	0	0	57	0	98	97
30–38	0	0	45	31	65	73
39–47	0	0	29	15	58	50
48–56	0	0	9	1	38	30
57–65	0	0	0	0	6	12
66–95	0	0	0	0	0	0

## Wealth distribution

- Model wealth distribution reasonably similar to the data
- Households with non-positive wealth: 11% (model) vs 10% (data)
- Better if data and model are truncated at \$3 million (98th pctile)



data source: SCF 2007

# Recession



# Great Recession

Great Recession (2007–2010) featured changes in

## 1. Labor market

- large decline in disposable labor income (8.7 percent)
- largest decline for young households
- increase (decrease) in downward (upward) risk, compared to 2002–2006

## 2. Asset market

- risky asset price index falls 23.6 percent
- increase in expected risky asset volatility
- real interest rates decreased

## Decline in disposable labor income

Table: Large decline in disposable labor income (2007–2010)

age	change (percent)
21–65	-8.7
21–29	-12.0
30–38	-11.0
39–47	-8.0
48–56	-10.1
57–65	-5.6

Source: CPS

Note: Detrended at 2 percent per year, adjusted for inflation

## Increase in earnings risk

- PSID available every 2 years
- Compared to 2002-2006, in 2006–2010: [▶ graph](#)
  - 4.0 p.p. increase in probability of  $\geq 30$  percent decrease
  - 4.0 p.p. decrease in probability of  $\geq 30$  percent increase
  - similar across age groups
- Compared to 2004-2006, in 2008-2010: [▶ graph](#)
  - 2.4 p.p. increase in probability of  $\geq 30$  percent decrease
  - 2.6 p.p. decrease in probability of  $\geq 30$  percent increase
  - similar across age groups (except for 57–65: no change)
  - reverts to normal in 2010–2012

## Labor market changes in the model

- Assume a 3 p.p. increase (decrease) in downward (upward) mobility
- Shock to labor income transition matrix  $\Gamma$ 
  - for  $i > 1$ ,  $z(i) - z(i - 1) \approx 0.3$
  - let  $\Delta = \min\{0.03, \Gamma\}$
  - $1 < i < N$ : set  $\hat{\Gamma}_{i,i+1} = \Gamma_{i,i+1} - \Delta_{i,i+1}$  and  $\hat{\Gamma}_{i,i-1} = \Gamma_{i,i-1} + \Delta_{i,i+1}$
  - set  $\hat{\Gamma}_{12} = \Gamma_{12} - \Delta_{12}$  and  $\hat{\Gamma}_{11} = \Gamma_{11} + \Delta_{12}$
  - set  $\hat{\Gamma}_{NN} = \Gamma_{NN} - \Delta_{NN}$  and  $\hat{\Gamma}_{N,N-1} = \Gamma_{N,N-1} + \Delta_{NN}$
- Adjust age-profile  $\hat{\eta}_j$  such that average decline matches data
- After 1 period, revert to non-recession  $\Gamma$  and  $\eta_j$ 
  - labor market shocks persistent

## Decline in risky asset price index [▶ graph](#)

- Risky asset price index: average inflation-adjusted changes of stock and housing prices, weighted by 2004 portfolio share
- 23.6 percent decrease from 2007 to 2010
- Fully recovered in 2014
- Model:
  - 23.6 percent decrease in period 1
  - return to steady state price in period 2

## Increase in risky asset volatility [▶ graph](#)

- 28.6 percent increase in VIX (expected 30-day standard deviation, annualized) from 2007 to 2010
- Returns to 2007 levels by 2013
- Model:
  - set  $\hat{\sigma}_\xi = 1.286 \times \sigma_\xi$
  - after 1 period, revert to non-recession  $\sigma_\xi$

## Change in real interest rates [▶ graph](#)

**Table:** Change in real rates (percent, relative to 2007)

	2010
10-year treasury	-0.7
15-year mortgage	-1.2
credit card	1.3

- Model:
  - feed in exact changes in period 1
  - return to steady state rates in period 2

## Timing and Information

- Sequence of events
  1. start  $t = 0$  in steady state
  2. MIT shock in  $t = 1$  : transition matrix  $\widehat{\Gamma}$  from  $t = 0$  to  $t = 1$ , income profile  $\widehat{\eta}_j$  and risky asset price  $\widehat{p}_x$  in  $t = 1$ , risky asset  $\widehat{\sigma}_\xi$  from  $t = 1$  to  $t = 2$
  3. return to steady state transition processes and prices in  $t = 2$
- Information set: households believe
  - $\theta$  probability that prices recover
  - $1 - \theta$  probability that prices persist forever
- Baseline: set  $\theta$  such that quantity of risky assets remains unchanged
- Alternatives: set  $\theta = 1$  or  $\theta = 0$





## Ex post value function

- Risky asset prices recover in period  $t = 2$
- Based on  $t = 1$  decisions, compute ex post value function:

$$v^*(j, a, z, k) = u_j(\widehat{c}(j, a, z, k)) + \beta \psi_j \sum_{z' \in Z} \sum_{\xi' \in \Xi} \Gamma_{z, z'}^j \widehat{\pi}(\xi') v(j+1, \widehat{a}'(p_x), z', k')$$

where

$$\widehat{a}'(p) = \widehat{b}'(j, a, z, k)(1 + \widehat{r}(b', x', j)) + p \widehat{x}'(j, a, z, k) e^{\xi'}$$

$$\widehat{k}' \text{ follows } (\widehat{1})$$

## Conditional welfare change (ex post)

- Utility function is homogeneous to degree  $1 - \sigma$
- Conditional welfare change (remaining lifetime consumption):

$$\delta(j, a, z, k) =$$

$$\left[ \frac{u_j(c(j, a, z, k)) + \beta \psi_j \sum_{z' \in Z} \sum_{\xi' \in \Xi} \widehat{\Gamma}_{z, z'}^j \pi(\xi') v^*(j+1, a'(\widehat{p}_x), z', k')}{v(j, a, z, k)} \right]^{\frac{1}{1-\sigma}} - 1$$

where

$$a'(p) = b'(j, a, z, k)(1 + r(b', x', j)) + px'(j, a, z, k)e^{\xi'}$$

$k'$  follows (1)

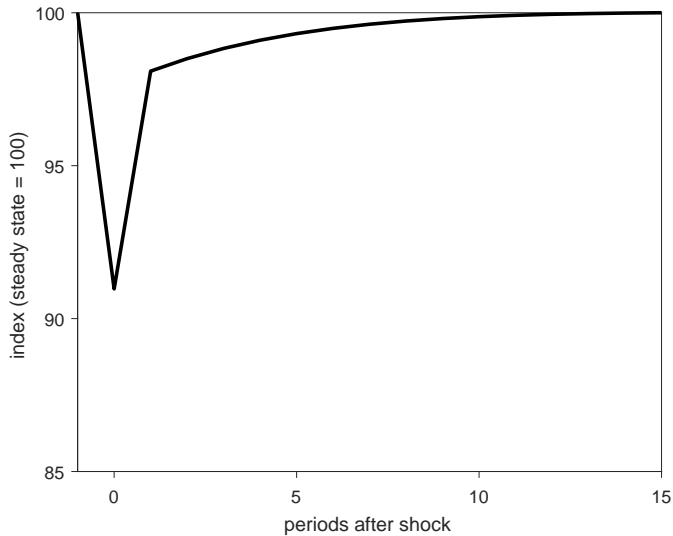
## One-period consumption equivalent

- Agents have different remaining lifetime
- Compute one-period consumption equivalent  $\delta^P(j, a, z, k)$  such that:

$$\begin{aligned}
 & u_j(c(j, a, z, k)(1 + \delta^P)) + \beta\psi_j \sum_{z' \in Z} \sum_{\xi' \in \Xi} \Gamma_{z, z'}^j \pi(\xi') v(j + 1, a'(\rho_x), z', k') \\
 & = u_j(c(j, a, z, k)) + \beta\psi_j \sum_{z' \in Z} \sum_{\xi' \in \Xi} \hat{\Gamma}_{z, z'}^j \pi(\xi') v^*(j + 1, a'(\hat{\rho}_x), z', k)
 \end{aligned}$$

where  $k'$  follows (1)

## Aggregate income



## Young suffer largest decline in net wealth

age	model (percent change)	data
21–29	–47.3	–59.2
30–38	–37.3	–44.1
39–47	–29.1	–18.9
48–56	–25.7	–19.6
57–65	–21.3	–11.7
66–74	–20.5	–20.0
75–83	–20.6	–9.2
84–92	–19.5	21.5

## Young risky participants suffer largest welfare loss

age	remaining lifetime	one-period cons. equivalent (percent)		
	cons. equivalent (percent)	all	risky participant	risky non-participant
21–29	–6.2	–18.1	–37.1	–9.2
30–38	–9.2	–22.3	–32.6	–6.4
39–47	–15.1	–24.2	–29.4	–2.5
48–56	–12.4	–21.5	–24.1	5.3
57–65	–5.1	–19.8	–21.4	15.2
66–74	–4.8	–19.9	–21.7	2.1
75–83	–4.8	–16.3	–19.3	0.0
84–92	–4.3	–10.0	–16.4	–0.1

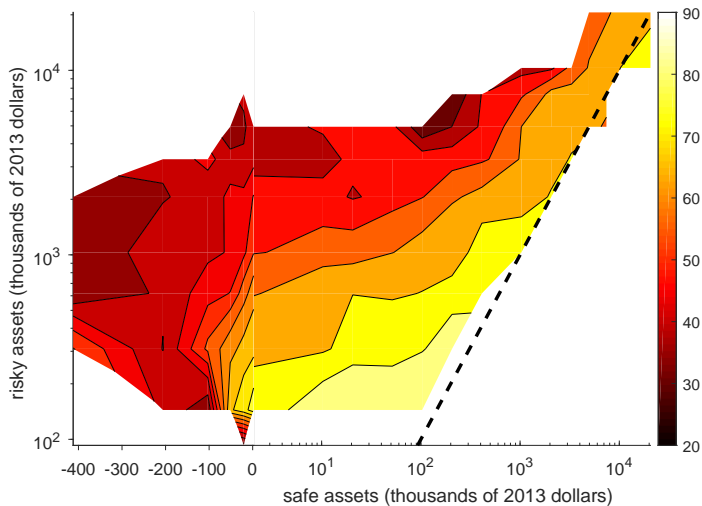
## Decomposition of welfare losses

age	labor market changes			risky asset changes			safe asset change		
	all	risky		all	risky		all	risky	
		> 0	= 0		> 0	= 0		> 0	= 0
21–29	78	32	136	24	67	-29	-2	2	-7
30–38	54	25	161	51	77	-47	-5	-3	-14
39–47	42	26	204	68	83	-85	-10	-9	-18
48–56	42	32	-902*	76	84	728*	-19	-15	274*
57–65	16	14	-24*	102	101	84*	-18	-15	40*
66–74	0	0	0*	113	112	58*	-13	-12	42*
75–83	0	0	n.a.	109	109	n.a.	-9	-9	n.a.
84–92	0	0	0	104	105	-43	-4	-5	143
all	39	19	179	71	90	-61	-10	-9	-18

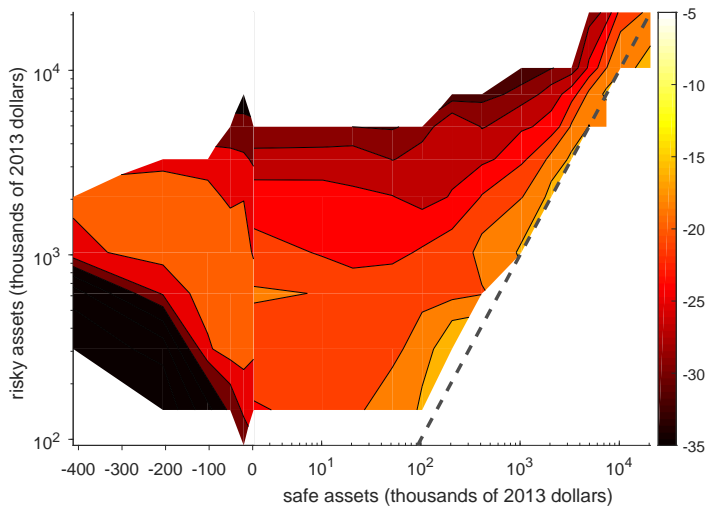
\* : decomposition of welfare gains.



## Young are more leveraged



## Biggest losers: the highly leveraged

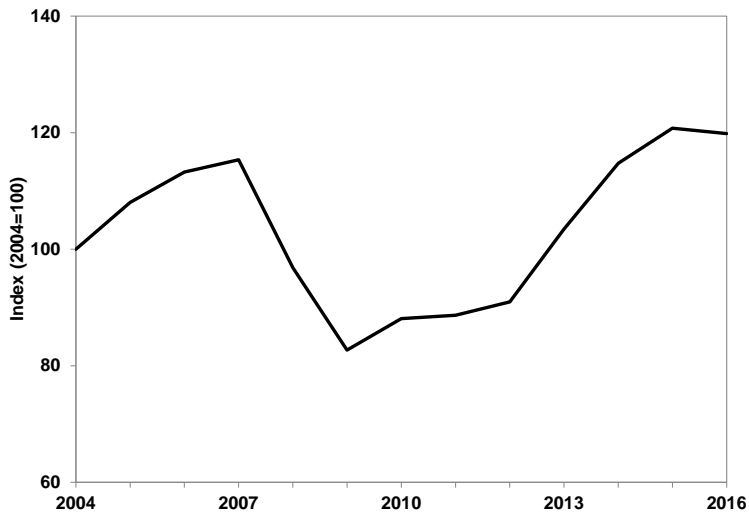


# Conclusion

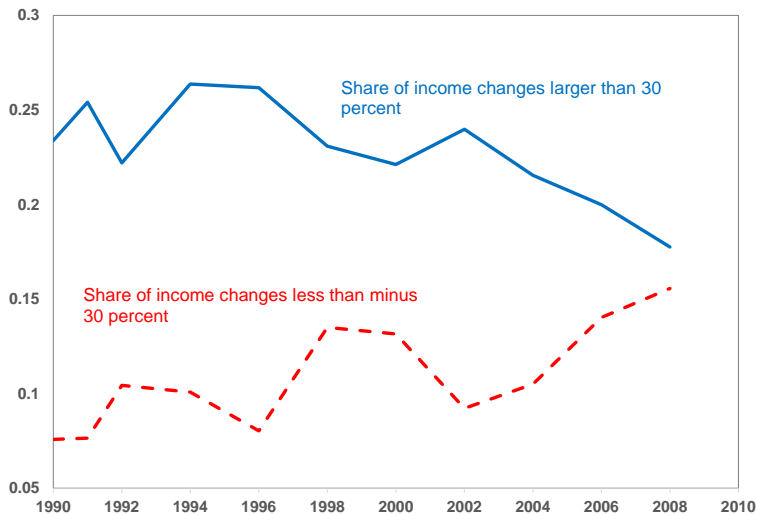
- Developed a model consistent with
  - age profiles of wealth, risky assets, risky asset participation, and leverage
  - distributions of household leverage and wealth
- Changes in labor and asset markets resembling Great Recession
- Results
  - young risky participants suffer the largest welfare losses
  - pre-retirement non-participants have the largest welfare gains

## Appendix

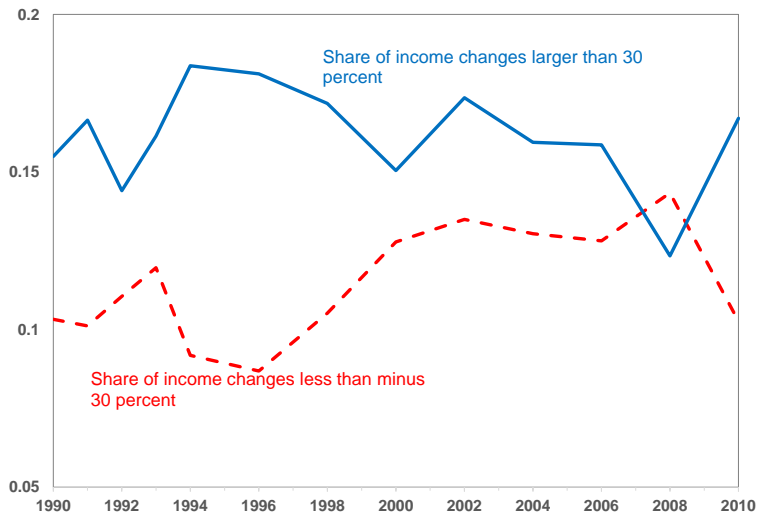
## Asset price index



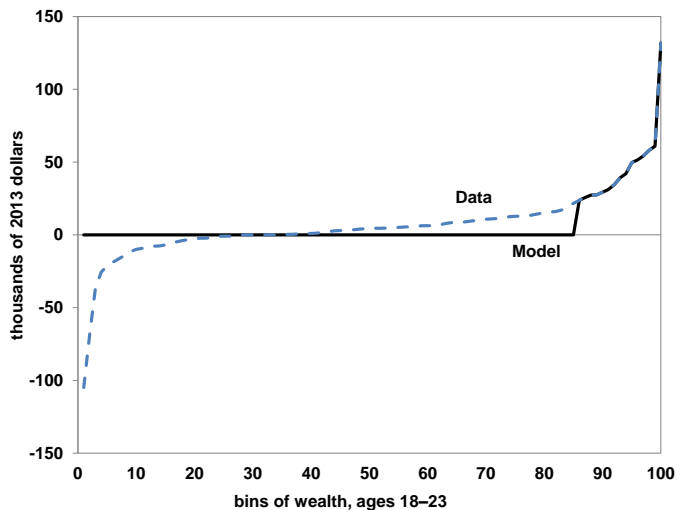
## 4-year changes in log real disposable income



## 2-year changes in log real disposable income

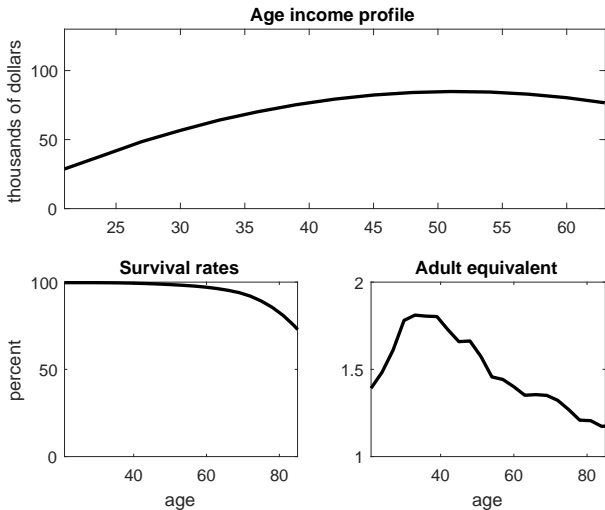


## Initial wealth endowments

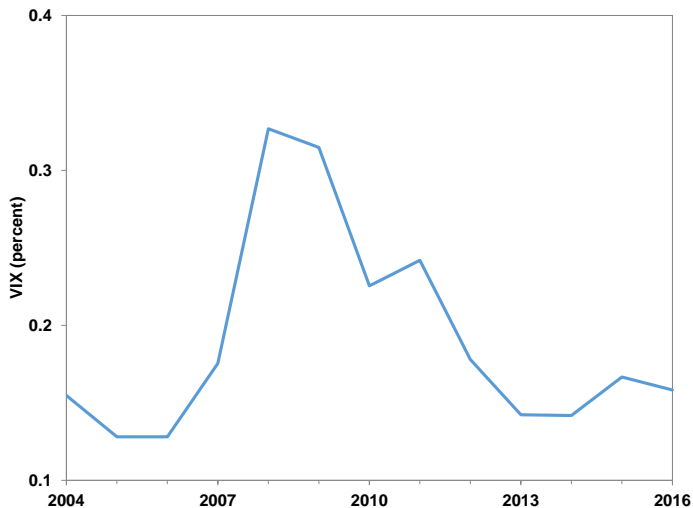




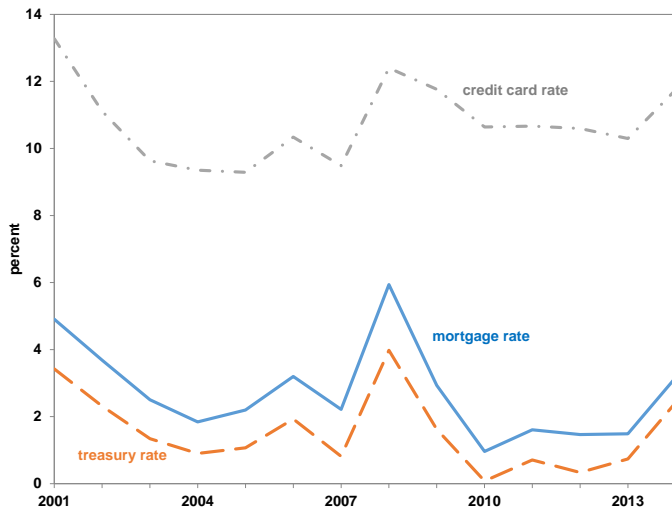
# Age income profile



# Stock Market Volatility Index



# Real interest rates



## Sensitivity for $\lambda = 0.745$ [▶ back](#)

- Model recalibrated

age	all	risky > 0	risky = 0
21–29	-17.9	-35.4	-9.1
30–38	-22.2	-29.7	-7.7
39–47	-23.2	-27.4	-3.1
48–56	-20.4	-22.3	3.2
57–65	-19.3	-20.3	10.8
66–74	-19.7	-21.0	2.3
75–83	-16.2	-18.4	0.0
84–92	-10.5	-15.9	-0.1
all	-19.6	-23.7	-5.5

## Sensitivity for $\sigma$ [▶ back](#)

- Model recalibrated

age	$\sigma = 2.5$			$\sigma = 3.5$		
	all	risky > 0	risky = 0	all	risky > 0	risky = 0
21-29	-21.5	-41.5	-10.5	-16.7	-32.1	-9.4
30-38	-26.0	-34.8	-8.3	-20.2	-28.7	-7.1
39-47	-26.2	-30.8	-2.3	-21.8	-26.3	-4.3
48-56	-22.0	-24.0	4.4	-22.1	-24.4	0.3
57-65	-19.7	-20.8	11.5	-20.5	-21.9	10.3
66-74	-18.9	-20.3	2.6	-20.4	-22.2	1.2
75-83	-15.1	-17.6	0.3	-17.1	-20.1	0.4
84-92	-9.6	-15.3	-0.1	-11.1	-16.8	-0.1
all	-21.2	-25.6	-6.1	-19.5	-24.2	-5.7