Inflation, Debt, and Default

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve Board, or the Federal Reserve System.
Motivation: U.S. inflation cyclicality discount

(a) Inflation and Consumption growth

(b) Real Interest Rates

(c) Co-movement and Real Interest Rates

Note: Inflation is the log difference between CPI in quarter t and CPI in quarter t-4. Consumption growth is the log difference in real personal consumption expenditures over the same interval. Real interest rates are nominal interest rates on government securities (from the IMF IFS database) minus expected inflation computed using a linear univariate forecasting model estimated on actual inflation.
Why inflation cyclicality matters

- The majority of sovereign debt in advanced economies is
  - subject to inflation risk (nominal)
  - held domestically

- The co-movement between inflation and consumption growth varies over time and across countries

- Procyclical inflation makes nominal debt
  - less risky to lender: less inflation in bad times $\rightarrow$ hedging $\uparrow$
  - more risky to borrower: more payout bad times $\rightarrow$ default $\uparrow$
  - procyclical/countercyclical inflation
    $\sim$ complement/substitute to default in bad times

- Inflation cyclicality jointly affects yields, debt and default
This paper

- We document inflation procyclicality discount
  - higher covariance associated with lower real rates
  - but not so much in bad states

- Use a simple 2-period model to demonstrate
  - inflation procyclicality discount
  - state-dependent default dynamics depend on cyclicality

- Calibrate richer quantitative default model and find
  (procyclical economy vs. countercyclical economy)
  - lower real rates in normal times
  - more debt crises and defaults in bad states
Related literature


- **Monetary unions**: Aguiar et al. (2013), Corsetti and Dedola (2013).
Empirical evidence
Evidence on real yields and inflation cyclicality

- Compute country-specific time-varying co-movement innovations to inflation and to consumption growth

- Follow Boudoukh (1993)’s country-by-country VAR

\[
\begin{bmatrix}
\pi_{it} \\
g_{it}^c
\end{bmatrix} = A_i \begin{bmatrix}
\pi_{i,t-1} \\
g_{i,t-1}^c
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{\pi it} \\
\varepsilon_{git}
\end{bmatrix}
\]

- Sample: 19 OECD countries; quarterly data 1985-2015
- Compute conditional co-movement between \( \varepsilon_{\pi it} \) and \( \varepsilon_{git} \) using overlapping ten-year windows

- Real yields adjust for expected future inflation
Real interest rates: the inflation cyclicality discount

<table>
<thead>
<tr>
<th>Real yield on government debt</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation consumption covariance</strong></td>
<td>$-1.797^{***}$</td>
<td>$-1.637^{***}$</td>
<td>$-1.804^{**}$</td>
</tr>
<tr>
<td></td>
<td>(0.539)</td>
<td>(0.380)</td>
<td>(0.636)</td>
</tr>
<tr>
<td>Lagged Debt</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Mean of $\pi$ and $g_c$ residuals</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Variance of $\pi$ and $g_c$ residuals</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.881</td>
<td>0.902</td>
<td>0.903</td>
</tr>
<tr>
<td>$N$</td>
<td>1726</td>
<td>1726</td>
<td>1726</td>
</tr>
</tbody>
</table>

Countries: AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JPN, KOR, NLD, NOR, PRT, SWE, USA.
Standard errors clustered by country. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
All regressions include country and time fixed effects

- One standard deviation increase in $\text{cov}(\varepsilon_{\pi it}, \varepsilon_{g_c it}) \sim 0.17$
  is associated with $\sim 31$ bp decrease in real sovereign yields
## Procyclicality discount only in good times

<table>
<thead>
<tr>
<th>Inflation consumption covariance</th>
<th>−1.804**</th>
<th>−1.159</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(.636)</td>
<td>(.683)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Covariance*1_{good times}</th>
<th>−1.834***</th>
<th>−2.994***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(.506)</td>
<td>(0.696)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Covariance*1_{bad times}</th>
<th>−1.159</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.683)</td>
</tr>
</tbody>
</table>

### Specifications:

<table>
<thead>
<tr>
<th></th>
<th>1_{good times}</th>
<th>other controls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>adj. $R^2$</th>
<th>0.903</th>
<th>0.910</th>
<th>0.910</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1726</td>
<td>1726</td>
<td>1726</td>
</tr>
</tbody>
</table>

$1_{good times} \equiv$ average residual cons. growth > 0.

Countries: AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JPN, KOR, NLD, NOR, PRT, SWE, USA.

Standard errors clustered by country. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time fixed effects.
Model
Model

- We develop a model of sovereign debt
  - builds on standard model (Arellano 2008)
  - inflation, exogenous
    (e.g. changes in monetary independence, changes in nature of supply/demand shocks in the economy)
  - risk-averse, domestic lenders hold nominal bonds

- Use calibrated model to investigate how inflation cyclicality affects interest rates, dynamics of debt and default
First, a simple model without default

- Two-period lived lender and borrower with endowments
  - first period: $y^\ell = 1 - \tau$ and $y^b = \tau$
  - second period: $y^\ell = (1 - \tau)x$ and $y^b = \tau x$

- Debt $b$ is nominal with price $q$

- Cyclical inflation is $(1 - \gamma x)^{-1}$ where $x$ is random
  - $\gamma$: cyclicality of inflation
  - if $\gamma > 0$, high $x \Rightarrow$ high inflation
First, a simple model without default

- Borrower solves

\[
\max_b u(\tau + qb) + \beta_b \int_X u(\tau x - b(1 - \gamma x)) \, dF(x)
\]

- Lender solves

\[
\max_b u(1 - \tau - qb) + \beta_l \int_X u((1 - \tau)x + b(1 - \gamma x)) \, dF(x)
\]
First, a simple model without default

- **Borrower solves**

\[
\max_b \quad u(\tau + qb) + \beta_b \int_X u(\tau x - b(1 - \gamma x)) \, dF(x)
\]

- **Lender solves**

\[
\max_b \quad u(1 - \tau - qb) + \beta_l \int_X u((1 - \tau)x + b(1 - \gamma x)) \, dF(x)
\]

- **Let** \( u(c) = Ac - \frac{\phi}{2} c^2, \mu_x = \int_X x \, dF(x) = 0, \) and \( \sigma_x^2 = \int_X x^2 \, dF(x) \)
Simple model – the inflation hedging discount

The lender’s Euler:

\[ qu' (1 - \tau - qb_\ell) = \beta_\ell \int_X \left( (1 - \gamma x) u'((1 - \tau) x + (1 - \gamma x) b_\ell) \right) dF(x) \]

\[ \uparrow \]

\[ q (A - \phi (1 - \tau - qb_\ell)) = \beta_\ell (A + \phi(1 - \tau)\gamma \sigma_x^2) - \beta_\ell \phi (1 + \gamma^2 \sigma_x^2) b_\ell \]

**Theorem**

*Near \( \gamma = 0 \), there is an inflation hedging discount on the interest rate:*

\[ \frac{\partial q}{\partial \gamma} > 0. \]
Simple model with default

- Suppose now that the borrower can default
  - during default: \( y_{\ell} = (1 - \tau)x_d \) and \( y_b = \tau x_d \)
- Borrower chooses to default if \( x < \hat{x} \equiv \frac{\tau x_d + b}{\tau + b\gamma} \)
  - ceteris paribus and for \( x_d \) low enough,
    default probability \( F(\hat{x}) \) increases with debt \((b)\) and inflation cyclicality \((\gamma)\)
Simple model with default

- Suppose now that the borrower can default
  - during default: $y_\ell = (1 - \tau)x_d$ and $y_b = \tau x_d$
- Borrower chooses to default if $x < \hat{x} \equiv \frac{\tau x_d + b}{\tau + b \gamma}$
  - ceteris paribus and for $x_d$ low enough, default probability $F(\hat{x})$ increases with debt ($b$) and inflation cyclicality ($\gamma$)
- Borrower solves

$$\max_b u(\tau + qb) + \beta_b \int_{X \setminus \hat{x}} u(\tau x - (1 - \gamma x) b) \, dF(x) + \beta_b u(\tau x_d) F(\hat{x})$$

- Lender solves

$$\max_b u(1 - \tau - qb)$$
$$+ \beta_\ell \int_{X \setminus \hat{x}} u((1 - \tau)x + (1 - \gamma x) b) \, dF(x) + \beta_\ell u((1 - \tau)x_d) F(\hat{x})$$
Simple model with default

- The lender’s Euler:

\[
\frac{d}{dt} (1 - \tau - qb) = \beta \int_{X \backslash X} (1 - \gamma x) \frac{d}{dt} ((1 - \tau)x + (1 - \gamma x) b) dF(x)
\]

\[
\Rightarrow
\]

\[
q (A - \phi (1 - \tau - qb)) = (1 - F(\hat{x})) \beta [A + \phi (1 - \tau) \gamma \sigma^2_R]
\]

+ other terms ...

where \( \sigma^2_R \equiv \frac{\int_{X \backslash X} x^2 dF(x)}{1 - F(\hat{x})} \)

- The interest rate now features a **default premium** and a **conditional inflation hedging discount**

--- state-dependent default premium and hedging discount
Model

- Closed economy, discrete time $t = 0, 1, 2, \ldots$, one good
- Endowments $y$ and inflation $\pi$ follow a joint Markov Process
- Agents
  - representative household (lenders)
  - government issues nominal bonds
Lenders

- Household (lender) preferences are given by

\[ E_0 \sum_{t=0}^{\infty} \beta_t^t u_\ell(c_t) \]

where \( 0 < \beta_\ell < 1 \) is the time discount factor

- Lenders receive \((1 - \tau)y\)
Government

- Government preferences are given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t_g u_g(g_t) \]

where \( 0 < \beta_g < \beta_\ell < 1 \) and \( g \) is government consumption

- Government revenue: \( \tau y \)
Government preferences are given by

\[ E_0 \sum_{t=0}^{\infty} \beta_t^g u_g(g_t) \]

where \( 0 < \beta_g < \beta_\ell < 1 \) and \( g \) is government consumption.

Government revenue: \( \tau y \)

Given the option to default, the government chooses

\[ V^o(B, s) = \max_{c,d} \{ V^c(B, s), V^d(B, s) \} \]

where \( B \) is incoming assets and \( s = (\pi, y) \).
Value of default

The value of default is given by

\[ V^d(B, s) = u_g \left( \tau \left( y - \phi^d(y) \right) \right) \]

\[ + \beta_g \mathbb{E}_{s'|s} \left[ \theta V^o \left( \frac{\lambda B}{1 + \pi'}, s' \right) + (1 - \theta) V^d \left( \frac{\lambda B}{1 + \pi'}, s' \right) \right] \]

\[ 0 \leq \lambda \leq 1 : \text{recovery rate,} \]
The value of default is given by

\[ V^d(B, s) = u_g \left( \tau \left( y - \phi^d(y) \right) \right) + \beta_g E_{s'|s} \left[ \theta V^o \left( \frac{\lambda B}{1 + \pi'}, s' \right) + (1 - \theta) V^d \left( \frac{\lambda B}{1 + \pi'}, s' \right) \right] \]

where

- \( 0 \leq \lambda \leq 1 \) : recovery rate,
- \( 0 \leq \theta \leq 1 \) : probability of regaining access to credit, and
Value of default

- The value of default is given by

\[ V^d(B, s) = u_g \left( \tau \left( y - \phi^d(y) \right) \right) \]

\[ + \beta_g E_{s|s'} \left[ \theta V^o \left( \frac{\lambda B}{1 + \pi'}, s' \right) + (1 - \theta) V^d \left( \frac{\lambda B}{1 + \pi'}, s' \right) \right] \]

- \( 0 \leq \lambda \leq 1 \): recovery rate,
- \( 0 \leq \theta \leq 1 \): probability of regaining access to credit, and

\[ \phi^d(y) = \max \left\{ 0, \frac{d_1}{d_0} y + \left( d_1 - \frac{d_1}{d_0} \right) y^2 \right\} \]

quadratic cost of default where

- default cost at mean is \( \phi^d(1) = d_1 \)
- default costs matter when \( \phi^d(y) > 0 \), when \( y < 1 + d_0 \)
Value of repayment

- The value, conditional on not defaulting, is given by

\[ V^c(B, s) = \max_{B'} \left\{ u_g(\tau y - q(B, s, B')B' + B) \right\} \]

\[ + \beta_g E_{s'|s} \left[ V^o \left( \frac{B'}{1 + \pi'}, s' \right) \right] \]

where \( q(B, s, B') \) is the bond price

- Real interest rate is stochastic (even w/o default)

- In bad times, countercyclical inflation \( \sim \) substitute to default
In this environment, the bond price satisfies

\[
q(B, s, B') = \beta_\ell E_{s'|s} \left[ 1 - d^* \left( \frac{B'}{1+\pi'}, s' \right) u'_\ell (c') \right] \frac{1}{1 + \pi'} \frac{u'_\ell (c)}{u'_\ell (c)}
+ \beta_\ell E_{s'|s} \left[ d^* \left( \frac{B'}{1+\pi'}, s' \right) q^d \left( \frac{B'}{1+\pi'}, s' \right) \frac{u'_\ell ((1 - \tau)(y - \phi^d(y')))}{u'_\ell (c)} \right]
\]

where \(q^d\) is the price of a bond in default.
Cyclicality of inflation and borrowing costs

- When $\lambda = 0$, the bond price can be written as

$$ q(B, s, B') = \beta E_{s'|s} \left[ \frac{1}{1 + \pi'} \right] E_{s'|s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] E_{s'|s} \left[ \frac{u'_\ell(c')}{u'_\ell(c)} \right] $$

$$ + \beta E_{s'|s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right), \frac{u'_\ell(c')}{u'_\ell(c)} \right] E_{s'|s} \left[ \frac{1}{1 + \pi'} \right] $$

$$ + \beta \text{cov}_{s'|s} \left[ \frac{1}{1 + \pi'}, \frac{u'_\ell(c')}{u'_\ell(c)} \right] E_{s'|s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] $$

$$ + \beta \text{cov}_{s'|s} \left[ \frac{1}{1 + \pi'}, 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] E_{s'|s} \left[ \frac{u'_\ell(c')}{u'_\ell(c)} \right] $$

- Default and inflation increase borrowing costs
Cyclicality of inflation and borrowing costs

When $\lambda = 0$, the bond price can be written as

$$q(B, s, B') = \beta \ell E_{s' | s} \left[ \frac{1}{1 + \pi'} \right] E_{s' | s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] E_{s' | s} \left[ \frac{u'_\ell(c')}{u'_\ell(c)} \right]$$

$$+ \beta \ell \text{cov}_{s' | s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right), \frac{u'_\ell(c')}{u'_\ell(c)} \right] E_{s' | s} \left[ \frac{1}{1 + \pi'} \right]$$

$$+ \beta \ell \text{cov}_{s' | s} \left[ \frac{1}{1 + \pi'}, \frac{u'_\ell(c')}{u'_\ell(c)} \right] E_{s' | s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right]$$

$$+ \beta \ell \text{cov}_{s' | s} \left[ \frac{1}{1 + \pi'}, 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] E_{s' | s} \left[ \frac{u'_\ell(c')}{u'_\ell(c)} \right]$$

Default and inflation increase borrowing costs

Countercyclical default *increases* borrowing costs
Cyclicality of inflation and borrowing costs

- When $\lambda = 0$, the bond price can be written as

$$q(B, s, B') = \beta_\ell E_{s' | s} \left[ \frac{1}{1 + \pi'} \right] E_{s' | s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] E_{s' | s} \left[ \frac{u'_\ell(c')}{u'_\ell(c)} \right]$$

$$+ \beta_\ell \text{cov}_{s' | s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right), \frac{u'_\ell(c')}{u'_\ell(c)} \right] E_{s' | s} \left[ \frac{1}{1 + \pi'} \right]$$

$$+ \beta_\ell \text{cov}_{s' | s} \left[ \frac{1}{1 + \pi'}, \frac{u'_\ell(c')}{u'_\ell(c)} \right] E_{s' | s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right]$$

$$+ \beta_\ell \text{cov}_{s' | s} \left[ \frac{1}{1 + \pi'}, 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] E_{s' | s} \left[ \frac{u'_\ell(c')}{u'_\ell(c)} \right]$$

- Default and inflation increase borrowing costs
- Countercyclical default increases borrowing costs
- Pro-cyclical inflation reduces borrowing costs
Quantitative experiment

- Calibrate model with zero covariance to match spreads and conditional default probabilities in advanced economies
- Assess impact of different inflation processes on interest rates, debt dynamics, and crises
Functional forms

- Preferences

\[ u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i} \quad \text{for } i = g, \ell \]

- Stochastic Process

\[
\begin{bmatrix}
\log y' \\
\pi'
\end{bmatrix} =
\begin{bmatrix}
\rho_y & \rho_{\pi,y} \\
\rho_{y,\pi} & \rho_{\pi}
\end{bmatrix}
\begin{bmatrix}
\log y \\
\pi
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_y \\
\varepsilon_{\pi}
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
\varepsilon_y \\
\varepsilon_{\pi}
\end{bmatrix} = N\left(\begin{bmatrix}0 \\ 0\end{bmatrix}, \begin{bmatrix}\sigma_y^2 & \sigma_{\pi,y} \\
\sigma_{\pi,y} & \sigma_{\pi}^2\end{bmatrix}\right)
\]
## Calibration – baseline with acyclical inflation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Joint targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov’t discount factor $\beta_g$</td>
<td>0.763</td>
<td>default prob. in good times: 1.02%*</td>
</tr>
<tr>
<td>Default cost cutoff $d_0$</td>
<td>−0.037</td>
<td>average spread: 0.74%**</td>
</tr>
<tr>
<td>Default cost at mean $d_1$</td>
<td>0.040</td>
<td>default prob. in bad times: 2.58%*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>VAR estimates – OECD cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence $\rho_y, \rho_\pi$</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Spillovers $\rho_\pi,y, \rho_y,\pi$</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Volatility $\sigma_y, \sigma_\pi$</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Covariance of innovations $\sigma_{\pi,y}$</td>
<td>0.00</td>
<td>acyclical baseline ±0.255e-4 (1.5 s.d.)</td>
</tr>
<tr>
<td>Lender discount factor $\beta_\ell$</td>
<td>0.99</td>
<td>risk-free rate: 1 percent</td>
</tr>
<tr>
<td>Gov’t risk aversion $\gamma_g$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Lender risk aversion $\gamma_\ell$</td>
<td>8</td>
<td>Storesletten, Telmer, Yaron (2007)</td>
</tr>
<tr>
<td>Probability of re-entry $\theta$</td>
<td>0.10</td>
<td>average exclusion: 10 quarters†</td>
</tr>
<tr>
<td>Recovery parameter $\lambda$</td>
<td>0.96</td>
<td>recovery rate: 50%†</td>
</tr>
<tr>
<td>Tax rate $\tau$</td>
<td>0.19</td>
<td>OECD gov’t consumption share</td>
</tr>
</tbody>
</table>

*: CDS-implied default probabilities 2001-2015 (threshold: 1 s.d. below trend consumption), **: Eurozone rates (in sample) less German rates 2001-2015, †: Richmond and Dias (2008), ‡: Benjamin and Wright (2009)
Results

- The pro-cyclical inflation regime has
  - lower borrowing costs
  - despite more default crises
  - similar debt levels

<table>
<thead>
<tr>
<th></th>
<th>Positive co-movement (+1.5 s.d.)</th>
<th>Negative co-movement (−1.5 s.d.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default prob. (percent)</td>
<td>1.31</td>
<td>1.14</td>
</tr>
<tr>
<td>Spreads (percent)</td>
<td>0.67</td>
<td>0.71</td>
</tr>
<tr>
<td>Debt (percent borrower income)</td>
<td>5.55</td>
<td>5.57</td>
</tr>
</tbody>
</table>

decomposition of bond price
Procyclicality not always good

- The pro-cyclical inflation economy has
  - lower borrowing costs during good times
  - higher borrowing costs during bad times, when output is more than 1 s.d. below mean
  - driven by larger increase in default probability

<table>
<thead>
<tr>
<th></th>
<th>Positive co-movement (+1.5 s.d.)</th>
<th>Negative co-movement (−1.5 s.d.)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spreads overall (percent)</td>
<td>0.67</td>
<td>0.72</td>
<td>−0.04</td>
</tr>
<tr>
<td>Spreads in good times (pct)</td>
<td>0.54</td>
<td>0.60</td>
<td>−0.06</td>
</tr>
<tr>
<td>Spreads in bad times (pct)</td>
<td>1.52</td>
<td>1.40</td>
<td>+0.12</td>
</tr>
<tr>
<td>Def. prob. in good times (pct)</td>
<td>1.05</td>
<td>0.94</td>
<td>+0.11</td>
</tr>
<tr>
<td>Def. prob. in bad times (pct)</td>
<td>2.81</td>
<td>2.33</td>
<td>+0.48</td>
</tr>
</tbody>
</table>
Higher lender risk aversion robustness ($\gamma_l = 16$)

- Higher risk aversion shapes both hedging motives and default
  - Stronger discount overall
  - But also fewer defaults

<table>
<thead>
<tr>
<th></th>
<th>Positive co-movement (+1.5 s.d.)</th>
<th>Negative co-movement (−1.5 s.d.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default prob. (percent)</td>
<td>0.89</td>
<td>1.15</td>
</tr>
<tr>
<td>Spreads (percent)</td>
<td>0.62</td>
<td>1.11</td>
</tr>
<tr>
<td>Debt (percent borrower income)</td>
<td>2.48</td>
<td>2.42</td>
</tr>
</tbody>
</table>

decomposition of bond price
Is procyclicality always good then?

- Government typically prefers countercyclicality
- While lenders prefers procyclicality
- Preferences strongly diverge in bad states

**Figure:** Welfare comparison of cyclicity regimes across states
Conclusion

- In good times, the procyclical economy enjoys lower real rates and accumulates more debt

- In bad times, the risk of default increases more for the procyclical economy which leads to higher real rates

- For monetary unions, recessions increase the contrast over monetary policy

- Provides an alternative explanation of the secular decline in real rates
Appendix
Domestic share of government debt is high

<table>
<thead>
<tr>
<th>Country</th>
<th>2004</th>
<th>2008</th>
<th>2012</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>83.3</td>
<td>85.6</td>
<td>61.9</td>
<td>76.9</td>
</tr>
<tr>
<td>Belgium</td>
<td>50.7</td>
<td>41.0</td>
<td>58.9</td>
<td>50.2</td>
</tr>
<tr>
<td>Canada</td>
<td>77.6</td>
<td>83.8</td>
<td>72.1</td>
<td>77.8</td>
</tr>
<tr>
<td>Denmark</td>
<td>74.5</td>
<td>75.2</td>
<td>70.9</td>
<td>73.5</td>
</tr>
<tr>
<td>Finland</td>
<td>23.1</td>
<td>38.1</td>
<td>25.9</td>
<td>29.0</td>
</tr>
<tr>
<td>France</td>
<td>57.9</td>
<td>57.8</td>
<td>51.5</td>
<td>55.7</td>
</tr>
<tr>
<td>Germany</td>
<td>68.6</td>
<td>53.5</td>
<td>41.4</td>
<td>54.5</td>
</tr>
<tr>
<td>Italy</td>
<td>59.9</td>
<td>60.9</td>
<td>66.1</td>
<td>62.3</td>
</tr>
<tr>
<td>Japan</td>
<td>95.7</td>
<td>91.9</td>
<td>92.1</td>
<td>93.3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>44.4</td>
<td>45.2</td>
<td>55.8</td>
<td>48.5</td>
</tr>
<tr>
<td>Norway</td>
<td>43.5</td>
<td>50.6</td>
<td>71.5</td>
<td>55.2</td>
</tr>
<tr>
<td>Portugal</td>
<td>24.0</td>
<td>27.3</td>
<td>35.9</td>
<td>29.0</td>
</tr>
<tr>
<td>Spain</td>
<td>55.7</td>
<td>62.6</td>
<td>78.1</td>
<td>65.5</td>
</tr>
<tr>
<td>Sweden</td>
<td>64.4</td>
<td>75.5</td>
<td>61.4</td>
<td>67.1</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>81.9</td>
<td>78.1</td>
<td>72.4</td>
<td>77.5</td>
</tr>
<tr>
<td>United States</td>
<td>80.8</td>
<td>78.0</td>
<td>73.3</td>
<td>77.3</td>
</tr>
<tr>
<td>Mean</td>
<td>61.6</td>
<td>62.8</td>
<td>61.8</td>
<td>62.1</td>
</tr>
</tbody>
</table>

Sources: BIS, Haver
Conditional correlation between inflation and consumption growth
The price of a bond in default satisfies

\[
q^d(B, s) = \beta_\ell \lambda \theta \mathbb{E}_{s'|s} \left[ \frac{1 - d^* \left( \frac{\lambda B}{1+\pi'}, s' \right)}{1 + \pi'} \frac{u'_\ell(c')}{u'_\ell(c_{def})} \right] \\
+ \beta_\ell \lambda \mathbb{E}_{s'|s} \left[ \frac{1 - \theta + \theta d^* \left( \frac{B'}{1+\pi'}, s' \right)}{1 + \pi'} q^d \left( \frac{\lambda B}{1+\pi'}, s' \right) \frac{u'_\ell(c'_{def})}{u'_\ell(c_{def})} \right]
\]
We measure spread as the real rate minus the risk-free rate:

\[ \text{spread}_t = \left( \frac{1}{q_{t+1} E_t [1 + \pi_{t+1}]} \right)^4 - \left( \frac{1}{q_{t+1}^{RF}} \right)^4 \]

where

\[ q_{t+1}^{RF} = \beta_\ell E_t \left[ \frac{u'_\ell (c_{t+1})}{u'_\ell (c_t)} \right] \]
The pro-cyclical inflation economy has lower borrowing costs

Driven by **procyclicality** of inflation, despite higher default

<table>
<thead>
<tr>
<th></th>
<th>Positive co-movement (+1.5 s.d.)</th>
<th>Negative co-movement (−1.5 s.d.)</th>
<th>difference (annual bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>price 100 × q</td>
<td>99.09</td>
<td>99.07</td>
<td>+10</td>
</tr>
<tr>
<td>no default 100 × (E[1 − d])</td>
<td>99.66</td>
<td>99.71</td>
<td>−17</td>
</tr>
<tr>
<td>(cov(irms, defl.) \ E[1 − d])</td>
<td>0.012</td>
<td>-0.016</td>
<td>+11</td>
</tr>
</tbody>
</table>

11 basis points accounts for 12 percent of the difference in data
The pro-cyclical inflation economy has lower borrowing costs

Driven by *procyclicality* of inflation and lower default

<table>
<thead>
<tr>
<th></th>
<th>Positive co-movement (+1.5 s.d.)</th>
<th>Negative co-movement (−1.5 s.d.)</th>
<th>difference (annual bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>price $100 \times q$</td>
<td>99.78</td>
<td>99.68</td>
<td>+40</td>
</tr>
<tr>
<td>no default $100 \times E[1-d]$</td>
<td>99.77</td>
<td>99.70</td>
<td>+29</td>
</tr>
<tr>
<td>cov (irms, defl.) $E[1-d]$</td>
<td>0.029</td>
<td>-0.033</td>
<td>+25</td>
</tr>
</tbody>
</table>

40 basis points accounts for [27] percent of the difference in data
Robust to alternative yield measures

<table>
<thead>
<tr>
<th>Yield source</th>
<th>Real sovereign yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IFS</td>
</tr>
<tr>
<td>Inflation consumption</td>
<td>-1.804**</td>
</tr>
<tr>
<td>covariance</td>
<td>(.636)</td>
</tr>
<tr>
<td>other controls</td>
<td>Yes</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.903</td>
</tr>
<tr>
<td>$N$</td>
<td>1726</td>
</tr>
</tbody>
</table>

Countries: AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JPN, KOR, NLD, NOR, PRT, SWE, USA.

Standard errors clustered by country. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time fixed effects.
Robust to alternative debt measures

<table>
<thead>
<tr>
<th>Debt source</th>
<th>Real sovereign yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Oxford &amp; OECD</td>
</tr>
<tr>
<td></td>
<td>(2) OECD</td>
</tr>
<tr>
<td></td>
<td>(3) Oxford</td>
</tr>
<tr>
<td></td>
<td>(4) OECD &amp; Oxford</td>
</tr>
<tr>
<td>Inflation consumption covariance</td>
<td>-1.804**</td>
</tr>
<tr>
<td></td>
<td>(.636) (1.594) (0.557) (0.640)</td>
</tr>
<tr>
<td>other controls</td>
<td>Yes</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.903</td>
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<tr>
<td>$N$</td>
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Countries: AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JPN, KOR, NLD, NOR, PRT, SWE, USA.

Standard errors clustered by country. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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