The Lost Generation of the Great Recession[☆]

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Abstract

This paper analyzes the effects of the Great Recession on different generations. While older generations suffered the largest decline in wealth due to the collapse in asset prices, younger generations suffered the largest decline in labor income. Potentially, some households may have benefited from the purchase of cheaper assets. To analyze the impact of these channels, I construct an overlapping-generations model with borrowing constraints in which households choose a portfolio of risky and risk-free assets. In response to shocks to labor income and asset markets resembling the Great Recession, young risky asset holders suffer the largest welfare losses, equivalent to a 33 percent reduction in one-period consumption.

Keywords: Great Recession, heterogeneous agents, overlapping generations, portfolio choice, borrowing constraints

JEL: D31, D91, E21, E32, G11

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1. Introduction

The Great Recession of 2007–2009 was the largest contraction in the United States since the Great Depression. However, the Recession did not impact all households equally, as documented in this paper and in Glover et al. (2011). On the one hand, older generations suffered the largest absolute decline in wealth due to the collapse in asset prices. On the other hand, younger generations suffered the largest decline in labor income. Potentially, some households who purchased cheaper assets during the Recession may have benefited in the long run. This paper provides a quantitative evaluation of the welfare effects of the changes in the labor and asset markets that took place during the Recession.

I construct an overlapping-generations model with borrowing constraints in which households choose a portfolio of risk-free and risky assets. Households are heterogeneous in portfolio, income, and wealth, both across and within age cohorts. The calibrated model fits the data well along the relevant dimensions, such as the age profiles of wealth, risky assets, risky asset participation, and household leverage (debt-to-risky-asset ratios), as well as the distributions of wealth and household leverage.

Several features of the Great Recession such as the decline in labor income, the decline in risky asset prices, the increase in uncertainty regarding the return on risky assets, and the decline in interest rates are introduced into the calibrated model exogenously. The welfare effects vary widely along two dimensions: age and risky asset participation prior to the Recession. On the age dimension, households between the ages of 30 and 38 suffered the largest welfare losses, equivalent to a 24 percent reduction in one-period consumption, while households between the ages of 84 and 92 suffered the smallest welfare losses, equivalent to an 11 percent decline in consumption.

On the risky asset participation dimension, households who owned risky assets prior to the Recession suffered significantly larger welfare losses, equivalent to a 23 percent reduction in consumption, compared to the 8 percent welfare loss for households who did not own risky assets. When jointly considering these factors, young risky asset participants (ages 21–29) suffered the largest welfare losses, equivalent to a 33 percent decline in consumption. This is because young households experience the largest decline in labor income and the largest relative decline in wealth since they are more leveraged than older households, as in the data. By contrast, pre-retirement households (ages 57–65) who did not own risky assets had the largest welfare gains, equivalent to a 6 percent increase in consumption. This is due to two reasons. The first is that, compared to older nonparticipants, pre-retirement households are still accumulating assets for retirement, and therefore are more likely to purchase risky assets. The second is that, compared to younger nonparticipants, they have more wealth, and thus are in a better position to take advantage of cheaper asset prices.

This paper is closely related to Glover et al. (2011), who also study the distributional consequences of the Great Recession, but find that older generations suffer the largest decline in welfare, while younger generations are close to welfare neutral, because of their ability to take advantage of depressed asset prices. This paper departs from Glover et al. (2011) along three dimensions. First, it considers borrowing constraints that may limit the extent to which households can benefit from cheaper asset prices. Second, it considers within-age heterogeneity. This allows further decomposing the welfare losses along other dimensions, such as risky asset participation and household leverage, in addition to age. Third, the features of the Great Recession are introduced into the model as an unanticipated shock. This is due to two reasons: the first is computational tractability and the second is that one can argue that the magnitude of the Recession was indeed unanticipated, given the focus on the Great Moderation by academics and policy-makers in the lead-up to the Recession.[1]

This paper is related to the literature on the welfare consequences of asset price declines, including Li and Yao (2007) who use a life-cycle model to show that housing price declines benefit young households, and Kiyotaki et al. (2011) who find a similar result if the housing price decline is driven by productivity shocks and not by interest rate shocks. It is also related to other studies that consider the distributional consequences of the Great Recession, such as Bell and Blanchflower (2011) and Elsby et al. (2010) who focus on labor outcomes, emphasizing the high unemployment among the young generation, Peterman and Sommer (2014) and Krueger et al. (2016b) who consider the role of Social Security and Unemployment insurance, respectively, in mitigating the adverse effects of the Great Recession, and Menno

[1]See, for example, Arias et al. (2007), Bernanke (2004), Blanchard and Simon (2001), Galí and Gambetti (2009), and Stock and Watson (2003).

and Oliviero (2014) who find that the collapse in housing prices during the Great Recession resulted in a large welfare loss for borrowers and a small welfare gain for savers.

This paper is structured as follows. Section 2 documents the changes in the labor and asset markets during the Great Recession which are used to model the Recession in the quantitative analysis. Section 3 presents a model economy that is used to formally analyze the lifetime welfare implications of the Recession. The calibration strategy and model fitness are discussed in Section 4. Section 5 presents the quantitative results and the welfare implications of the Great Recession. Section 6 concludes.

2. Empirical Analysis

This section documents the changes in the labor and asset markets over the Great Recession that are introduced into the model in subsequent sections. The statistics documenting changes in disposable labor income are computed using the Current Population Survey (CPS) March supplements (2008 and 2011). The changes in earnings risk are computed using the Panel Study of Income Dynamics (PSID). The changes in asset prices, expected asset volatility, and interest rates are downloaded from the Federal Reserve Economic Data (FRED).

2.1. Disposable Labor Income

In this section, I document the changes in aggregate disposable labor income by age during the Great Recession. Disposable labor income is defined as the sum of wages, salaries, and two-thirds of self-employment income plus transfers minus tax liabilities.[2] Table 1 reports the percent changes in real disposable labor income, linearly detrended at 2 percent per year, from 2007 to 2010. Young households, defined as households whose head is between the ages of 21 and 29, suffered the largest decline in disposable labor income during the Great Recession, followed by households aged 30–38.

[2]Since the model abstracts from taxes, transfers, and unemployment, disposable labor income is the relevant variable of interest. Tax liabilities are computed for each household using TAXSIM (v9).

Table 1: Changes in real disposable labor income.

age	real disposable labor income (percent change from 2007 to 2010)
21–29	−12.0
30–38	−11.0
39–47	−8.0
48–56	−10.1
57–65	−5.6
21–65	−8.7

Disposable labor income (sum of wages, salaries, and two-thirds of self-employment income plus transfers minus tax liabilities) has been adjusted for inflation and detrended by 2 percent per year.

Source: CPS March Supplements (2008, 2011).

2.2. Earnings Risk

The Great Recession also featured an increase in downward earnings risk, defined as the share of households that experience a reduction in disposable labor income (after controlling for quadratic age effects) that exceeds 25 percent in a 4-year window. Specifically, the last row of Table 2 shows that downward earnings risk increased by 4.1 percentage points from an average of 20.9 percent in 1996–2006 to 25.0 percent in 2006–2010. Table 2 also shows that all age groups experienced an increase in downward earnings risk, with the percentage point increases ranging from 3.1 to 5.2 percent.[3]

Similarly, there was a decrease in upward earnings risk, defined as the share of households that experience an increase in disposable labor income (after controlling for quadratic age effects) that exceeds 25 percent in a 4-year window. The last row of Table 3 shows that upward earnings risk decreased by 5.1 percentage points from an average of 24.0 percent in

[3] Note that I report 4-year income changes for households aged 21–61, instead of 3-year changes for households aged 21–65 as in the previous subsection. This is because the PSID is available every two years, and households aged 62–65 are dropped since most of them will have retired by the end of the 4-year window. In Tables 14 and 15 (Appendix D), I also report similar findings for 2-year changes.

Table 2: Increase in downward earnings risk for 4-year changes.

age	downward earnings risk (percent)		
	1996–2006	2006–2010	difference
21–29	21.6	25.7	4.1
30–38	24.6	27.7	3.1
39–47	20.3	24.6	4.3
48–61	17.0	22.2	5.2
21–61	20.9	25.0	4.1

Downward earnings risk is defined as the share of households that experience a reduction in disposable labor income (after controlling for quadratic age effects) that exceeds 25 percent in a 4-year window. The column for 1996–2006 reports the average for multiple 4-year windows.
Source: PSID.

1996–2006 to 18.9 percent in 2006–2010. All age groups experienced this decrease in upward earnings risk, with the changes ranging from −8.9 percent to −3.3 percent.

These facts are consistent with Guvenen et al. (2014), who document that large upward income movements become less likely, while large drops in income become more likely during recessions.

2.3. Risky Asset Market

The Great Recession featured a large decline in risky asset prices. Risky assets are defined as risky financial assets (including stocks and business assets) plus risky nonfinancial assets (including housing and other real estate). I construct a risky asset price index, which is a weighted average of housing prices, as measured by the Case-Shiller national home price index and adjusted for inflation, and stock prices, as measured by the Wilshire 5000 Index and adjusted for inflation, using the 55 percent risky nonfinancial asset portfolio share of 2004 as the weight on housing prices. Figure 1 shows that the risky asset price index declined by 23.6 percent from 2007 to 2010 and returned to its 2007 level by 2014.

The Recession also featured a sharp increase in uncertainty regarding risky asset prices. Figure 2 shows that the VIX index, which reflects expected future volatility of the S&P

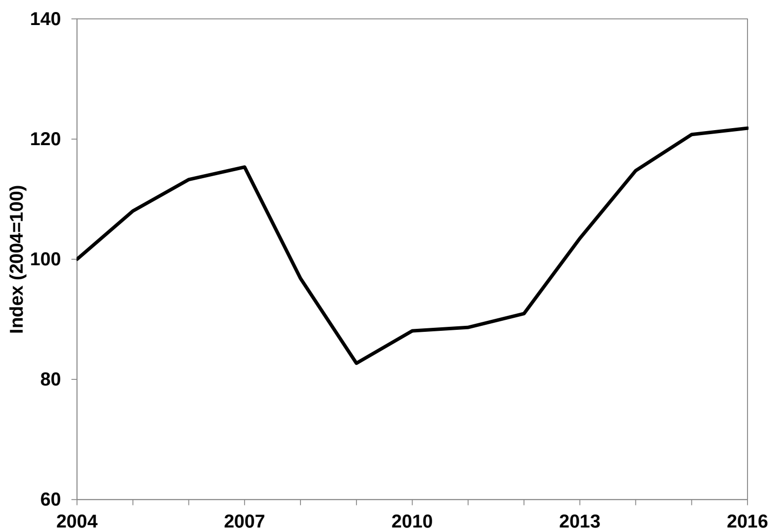
Table 3: Decrease in upward earnings risk for 4-year changes.

age	upward earnings risk (percent)		
	1996–2006	2006–2010	difference
21–29	21.7	18.4	−3.3
30–38	22.0	18.1	−3.9
39–47	24.3	20.0	−4.4
48–61	28.3	19.4	−8.9
21–61	24.0	18.9	−5.1

Upward earnings risk is defined as the share of households that experience an increase in disposable labor income (after controlling for quadratic age effects) that exceeds 25 percent in a 4-year window. The column for 1996–2006 reports the average for multiple 4-year windows.

Source: PSID.

Figure 1: Risky asset price index.

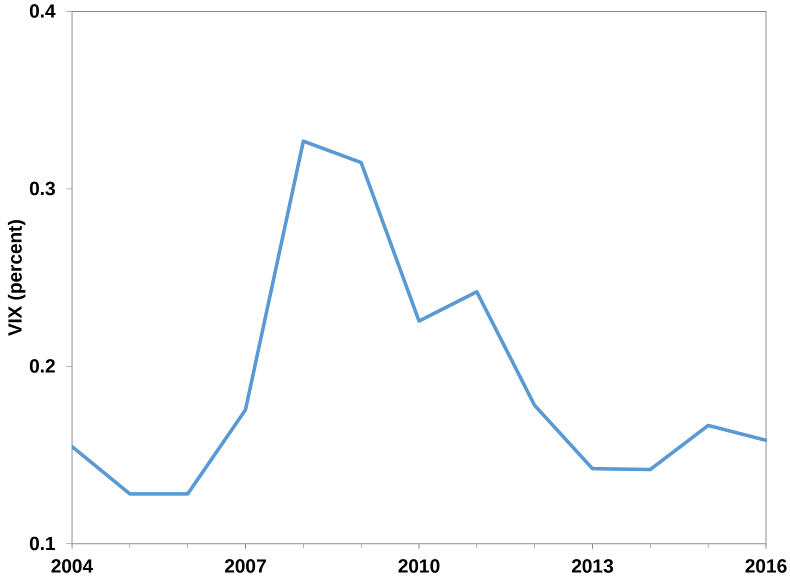


The risky asset price index is a weighted average of housing prices (Case-Shiller national home price index), adjusted for inflation, and stock prices (Wilshire 5000 Full Cap Price Index), adjusted for inflation, using the 55 percent risky nonfinancial asset portfolio share of 2004 as the weight on housing prices.

Sources: FRED, SCF 2004.

500 Index and is widely used as a measure of uncertainty (see, for example, Bloom 2009), increased by 28.6 percent from 2007 to 2010 and returned to its 2007 level by 2013.

Figure 2: Stock market volatility index.



VIX measures market expectation of near term volatility conveyed by stock index option prices.

Source: FRED.

2.4. Real Interest Rates

Real interest rates on savings, measured as the nominal rates on 10-year treasuries minus CPI inflation, declined by 73 basis points from 2007 to 2010, as can be seen in Table 4. While real interest rates on secured debt (nominal rates on a 15-year mortgage minus CPI inflation) declined by 125 basis points from 2007 to 2010, the real interest rate on unsecured debt (nominal rates on credit card debt minus CPI inflation) actually increased by 116 basis points over the same period.

Table 4: Real interest rates.

asset	real interest rate (percent)			
	2004	2007	2010	2013
10-year treasury	0.90	0.82	0.08	0.74
15-year mortgage	1.84	2.22	0.96	1.49
credit card	9.35	9.49	10.64	10.30

Real interest rates are nominal interest rates on each asset minus yearly CPI inflation.

Source: FRED.

In sum, the Great Recession featured large changes in the labor and asset markets. In the labor market, there was a large decline in disposable labor income, particularly for young households, an increase in downward earnings risk, and a decrease in upward earnings risk. In the asset market, there was a large decline in asset prices, an increase in uncertainty (expected volatility) regarding risky asset prices, a decline in the real interest rate on savings and secured debt, and an increase in the real interest rate on unsecured debt. The next section presents a model that provides a framework to evaluate the welfare consequences of these changes.

3. Model

This section presents a model economy that is used to quantitatively analyze the lifetime welfare implications of the Recession. The setting is similar to those in recent works that use calibrated life-cycle heterogeneous-agent models with borrowing constraints (see, for example, Conesa et al. 2009, Favilukis et al. 2016, Fernandez-Villaverde and Krueger 2011, Heathcote et al. 2010, and Iacoviello and Pavan 2013). The analysis is done in partial equilibrium, where all prices are set exogenously. The economy is inhabited by overlapping generations of finitely lived households. Households face borrowing constraints and choose portfolios of risky and risk-free assets. There are idiosyncratic shocks to labor income and returns to the risky asset that generate heterogeneity in wealth and portfolio holdings. This heterogeneity is crucial: not all old households have large risky asset holdings, and not all young households are credit constrained. I now describe the environment in more detail.

3.1. Households

There is a continuum of finitely lived households indexed by i. Households of age $j \in J \equiv \{1, 2, .., \overline{J}\}$ face conditional survival probabilities given by $\{\psi_j\}$. Newborns are endowed with initial wealth, $\{\omega_i\}$, which is exogenous and time invariant. The aggregate measure of households is normalized to one. Preferences are given by

$$\mathbf{E}\left[\sum_{j\in J}\beta^{j-1}\left(\prod_{a=1}^{j-1}\psi_a\right)u_j(c_{ij})\right]$$

where c_{ij} is consumption at age j and β is the time discount factor. Note that the period utility function u_j depends on age. This is intended to capture the change in the consumption needs of different household sizes along the life cycle.[4] Changes in household size are exogenously given. I assume that $u_j(c_{ij}) = u(c_{ij}/e_j)$ where e_j is the number of adult equivalents in age j households, discussed in more detail in Section 4.1.

3.2. Labor Income

The model abstracts from taxes and transfers, with the exception of retirement transfers. Thus, labor income in the model corresponds to disposable labor income in the data. Household endowments of labor income have two determinants: a deterministic age-specific component $\{\eta_j\}$ and an idiosyncratic component $z \in Z$, which follows a Markov process with transition matrix Γ. The age-specific component η_j captures the labor income profile of households over the life cycle, while the idiosyncratic component z captures the heterogeneity of labor income within age cohorts and the risky nature of labor income over time. There is mandatory retirement at age R, after which households receive retirement benefits of $s(k)$, which depend on past earnings k. Thus, household i of age j with shock z and past earnings k earns:

$$y_j(z,k) = \begin{cases} e^z \eta_j & \text{if } j < R \\ s(k) & \text{otherwise.} \end{cases}$$

3.3. Retirement Income

Retirement income is proportional to previous earnings, summarized by k. Consistent with the U.S. social security system and following Huggett and Parra (2010), marginal benefit rates are

$$\kappa_1 \text{ for } 0 < k \leq k_1$$
$$\kappa_2 \text{ for } k_1 < k \leq k_2$$
$$\kappa_3 \text{ for } k_2 < k \leq \bar{k}$$

[4]See Bick and Choi (2013) for a discussion on modeling household size and economies of scale within households.

where k_1 and k_2 are bend points, and $\bar{k}$ is the maximum earnings that count toward retirement income.

Following Kitao (2014) and Peterman and Sommer (2014), previous earnings are summarized by

$$k'(j,z,k) = \begin{cases} \dfrac{\min\{g(e^z\eta_j)), \bar{k}\} + (j-1)k}{j} & \text{if } j \leq j^* \\ \max\left\{k, \dfrac{\min\{g(e^z\eta_j), \bar{k}\} + (j-1)k}{j}\right\} & \text{if } j^* < j < R \\ k & \text{if } j \geq R \end{cases} \tag{1}$$

where g transforms disposable to gross labor income, and notice that k' does not fall below the average of the first j^* working periods.[5]

3.4. Portfolio Choice

Households can choose a portfolio that consists of a risky asset and a risk-free one. The risky asset, denoted by x, is subject to an idiosyncratic shock e^ξ with $\xi \in \Xi$ and probability $\pi(\xi)$.[6] Holding this asset requires a participation cost of f, which is intended to capture the limited participation in the risky asset market observed in the data, and is a reduced-form way of modeling monitoring costs, transaction fees, etc.[7] The price of the risky asset is given by p_x.

Households also have access to a risk-free asset b, which yields an exogenously given interest rate r_s if $b \geq 0$. Additionally, households pay *marginal* interest rates of r_b on debt not exceeding $\lambda p_x x$ and r_h/ψ_j on the portion of debt that exceeds $\lambda p_x x$. Here, λ denotes the

[5] This captures the spirit of social security, where benefits are computed based on the average of the 35 highest-earning years, while keeping the model computationally tractable.

[6] Mendoza et al. (2009) also use idiosyncratic shocks in modeling risky assets. It is worth noting that household investment in equity, especially in private equity, is highly concentrated, as documented by Moskowitz and Vissing-Jorgensen (2002). Non-diversification can be an important source of idiosyncratic returns, as argued by Angeletos (2007).

[7] See, for example, work by Attanasio and Paiella (2011), Khorunzhina (2013) and Vissing-Jorgensen (2002) that document the significance of participation costs in accounting for limited stock market participation.

fraction of the value of risky assets that can be collateralized. Unsecured debt that exceeds the collateral value is charged a higher interest rate, especially for households with a lower survival probability. Let $r(b, x, j)$ denote the *average* interest rate.

3.5. Household Problem

The problem of the household of age j with beginning-of-period wealth a, labor income shock z, and prior earnings k can be written recursively as:

$$
\begin{aligned}
v(j,a,z,k) = \quad & \max_{c,b',x'} \quad u_j(c) + \beta\psi_j \sum_{z'\in Z}\sum_{\xi'\in\Xi} \Gamma^j_{z,z'}\pi(\xi')v(j+1,a',z',k') \qquad (2)\\
& s.t. \quad c + p_x x' + b' \le y_j(z,k) - \mathbb{1}_{x'>0} f + a\\
& \qquad\quad a' = b'(1 + r(b',x',j)) + p_x x' e^{\xi'}\\
& \qquad\quad k' \text{ follows (1)}\\
& \qquad\quad c \ge 0,\ x' \ge 0
\end{aligned}
$$

where $v(\overline{J}+1,\cdot,\cdot,\cdot) = 0$, $\Gamma^j = \Gamma$ for $j < R-1$, and $\Gamma^j = I$ for $j \ge R$ where I is the identity matrix. The solution to this problem can be represented by policy functions for consumption $c(j,a,z,k)$, risky assets $x'(j,a,z,k)$, and risk-free assets $b'(j,a,z,k)$.

The formal definition of the equilibrium can be found in Appendix B.1.

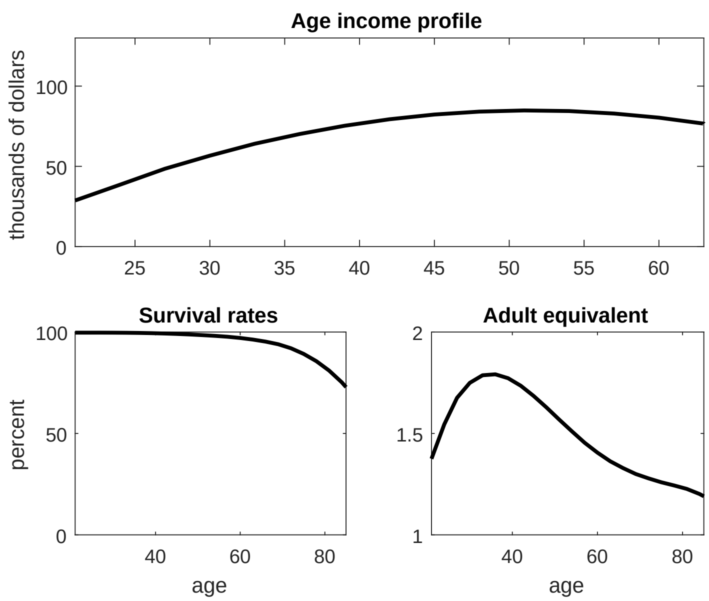
4. Calibration

This section explains the calibration of the model. The steady state equilibrium of the model is calibrated to the US economy in 2007, prior to the Great Recession. Section 4.1 discusses the parameters set outside the model, followed by parameters that require solving for equilibrium allocations in Section 4.2. Section 4.3 then shows that the calibrated model matches the data along important dimensions.

4.1. Parameters and Functional Forms

A period in the model is 3 years. Households enter the labor market at age 21 (model age $j = 1$), retire at age 66 ($R = 16$), and can live up to age 95 ($\overline{J} = 25$). Survival probabilities $\{\psi_j\}_{j\in J}$ are taken from the 2007 US Life Tables. Adult equivalent sizes $\{e_j\}_{j\in J}$ are calculated using household characteristics from the Survey of Consumer Finances (SCF 2007), obtained by first assigning a value of 1 to the household head, of 0.5 to each additional adult member, and of 0.3 to each child, and then fitting a fourth order polynomial in age.[8] The age-specific component of labor income $\{\eta_j\}_{j \in J}$ is obtained by fitting a quadratic in age to household disposable labor income from the SCF 2007. Note that since the model abstracts from taxes and transfers (excluding retirement transfers), the relevant data equivalent is disposable labor income. Figure 3 summarizes the key demographics and income parameters.

Figure 3: Life-cycle parameters.

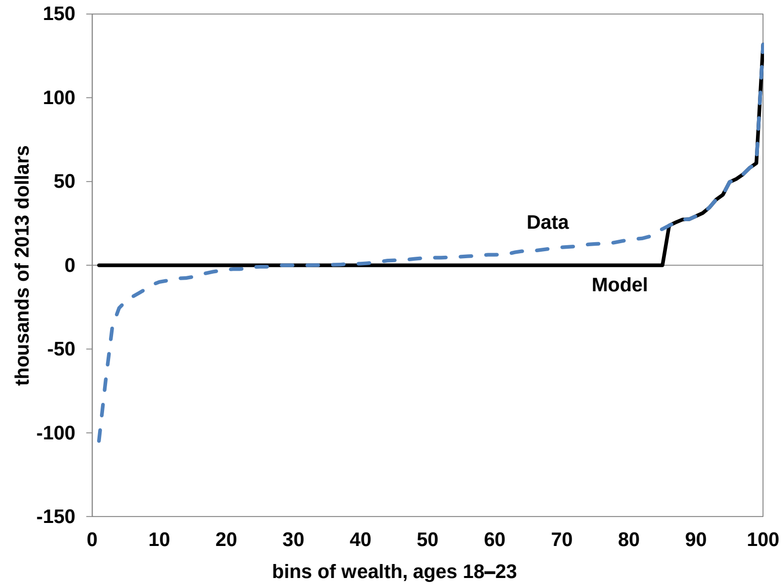


Sources: SCF 2007 and 2007 US Life Tables.

Initial wealth. The initial wealth endowments ω_i are set to resemble the wealth of households aged 18–23, calculated from the SCF 2007. However, to prevent agents in the model from starting with large negative wealth, I match only the top 15 percent, and set the rest to begin with zero wealth. Since the bottom 85 percent of the wealth distribution of households aged 18–23 have a cumulative net wealth that is close to zero, the total wealth endowment in the model equals the total wealth of households aged 18–23 in the data. Figure 4 depicts the net wealth of households aged 18–23 and the initial wealth endowments used in the model.

[8]This is the OECD-modified equivalence scale, proposed by Hagenaars et al. (1996).

Figure 4: Initial wealth endowments.



Source: SCF 2007.

Preferences. Preferences are given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

where σ is risk aversion. The baseline value for σ is set to 3, with sensitivity analyses reported in Section 5.4.

Labor income process. The process for labor income shocks is estimated using the Panel Survey of Income Dynamics (PSID). First, I compute three-year disposable labor earnings on the PSID 1970–1997 core sample of households (ages 21–65). Second, I obtain residuals from a model with age and time fixed effects. Finally, I estimate, using the generalized method of moments (GMM), the following model specification:

$$\begin{aligned} \log(y_t) &= z_t + \nu_t, \qquad && \nu_t \sim N(0, \sigma_\nu^2), \\ z_t &= \rho z_{t-1} + \varepsilon_t, && \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \end{aligned} \tag{3}$$

where y_t is residual three-year disposable labor income, ν_t is the transitory shock, and ε_t is the persistent shock. The estimation yields a persistence of $\rho = 0.8629$, persistent shock variance of $\sigma_\varepsilon^2 = 0.0929$, and a transitory shock variance of $\sigma_\nu^2 = 0.0001$. Details on the estimation can be found in Appendix A.

In the application to the model, I treat the transitory shock as measurement error and set $\sigma_\nu^2 = 0$. This process is approximated with an 11-state Markov process, using the Rouwenhorst procedure described in Kopecky and Suen (2010).

Retirement income. Retirement income depends on previous earnings k, which evolve according to equation (1). For converting disposable labor income y to gross labor income $g(y)$, which is used to calculate previous earnings in equation (1), the following expression,

$$y = \beta_0 + \beta_1 g(y),$$

is estimated using OLS from the SCF 2007. The estimated coefficients are $\beta_0 = 0.095$ and $\beta_1 = 0.717$, where average working age (21–65) disposable labor earnings has been normalized to 1. I set $j^* = 11$, the age after which average earnings cannot fall, to correspond to 35 working years.[9] The marginal replacement rates are

$$\begin{aligned}
&0.90 \text{ for } 0 < k \leq 0.27\\
&0.32 \text{ for } 0.27 < k \leq 1.65\\
&0.15 \text{ for } 1.65 < k \leq 3.10,
\end{aligned}$$

which, after adjusting for units, correspond exactly to the social security replacement rates and bend points documented by Huggett and Parra (2010).[10]

Asset market. The spread on secured debt $r_b - r_s$ is set to 1.3 percent, annualized, to match the average of the 15-year mortgage rate minus the 10-year treasury rate from 2001 to 2007. Similarly, the spread on unsecured debt $r_h - r_s$ is set to 8.7 percent, annualized, to match the credit card rate minus the 10-year treasury rate over the same period. The collateral constraint λ for secured debt is set to 0.8, which corresponds to a maximum loan-to-value

[9]This is a technique introduced by Kitao (2014) to capture the spirit of social security, where benefits are computed based on the average of the 35 highest-earning years, while keeping the model computationally tractable.

[10]Huggett and Parra (2010) report bend points of 0.21, 1.29, and 2.42 times average earnings. Average labor income is 1.28 times average disposable labor income in the SCF 2007.

ratio of 80 percent.[11] Given these values, for example, a household with a debt-to-value of 90 percent would pay 82 more basis points annually than the household with a debt-to-value less than 80 percent, not including the premium from death risk.[12]

The price of the risky asset, p_x, is normalized to 1. The risky asset shock ξ is drawn from a normal distribution with mean μ_ξ and variance σ_ξ^2. These values are described below.

4.2. Jointly Calibrated Parameters

The discount factor β, risky asset participation cost f, the savings interest rate r_s, and the mean and variance of the risky asset shock, μ_ξ and σ_ξ^2, are jointly calibrated to match five moments: household indebtedness, risky asset participation, the 90th-percentile-to-median wealth ratio, net safe assets, and the total value of risky assets.[13] Of particular importance is the indebtedness of households, since it disciplines the extent to which households are constrained. Using the Survey of Consumer Finances 2007, I find that the average leverage ratio, a measure of household indebtedness, is 23 percent.[14] I also find that 81 percent of all households hold positive amounts of risky assets, that the 90-to-median wealth ratio is 7.43, and that net safe assets and the total value of risky assets is 0.40 times and 8.78 times disposable labor income per working age household, respectively. The main parameters used in the model are summarized in Table 5.

[11] This is motivated by the fact that (a) private mortgage insurance is required on residential mortgages with debt-to-values exceeding 80 percent (Curtis 2016), making the effective interest rate higher; (b) commercial loans typically have higher interest rates and lower debt-to-value ratios (Folger 2017); (c) the IRS only allows loans of up to 50 percent of qualifying retirement account balances (IRS 2017); and (d) loans based on stocks can be costly due to higher interest rates in the case of stock-based loan programs (FINRA 2016) or large potential losses in the case of margin loans (Salisbury 2012) or securities-backed lines of credit (SEC 2015). The results using an alternatively calibrated value of λ are discussed in Section 5.4.

[12] The average interest rate for a household with a debt-to-value of 90 percent is $r_b + (r_h/\psi_j - r_b)/9$

[13] Risky assets are defined as risky financial assets (including stocks and business assets) plus risky non-financial assets (including housing and other real estate). Safe assets are defined as net wealth minus risky assets.

[14] Leverage is defined as minus safe assets divided by risky assets, bounded by 0 and 1, if risky assets are positive. If not, then leverage is 0 if safe assets are positive and 1 if safe assets are negative.

Table 5: Parameters and calibration.

Variables	Value	Target/Source
Discount factor $\beta^{\frac{1}{3}}$	0.990	average leverage[a]: 0.23
Risky shock variance σ_ξ^2	0.087	90-50 wealth ratio: 7.43
Participation cost f	0.045[b]	risky asset participation: 0.81
Risky return $E(e^\xi)^{\frac{1}{3}} - 1$	8.6%	risky assets: 8.78[b]
Interest rate $(1 + r_s)^{\frac{1}{3}} - 1$	2.8%	safe assets: 0.40[b]
Risk aversion σ	3	sensitivity in Section 5.4
Income persistence ρ_z	0.863	estimation
Persistent shock variance σ_ε^2	0.093	estimation
Collateral constraint λ	0.80	discussion in footnote 11
Secured debt spread, annual	1.3%	15-year mortgage rate[c]
Unsecured debt spread, annual	8.7%	credit card rate[c]
Number of cohorts J	25	ages 21–95 (3-year intervals)
Retirement R	16	ages 66–68
Adult equivalents $\{e_j\}$	Figure 3	SCF 2007
Initial wealth $\{\omega_i\}$	Figure 4	wealth, ages 18–23 (SCF 2007)
Endowment profile $\{\eta_j\}$	Figure 3	disp. labor income (SCF 2007)
Survival probabilities $\{\psi_j\}$	Figure 3	2007 US Life Tables
Retirement parameters		section 4.1

[a] : leverage $= \left[\frac{-\text{safe}}{\max\{0,\text{risky}\}}\right]_0^1$, [b] : in units of disposable labor income per 21–65, [c] : less 10-year treasury rate (2001–2007).

4.3. Steady State

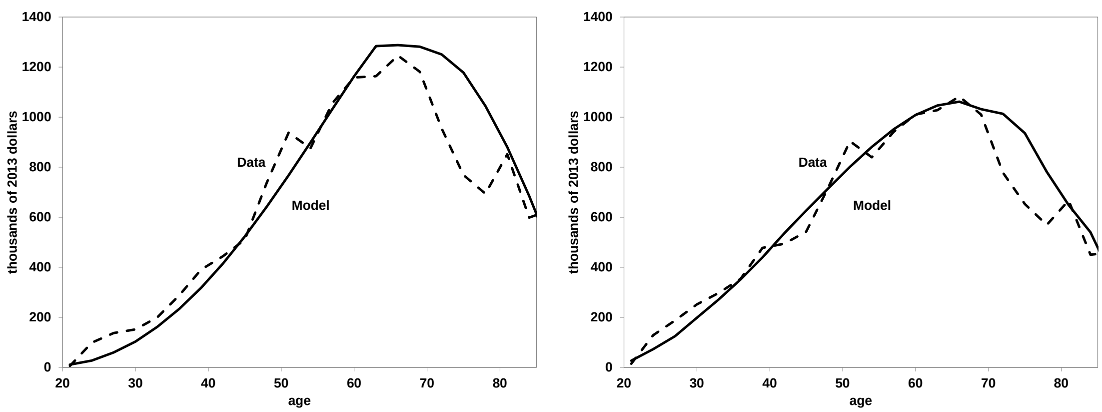
Before moving on to the quantitative analysis of the Great Recession, it is important to verify that the calibrated steady state equilibrium is consistent with the relevant dimensions of the data. Indeed, the calibrated model generates age profiles of wealth, risky assets, risky asset participation, and leverage that fit the data very well, as well as distributions of wealth and household leverage that match the data reasonably well.

Panels (a) and (b) of Figure 5 show that the age profiles of wealth and risky assets generated by the model closely resemble those in the data (SCF 2007). Panels (a) and

(b) of Figure 6 show that the age profiles of risky asset participation and leverage in the model are also quite close to those in the data (SCF 2007). Notice, though, that the model underpredicts risky asset participation for younger households and overpredicts that of older households. As will be seen in the next section, risky asset participation exacerbates the welfare losses suffered due to the Great Recession.

Figure 5: Household wealth.

(a) Net wealth (b) Risky assets

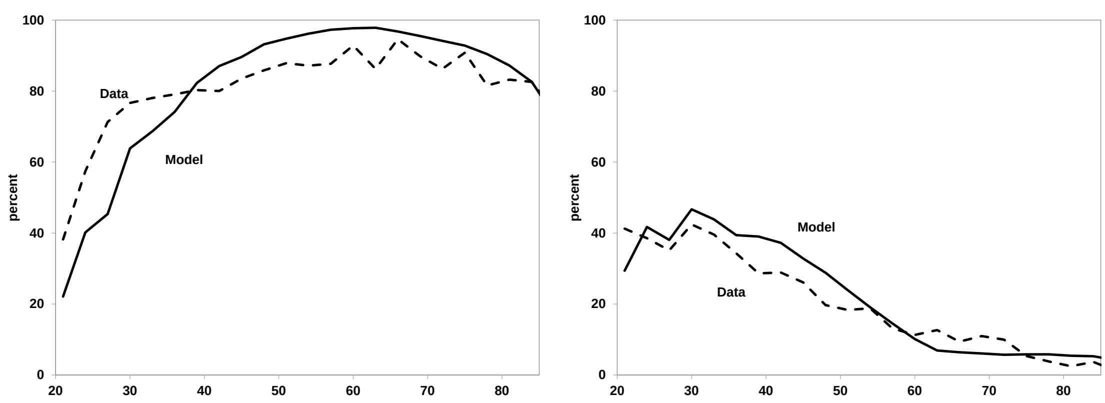


Net wealth is total assets minus total debt, and risky assets include stocks (direct and indirect), real estate, and noncorporate business assets.

Source: SCF 2007.

Figure 6: Household portfolios.

(a) Risky participation (b) Leverage



Leverage is minus safe assets divided by risky assets, bounded by 0 and 1, if risky assets are positive. If not, then leverage is 0 if safe assets are positive and 1 if safe assets are negative.

Source: SCF 2007.

In addition to matching the age profile of household leverage, the model also generates cross sections of leverage by age groups that resemble the data, as can be seen in Table 6. The model replicates the fact that the fraction of households with low leverage (less than 25 percent) is U-shaped with respect to age, that the fractions of households with moderate levels of leverage (between 25 and 75 percent) are inverted U-shapes with respect to age, and that the fraction of households with high leverage (75 percent or higher) is monotonically decreasing in age. The model also matches the fact that the vast majority of old households (ages 57 and above) have very little leverage. However, for all age groups, the model overpredicts the fraction of households with moderately high leverage (between 50 and 75 percent), and underpredicts the fraction of households with high leverage (75 percent or higher). While the cross-sectional distribution of leverage does not perfectly fit the data, it is assuring that the model is underpredicting high leverage since I will show in the following section that high leverage can exacerbate welfare losses.

Table 6: Distribution of household leverage.

	Percent of households with							
	low leverage $[0.00, 0.25)$		medium-low leverage $[0.25, 0.50)$		medium-high leverage $[0.50, 0.75)$		high leverage $[0.75, 1.00]$	
age	model	data	model	data	model	data	model	data
21–29	51	56	2	5	27	8	20	31
30–38	31	46	15	16	39	15	15	24
39–47	39	59	21	16	31	12	8	13
48–56	61	72	18	13	18	8	3	7
57–65	85	81	9	11	5	3	1	5
66–95	92	90	5	5	2	1	0	3

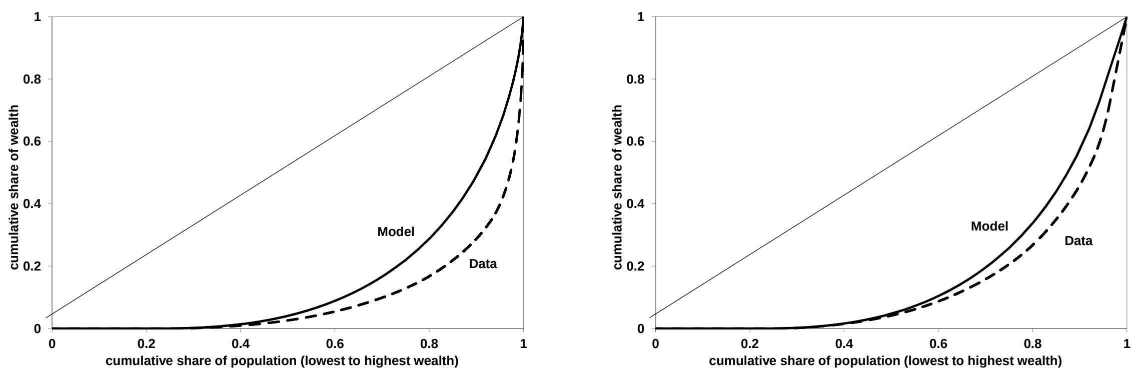
Source: SCF 2007. Units: percent.

The model also generates a distribution of wealth that is reasonably close to the data. Panel (a) of Figure 7 shows the Lorenz curves of the wealth distribution based on the data and on the model, while panel (b) shows the Lorenz curves for which both the data and the model have been top-coded at 3 million dollars. The curves are better aligned in panel (b), because of the inability of the model to match the wealth of the extremely rich, a common shortcoming in existing models.[15] The model also generates 13 percent of households with nonpositive wealth, which is close to the 10 percent of such households in the data.

Figure 7: Wealth distribution.

(a) Lorenz curve

(b) Lorenz curve, truncated



Source: SCF 2007.

5. Quantitative Analysis

5.1. Great Recession

This section describes how the changes in the labor and asset markets that occurred during the Great Recession are introduced into the model, and how timing and information sets are treated.

Labor market. To summarize the changes in the labor market documented in Section 2, there was (a) a large decline in disposable labor income, especially for young households; and (b) an increase in downward earnings risk and a decrease in upward earnings risk. In particular, Tables 2 and 3 show a 4.1 percentage point increase in downward risk and a 5.1 percentage point decrease in upward risk using 4-year changes, and Tables 14 and 15 (Appendix D) show a 2.1 percentage point increase in downward risk and a 2.6 percentage point decrease in upward risk using 2-year changes.[16]

[15]In principle, one could plausibly generate extremely rich households by introducing highly productive income states as in Kindermann and Krueger (2014) or heterogeneous discount factors as in Krusell and Smith (1998).

[16]The PSID is available every two years since 1997.

Since the model period is 3 years, I approximate these changes in earnings risk by assuming a 3 percentage point increase in downward earnings risk and a 3 percentage point decrease in upward earnings risk. This is implemented by converting the transition matrix for labor income from Γ to $\widehat{\Gamma}$ by shifting the appropriate probability mass from upward transitions to downward transitions. See Appendix C.2 for details. This leads to a 2.4 percent decline in average earnings. The age profile of income η_j is scaled downward to $\widehat{\eta}_j$ so that the total decline in income by age group matches the data in Table 1. The induced changes to earning risk is shown in Tables 16 and 17 in Appendix D.

Both the transition matrix and the age profile of income are assumed to revert to the steady state processes after one period. Compared to a recession in which *all* of the decline is driven by a downward scaling of the age-income profile, a recession in which *some* of the decline is driven by changes to the transition matrix has some persistence, which I describe in more detail in Section 5.3.1.[17]

Asset market. Section 2 documented a 23.6 percent decline and subsequent recovery in the price of the risky asset. Thus, the risky asset price is set to $\widehat{p}_x = 0.764$ in the period of the recession, with a return to $p_x = 1$ after one period. Section 2 also documented a 28.6 percent increase in VIX, which measures the market's expected volatility of the S&P 500. This is implemented as a 28.6 percent increase in the risky asset shock standard deviation $\widehat{\sigma}_\xi = 1.286 \times \sigma_\xi$.[18]

For real interest rates, I feed in the exact changes from 2007 to 2010: a decline of 73 basis points in the marginal rate on savings $\widehat{r}_s$, a decline of 125 basis points in the marginal rate

[17]The main results are robust to assuming all of the decline in income is implemented by a temporary downward scaling in the age-income profile. These results are discussed in Section 5.4.

[18]Several caveats are in order. Interpreting VIX as the expected standard deviation requires two assumptions. First, one needs to assume that log returns are normally distributed, which is true in the model. Second, one needs to assume that the expected return is zero, which is not necessarily true in the model. See Whaley (2009) for details. Moreover, VIX is quoted as an annual standard deviation (see CBOE 2017), while the model period is three years. Another caveat is that there is no equivalent measure for housing or business assets, which account for a large portion of risky assets. The main findings are robust to assuming there is no change to the risky asset shock standard deviation. These results are discussed in Section 5.4.

on secured debt $\widehat{r}_b$, and an increase of 116 basis points in the marginal rate on unsecured debt $\widehat{r}_h$. As before, let $\widehat{r}(b, x, j)$ denote the average interest rate on the safe asset in the recession.[19]

Timing and information sets. The sequence of events is as follows. The economy begins $t = 0$ in steady state. The Great Recession is introduced as a one-time unanticipated shock in $t = 1$: disposable labor income transits according to $\widehat{\Gamma}$ from $t = 0$ to $t = 1$, the age-income profile is given by $\widehat{\eta}_j$ in $t = 1$, the risky asset price is $\widehat{p}_x$ in $t = 1$, the real interest rate is given by $\widehat{r}$ in $t = 1$, and the risky asset shock variance is given by $\widehat{\sigma}^2_\xi$ from $t = 1$ to $t = 2$. The economy is assumed to revert to steady state processes and prices in $t = 2$.

Even though households have perfect foresight regarding the transition matrix, the age-income profile, the interest rates, and the risky asset shock variance after $t = 1$, I assume that households are uncertain about the risky asset price. In particular, households believe that prices recover to (and stay at) $p_x = 1$ in $t = 2$ with probability θ or that the low risky asset price $\widehat{p}_x$ persists forever with probability $1 - \theta$. In the baseline analysis, θ is set such that the price-adjusted quantity of risky assets remains unchanged from $t = 0$ to $t = 1$. For robustness, I also report the results for $\theta = 1$, corresponding to perfect foresight, and $\theta = 0$.

[19] To get a sense of the magnitude of these changes, consider the following example. A household with a debt-to-value of 90 percent in the recession pays an average interest rate of $\widehat{r_b} + (\widehat{r_h} - \widehat{r_b})/9$, which is 3.6 percent in annual terms (not including the death risk premium), and is 1 percentage point cheaper than what the same household would have paid in the steady state.

5.2. Household Problem in Recession

Given the assumptions above, the $t = 1$ problem of the household of age j with beginning-of-period wealth a, labor income shock z, and prior earnings k can be written as:

$$
\begin{aligned}
&\widehat{v}(j,a,z,k) = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4)\\
&\max_{c,b',x'} \left\{ \begin{aligned} u_j(c) + \beta\psi_j \sum_{z'\in Z}\sum_{\xi'\in\Xi} \Gamma^j_{z,z'}\widehat{\pi}(\xi') \Big[&\theta v(j+1, \widehat{a}'(p_x), z', \widehat{k}') \\ &+ (1-\theta) v(j+1, \widehat{a}'(\widehat{p}_x), z', \widehat{k}') \Big] \end{aligned} \right\} \\
&\text{s.t. } c + \widehat{p}_x x' + b' \le \widehat{y}_j(z,k) - \mathbb{1}_{x'>0} f + a \\
&\quad \widehat{a}'(p) = b'(1 + \widehat{r}(b', x', j)) + p x' e^{\xi'} \\
&\quad \widehat{k}' \text{ follows } (\widehat{1}) \\
&\quad c \ge 0,\ x' \ge 0
\end{aligned}
$$

where $v(\overline{J}+1,\cdot,\cdot,\cdot) = 0$,

$$
\widehat{y}_j(z,k) = \begin{cases} e^z \widehat{\eta}_j & \text{if } j < R \\ s(k) & \text{otherwise,} \end{cases}
$$

and $(\widehat{1})$ is analogous to (1) but uses the recession income profile $\widehat{\eta}_j$. The policy functions are denoted $\widehat{c}(j,a,z,k)$, $\widehat{b}'(j,a,z,k)$, and $\widehat{x}'(j,a,z,k)$. Note that households evaluate the continuation values using the steady state value function v regardless of the belief on prices. This is because prices only affect the beginning-of-period wealth a' but otherwise are irrelevant, given the assumption that prices remain constant from $t = 2$ onwards.

5.3. Consequences of the Great Recession

5.3.1. Changes in Income and Wealth

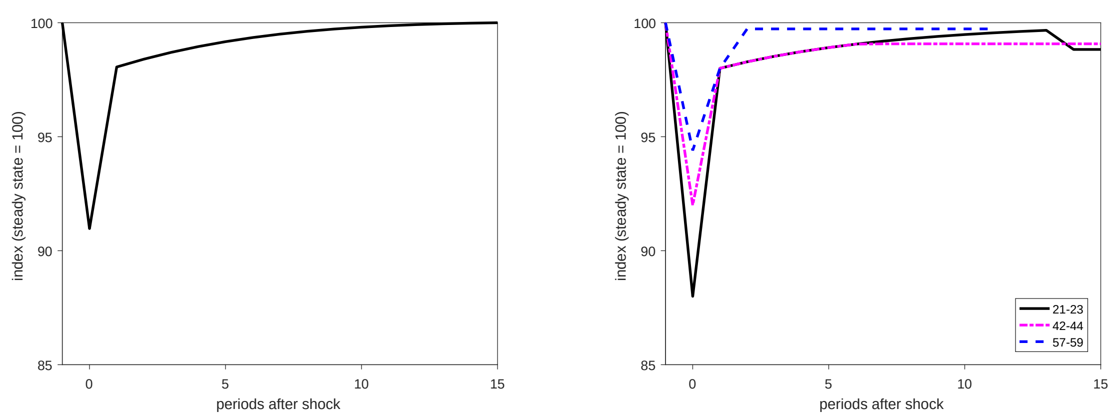
This section evaluates the consequences of the Great Recession. First, I report the persistent effects of the labor market shocks. Panel (a) of Figure 8 plots aggregate labor income and panel (b) plots labor and retirement income for a few age cohorts over time. Because the decline in labor income is a result of a shift in the transition matrix, which has persistent effects, and a downward scaling of the age-income profile, which is temporary, much of the decline in labor income recovers after one period, for all cohorts. However, the long-run effects are very different across cohorts. For example, households aged 57–59 in

$t = 0$ experience the smallest decline in labor income in $t = 1$ and face a negligible decline in retirement income when they retire 2 periods after the recession. This is because retirement income is based on lifetime earnings, whereas this cohort suffers only 2 periods of below-trend income.[20] The long-run effects are much larger for households aged 21–23 in $t = 0$. These households face the largest decline in labor income in $t = 1$, continue to have income levels below steady state throughout their working years, and face a permanent 1 percent decline in retirement income relative to steady state when they retire 14 periods after the recession. These effects seem plausible considering the long-term effects of recessions that have been documented in the literature.[21]

Figure 8: Labor income dynamics.

(a) Aggregate labor income

(b) Income for select age cohorts



Furthermore, the model generates a persistence of earnings losses that is similar to the data. In the model, among households who had a large income decline (greater than 25 percent), after controlling for quadratic age effects, 23 percent recovered their pre-recession income in the period following the recession. In the data, among households who had a large

[20]To be precise, retirement earnings are based on a summary of previous earnings, k, which is calculated according to (1), where k is nondecreasing after 11 model periods, to capture the spirit of social security where retirement income is based on the 35 highest-earning years.

[21]For example, Kahn (2010) finds that cohorts who graduate college in recessions, corresponding to the youngest cohorts in the model, have large wage losses which remain statistically significant even 15 years after graduating college. Other papers that study the long-term effects of recessions include Barnette and Michaud (2012), Oreopoulos et al. (2012), and Von Wachter et al. (2009).

income decline from 2006 to 2010, 21 percent recovered their 2006 income by 2014. Similarly, 16 percent of households who had a large income decline from 2008 to 2010 recovered their 2008 income by 2012.

Next, I report the effects of the Recession on household wealth. As can be seen in Table 7, the young households (21–29) suffer the largest percentage decline in wealth, both in the model and in the data. In fact, the percentage decline in wealth by age group in the model matches quite well with the data, except for the increase in wealth observed in the data for the oldest group.[22]

Table 7: Changes in household wealth.

age	model (percent change)	data
21–29	−50.6	−59.2
30–38	−38.0	−44.1
39–47	−30.1	−18.9
48–56	−26.2	−19.6
57–65	−21.5	−11.7
66–74	−19.9	−20.0
75–83	−20.0	−9.3
84–92	−19.7	21.5

The model and data report the percent change of household wealth from period $t = 0$ to $t = 1$ and from 2007 to 2010, respectively.
Source: SCF 2007, 2010.

5.3.2. Main Welfare Results

The welfare consequences of the Great Recession are the main focus of the paper, and are presented in Table 8. The first two columns show that households between the ages of 30 and 38 suffer the largest welfare losses, equivalent to a 10.1 percent reduction in remaining lifetime

[22] The number of observations in the SCF drops considerably for ages greater than 83.

consumption and a 23.7 percent reduction in one-period consumption.[23] The last row of the third and fourth columns, which report the welfare changes in one-period consumption for risky asset participants and nonparticipants for all ages, shows that households who owned risky assets at the end of $t = 0$ suffered the equivalent of a 22.9 percent reduction in one-period consumption, which is much larger than the 7.9 percent welfare loss suffered by those who did not own risky assets. Young households (ages 21–29) who owned risky assets prior to the Recession were particularly vulnerable, suffering the equivalent of a 33.2 percent decline in one-period consumption. These households were the most leveraged coming into the Recession (Figure 6 panel b) and suffered the largest percent declines in labor income (Table 1) and wealth (Table 7).

Table 8: Welfare changes.

age	remaining lifetime cons. equivalent (percent)	one-period cons. equivalent (percent)		
		all	risky participant	risky nonparticipant
21–29	−7.4	−19.1	−33.2†	−11.2†
30–38	−10.1†	−23.7†	−30.2	−9.1
39–47	−6.8	−21.1	−23.5	−5.8
48–56	−5.0	−20.2	−21.2	−2.6
57–65	−4.2	−19.9	−20.6	6.1*
66–74	−4.7	−19.7	−20.8	4.2
75–83	−4.9	−16.5	−18.3	1.0
84–92	−4.9	−11.1	−15.4	−0.1
all ages	−6.2	−19.8	−22.9	−7.9

† : largest welfare loss. * : largest welfare gain.

The detailed welfare results of finer age groups can be found in Table 18 (Appendix D).

As the last column of Table 8 shows, households who did not own risky assets fared

[23] I report the welfare changes in terms of remaining lifetime consumption and one-period consumption. Since households may have different expected remaining lifetimes, I use the one-period consumption equivalent as the baseline measure of welfare change. Both of these measures are formally defined in Appendix B.3.

much better. In fact, nonparticipant households aged 57–65 gained the equivalent of a 6.1 percent *increase* in one-period consumption. Compared to older nonparticipant households, these households are able to take better advantage of cheaper assets because they are still saving for retirement, which can be seen in the wealth profiles in Figure 5. Among nonparticipants, young households (ages 21–29) suffered the largest welfare losses, equivalent to a 11.2 percent reduction in one-period consumption. Not only do young households suffer the largest percent decline in labor income, but many of them are also not in the financial position to take full advantage of cheaper assets, given that younger nonparticipants have less savings, in the model and in the data, as can be seen in Table 9. Of course, it should be emphasized that while the youngest households suffer the largest welfare losses within the group of risky participants and within the group of nonparticipants, they also have the lowest rate of participation, and therefore do not suffer the largest welfare losses overall.

Table 9: Safe assets (nonparticipants).

age	model ($t = 0$)	data (2007)
	(thousands of 2013 dollars)	
21–29	1.5	2.2
30–38	1.3	5.6
39–47	3.7	1.4
48–56	6.2	6.5
57–65	8.8	8.4
66–74	11.9	20.8
75–83	14.9	22.4
84–92	26.4	24.2

Source: SCF 2007.

Given that risky asset investment is an important determinant of the welfare effects, I report the changes in risky asset shares and risky asset participation rates in the model and in the data in Table 10. The first two columns show that the share of total risky assets held by households aged 21–38 decreased during the model recession, as in the data. The model also matches the fact that all age groups reduced their risky asset participation, except for those above 75, as in the data. While the model changes in risky asset investment, especially in shares, are not perfectly aligned with the data, it is reassuring that the welfare effects are not driven by any counterfactually large increases or decreases in risky asset investment.

Table 10: Changes in household risky assets.

	risky asset shares		risky asset participation	
age	model	data	model	data
		(percentage point change)		
21–29	−0.2	−1.0	−1.8	−3.7
30–38	−0.3	−1.7	−2.6	−3.3
39–47	0.3	−0.7	−0.9	−0.8
48–56	0.5	−1.6	−0.5	−2.3
57–65	0.2	2.3	−0.1	−1.2
66–74	−0.2	0.5	−0.0	−2.7
75–83	−0.1	1.5	0.5	1.8
84–92	−0.2	0.2	2.3	4.1

The model and data report the percentage point change of household risky asset shares and risky asset participation from period $t = 0$ to $t = 1$ and from 2007 to 2010, respectively.
Source: SCF 2007, 2010.

5.3.3. Welfare Decomposition

Next, I provide a decomposition of the welfare losses, measured in units of one-period consumption, by turning on and off the different changes to the labor markets, risky asset markets, and interest rates. First, let's consider the effects of the labor market changes, reported in the first three columns of Table 11. Not surprisingly, the labor market changes adversely affect the welfare of young households, accounting for 75 and 50 percent of the welfare losses for households aged 21–29 and 30–38, respectively.

Second, consider the effects of the changes in the risky asset market, shown in the middle three columns of Table 11. As expected, the risky asset market shocks worsen the welfare of risky participants and improve the welfare of nonparticipants, accounting for 86 and −26 percent of the welfare losses of participants and nonparticipants, respectively.

Table 11: Decomposition of welfare losses.

	labor market changes			risky asset market changes			safe asset market change		
age	all	risky>0	risky$=0$	all	risky>0	risky$=0$	all	risky>0	risky$=0$
21–29	75	34	117	28	70	−15	−3	−4	−2
30–38	50	31	129	57	76	−24	−7	−7	−5
39–47	49	40	164	63	72	−51	−12	−12	−13
48–56	45	40	231	70	74	−101	−15	−15	−30
57–65	16	14	231	96	97	−235	−12	−11	−95
66–74	0	0	0	111	110	−53	−11	−10	−47
75–83	0	0	0	109	108	−57	−9	−8	−43
84–92	0	0	0	104	104	−25	−4	−4	125
all	38	24	132	71	86	−26	−9	−10	−6

Units: percent. The welfare effects of changes to the labor market, risky asset market, and safe asset market sum to 100 for welfare losses and −100 for welfare gains.

A description of how the decomposition is computed is provided in Appendix C.4.

Third, we can consider the effects of changes to interest rates, reported in the last three columns of Table 11. The changes to interest rates improve the welfare of almost all groups. This is because lower interest rates on debt make the purchase of risky assets more affordable (income effect), and lower interest rates on savings induce larger investments in the risky asset (substitution effect), a strategy that pays high dividends ex post when prices recover. The shocks to interest rates do adversely affect the oldest nonparticipant households (84–92), for whom the negative income effect (lower return on savings) dominates the positive substitution effect (more risky investment).

Using this decomposition, we can ask whether the decline in risky asset prices help offset the adverse labor market outcomes for young households (21–29). Not in general, as the risky asset market changes account for 28 percent of the welfare losses of young households. The risky asset market changes do improve the welfare outcomes for nonparticipants, but the improvement is largest for older households (ages 48–65). As before, this is because (a) compared to older nonparticipants, pre-retirement households (48–65) are still saving for retirement; and (b) compared to younger nonparticipants, they have more savings (Table 9) and thus are in a better position to take advantage of cheaper asset prices.

5.3.4. Beliefs Regarding Prices

The extent to which households could take advantage of cheap assets depends on whether households believe that the price decline is temporary or permanent. In the baseline results reported in Table 8, the prior that the price would recover in one period, θ, was set to 0.19 so that the quantity of risky assets demanded would remain constant from $t = 0$ to $t = 1$. Instead, Tables 12 and 13 report the welfare results for which θ is equal to 1, corresponding to perfect foresight, and 0, respectively. With perfect foresight, the welfare results improve for all groups. The welfare of households aged 57–65 improves the most, improving from a 19.9 percent loss to a 5.6 percent loss. Even with perfect foresight, young households suffer large losses, especially those that owned risky assets, suffering a 30.9 percent welfare loss. The perfect foresight scenario leads to a counterfactually high 79 percent increase in the quantity of risky assets held by households. When households believe that the price decline is permanent ($\theta = 0$), the welfare losses are larger for all groups. Regardless of the beliefs, it is a robust result that households between the ages of 30 and 38 suffer the largest overall welfare losses, and that young risky participants (21–29) suffer the largest welfare losses.

5.3.5. Leverage

Leverage is another important dimension in which household welfare changes differ. As can be seen in Figure 9, which plots the welfare gains by portfolio prior to the Recession, households whose portfolios are more leveraged tend to suffer larger welfare losses. For example, fixing the level of risky assets at 200 thousand dollars, the welfare losses get larger (shades get darker) as we move to the left, representing more leverage. As documented in Section 4.3, younger households are typically more leveraged than older households. The model also generates a cross section of portfolio holdings in which the young are more likely to be leveraged, as can be seen in Figure 10, which plots the average age by portfolio. The fact that young households are highly leveraged at the onset of the Great Recession, coupled with the fact that the young suffer the largest declines in labor income, induces large welfare

Table 12: Welfare changes for $\theta = 1$ (perfect foresight).

age	remaining lifetime cons. equivalent (percent)	one-period cons. equivalent (percent)		
		all	risky participant	risky nonparticipant
21–29	−6.8	−16.1	−30.9†	−7.8†
30–38	−9.2†	−19.4†	−25.9	−4.8
39–47	−5.2	−13.7	−15.6	−2.0
48–56	−2.5	−8.1	−8.5	0.8
57–65	−1.1	−5.6	−6.1	13.3
66–74	−1.8	−8.5	−9.6	15.2*
75–83	−2.4	−8.5	−10.4	9.0
84–92	−3.5	−7.9	−11.6	1.5
all ages	−4.3	−11.6	−13.6	−3.9

† : largest welfare loss. * : largest welfare gain.

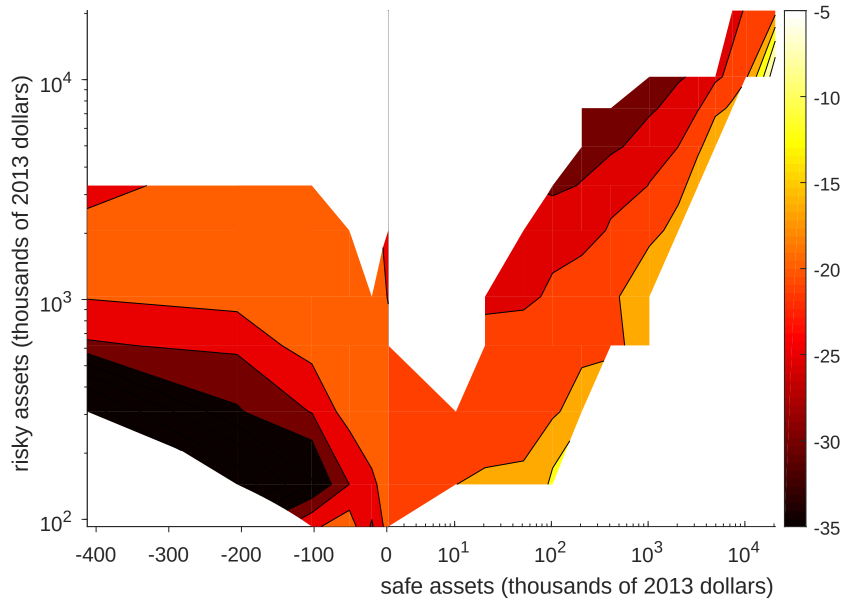
Table 13: Welfare changes for $\theta = 0$.

age	remaining lifetime cons. equivalent (percent)	one-period cons. equivalent (percent)		
		all	risky participant	risky nonparticipant
21–29	−7.6	−20.1	−34.0†	−12.3†
30–38	−10.4†	−25.1†	−31.8	−10.4
39–47	−7.5	−23.9	−26.3	−8.7
48–56	−5.9	−23.7	−24.7	−6.0
57–65	−5.3	−24.1	−24.7	0.1*
66–74	−6.0	−23.8	−24.9	−0.2
75–83	−6.0	−19.4	−21.4	−0.2
84–92	−5.5	−12.3	−17.1	−0.1
all ages	−7.0	−22.5	−25.9	−9.4

† : largest welfare loss. * : largest welfare gain.

losses for the young.[24]

Figure 9: Welfare gains by portfolio.



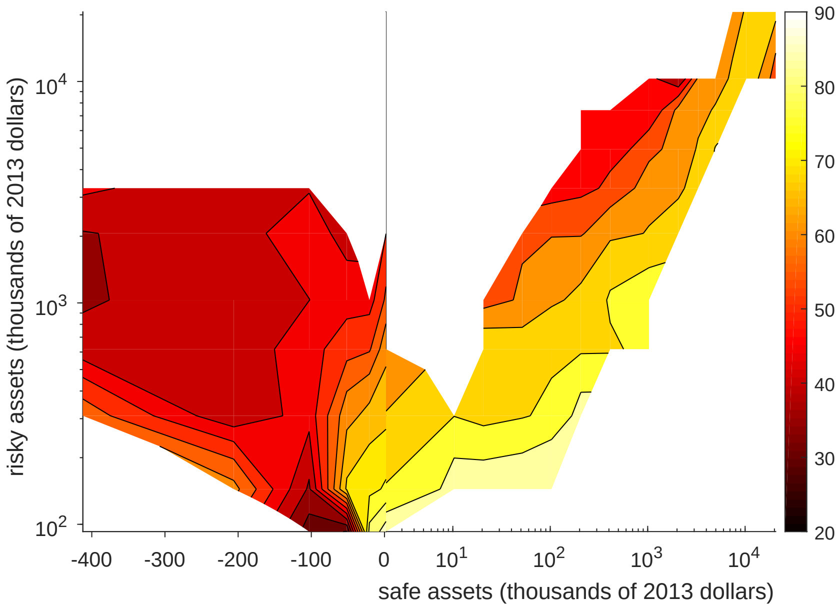
5.4. Sensitivity

Sensitivity analysis indicates that the main results are robust to different parameter specifications. Specifically, the result that young (21–29) risky participants suffer the largest welfare losses and that pre-retirement (57–65) nonparticipants experience the largest welfare gains are robust to changes in risk aversion σ and the collateral constraint λ, as can be seen in Table 19 (Appendix D).

Moreover, the main results are robust to a number of different exercises. The first six columns of Table 20 (Appendix D) report sensitivity analyses for a counterfactual in which *all* of the drop in labor income is modeled as a temporary downward scaling of the income profile

[24]These facts are consistent with Hurd and Rohwedder (2010) who document that 48 percent of households under age 50 are under financial distress, compared to 16 percent for ages above 64, where financial distress is as an indicator for any of the following: unemployed, negative equity in house, behind more than two months on mortgage, or in foreclosure. The results are also related to recent empirical work by Emmons and Noeth (2013) who find that demographic groups who are more likely to be economically vulnerable are also more likely to be leveraged.

Figure 10: Average age by portfolio (model).



and for a counterfactual in which there is no change to the risky asset shock variance σ_ξ^2. In both of these exercises, young risky participants suffer the largest welfare losses. Interestingly, when the risky asset price decline is not accompanied by an increase in uncertainty, pre-retirement households (57–65) experience large welfare gains, including those that held risky assets. This is because those households are financially in the best position to take advantage of cheaper asset prices, as discussed in Section 5.3.3. The last three columns of Table 20 (Appendix D) report the results for the counterfactual in which households have the option to default on their $t = 1$ debt. While this innovation disproportionately improves the welfare of young risky participants, it is still the case that young risky participants suffer large losses, equivalent to a 19 percent reduction in consumption, and that young nonparticipants suffer the largest welfare losses among nonparticipants. I leave for future research a more nuanced model of household default and how it interacts with large changes in the labor and asset markets.

6. Conclusion

This paper develops a model of the Great Recession that is consistent with the age profiles of wealth, risky assets, risky asset participation, and leverage, and the distributions of wealth and leverage. This model is used to quantitatively evaluate the welfare consequences of the various changes that occurred during the Recession. I find that household age and risky asset participation are important factors that determine the welfare losses. The youngest households (ages 21–29) suffer large welfare losses, equivalent to a 19 percent reduction in consumption, that are almost twice those of the oldest households (ages 84–92), who suffer a 11 percent welfare loss. Households who owned risky assets have significantly worse outcomes (23 percent welfare loss) than those who did not (8 percent welfare loss). When jointly considering these factors, young risky asset participants suffered the largest welfare losses, equivalent to a 33 percent decline in consumption, while pre-retirement households (ages 57–65) who did not own risky assets had the largest welfare gains, equivalent to a 6 percent increase in consumption.

This paper has important policy implications. Policies such as the Making Home Affordable Program have helped alleviate the welfare losses of the most vulnerable group, young risky asset owners, by providing mortgage relief to households that are "underwater" or delinquent, possibly because of adverse labor market outcomes and asset price declines. Another example is the Troubled Asset Relief Program, which may have contributed to preventing a further collapse in asset prices, helping risky asset owners. Other policies may have exacerbated the welfare losses of the young. For example, there has been a dramatic increase in government debt since the onset of the Great Recession, shifting the tax burden to younger generations. More policies geared toward the young and highly leveraged may be needed to help the "lost generation" recover from the Great Recession.

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A. Data Appendix

Estimation of disposable income process. The estimation procedure closely follows the procedure described in Krueger et al. (2016a). I use annual household income data from the PSID core sample (1970–1997). I select all households whose head is aged between 21 and 65, dropping the observations where the head's wage (head's labor income divided by the head's total hours worked) is below half the minimum wage for that year, or where the head is not in the labor force (head was not unemployed at any time during the year but worked less than 520 hours). For each household, I compute total household labor income as the sum of labor income of the head and spouse, income from farm and from business, plus transfers. Next, I construct household disposable labor income as total household labor income minus tax liabilities, computed for each household using the TAXSIM (ver 9) tax calculator. I then deflate disposable labor income using the CPI. I compute total real disposable labor income over a 3-year window. On this sample, I regress the log of 3-year real disposable income on age and time dummies. I then exclude all household income sequences that are are shorter than 12 years, leaving a final sample of 2657 households, with an average length of 21 years. On these data, I compute the autocovariance matrix of the residuals. The stochastic process in equation (3) is estimated using GMM, targeting the covariance matrix, where the weighting matrix is the identity matrix. I thank Chris Tonetti for providing the Matlab routines that perform the estimation.

Details regarding the construction of other data can be found in the Online Data Appendix.

B. Formal Definitions

B.1. Equilibrium

Let $S \equiv J \times A \times Z \times K$ denote the state space, and $\mathcal{B}$ denote the associated Borel σ-algbra. A *stationary equilibrium* is a value function $v : S \to \mathbb{R}$, policy functions for the households $c : S \to \mathbb{R}_{++}$, $b' : S \to \mathbb{R}$, and $x' : S \to \mathbb{R}_{+}$, and a stationary measure $\mu : S \to \mathbb{R}_{+}$ such that, given prices and stochastic processes:

1. the policy functions solve the households' problem in (2) and v is the associated value function.
2. for all $(\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) \in \mathcal{B}$, the invariant measure μ satisfies

$$\mu(\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) = \int_{J \times A \times Z \times K} Q((j,a,z,k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) d\mu(j,a,z,k) \tag{5}$$

   where Q is the transition function defined below and the distribution for newborns is given.

B.2. Transition Function in the Steady State

Let $S \equiv J \times A \times Z \times K$ denote the state space, and $\mathcal{B}$ denote the associated Borel σ-algbra. For any set $\mathcal{S} \in \mathcal{B}$, let $\mu(\mathcal{S})$ denote the measure of agents in $\mathcal{S}$. Let $Q((j,a,z,k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K})$ be the probability that a household with state (j,a,z,k) transits to $\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}$ next period. Formally, $Q : S \times \mathcal{B} \to [0,1]$ and

$$\begin{aligned} &Q((j,a,z,k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) = \\ &\psi_j \mathbb{1}_{\{k'(j,z,k) \in \mathcal{K}\}} \mathbb{1}_{\{j+1 \in \mathcal{J}\}} \sum_{z' \in \mathcal{Z}} \sum_{\xi \in \Xi} \mathbb{1}_{\{a'(j,a,z,k;\xi) \in \mathcal{A}\}} \pi(\xi) \Gamma^{j}_{z,z'} \end{aligned} \tag{6}$$

where $\mathbb{1}$ is the indicator function, $a'(j,a,z,k;\xi) = b'(j,a,z,k)(1+r(b',x',j)) + p_x x'(j,a,z,k)e^{\xi}$, and $b'(j,a,z,k)$ and $x'(j,a,z,k)$ are the policy functions.

B.3. Computing Welfare

The welfare changes can be measured in two ways. I discuss these below.

It is useful to define the ex post value function:

$$v^*(j,a,z,k) = u_j(\widehat{c}(j,a,z,k)) + \beta \psi_j \sum_{z' \in Z} \sum_{\xi' \in \Xi} \Gamma^{j}_{z,z'} \widehat{\pi}(\xi') v(j+1, \widehat{a}'(p_x), z', \widehat{k}')$$

where $\widehat{a}'(p) = \widehat{b}'(j,a,z,k)(1+\widehat{r}(b',x',j)) + p\widehat{x}'(j,a,z,k)e^{\xi'}$ and $\widehat{k}'$ follows $(\widehat{1})$. This is the ex post value function in $t=1$, in the sense that it measures the average discounted utility of households that chose to consume $\widehat{c}(j,a,z,k)$ and invest $\widehat{b}'(j,a,z,k)$ and $\widehat{x}'(j,a,z,k)$ in $t=1$, given that prices recovered in period $t=2$.

The first way of measuring welfare is to compute the consumption equivalent of remaining lifetime consumption. Since the utility function is homogenous to degree $1-\delta$, this can be easily computed as:

$$\delta(j,a,z,k) = \left[\frac{u_j\left(c(j,a,z,k)\right) + \beta\psi_j \sum_{z'\in Z}\sum_{\xi'\in\Xi}\widehat{\Gamma}^j_{z,z'}\pi(\xi')v^*(j+1,a'(\widehat{p}_x),z',k')}{v(j,a,z,k)}\right]^{\frac{1}{1-\sigma}} - 1$$

where $a'(p) = b'(j,a,z,k)(1+r(b',x',j)) + px'(j,a,z,k)e^{\xi'}$ and k' follows (1). This measures the fraction of remaining lifetime steady-state consumption an agent would be willing to give up to remain in the steady state when learning about the Recession after $t=0$ decisions have been made. This is a common method of computing conditional welfare changes.

However, in overlapping generations models, the consumption equivalent of a young household may not have the same interpretation as the consumption equivalent of an old household. Therefore, as a second, and preferred, way of measuring welfare changes, I compute the $t=0$ (one-period) consumption equivalent, $\delta^p(j,a,z,k)$, which is defined implicitly:

$$\begin{aligned}
u_j\left(c(j,a,z,k)(1+\delta^p(j,a,z,k))\right) &+ \beta\psi_j \sum_{z'\in Z}\sum_{\xi'\in\Xi}\Gamma^j_{z,z'}\pi(\xi')v(j+1,a'(p_x),z',k') \\
&= u_j\left(c(j,a,z,k)\right) + \beta\psi_j \sum_{z'\in Z}\sum_{\xi'\in\Xi}\widehat{\Gamma}^j_{z,z'}\pi(\xi')v^*(j+1,a'(\widehat{p}_x),z',k)
\end{aligned}$$

where k' follows (1). This measure is similar to the remaining lifetime consumption equivalent except that it is in units of $t=0$ consumption.[25]

[25] Asturias et al. (2016), Ganelli and Tervala (2015), and Sims and Wolff (2013) also report welfare changes in terms of one-period consumption.

C. Computational Appendix

The computation strategy involves jointly solving for equilibrium and calibration procedures. The household problem is characterized by four states: (i) age, (ii) wealth, (iii) labor productivity shock, and (iv) previous earnings, and two decisions: (i) risk-free assets and (ii) risky assets. The income process is approximated with an eleven-state Markov process using the Rouwenhorst procedure described in Kopecky and Suen (2010).

I discretize the state space for wealth, previous earnings, and decision variables by choosing a finite grid, and use linear interpolation when the levels of next-period wealth and earnings are not on the grid. For computational efficiency, I guess and verify that above a certain cash-on-hand threshold, the household's optimal portfolio can be represented by savings rates between 0 and 1, and risky asset shares between 0 and 20. Below that threshold, I use standard grids for risky and safe assets. The measure of households over age, wealth, labor productivity, and previous earnings, denoted by $\mu(j, a, z, k)$ can then be represented by a finite-dimensional array. In computing this large dimensional problem, I have benefited greatly from the parallel computing resources at Pittsburgh Supercomputing Center as well as the Center for Simulation and Modeling at the University of Pittsburgh. All codes used in the paper available online.

C.1. Algorithm for Solving Steady State Equilibrium and Calibration

1. Guess a vector of parameters $\{\beta, f, \mu_\xi, \sigma_\xi, r_s\}$.
2. Starting from age $\overline{J}$ backward, compute the steady state value function $v(j, a, z, k)$ and policy functions $c(j, a, z, k), b'(j, a, z, k), x'(j, a, z, k)$ by solving the household problem in (2) for all j, a, z, k.
3. Using the policy functions obtained in step 2 and given the distribution of initial wealth, calculate the distributions $\mu(j, a, z, k)$ using the law of motion in (5).
4. Using the policy functions obtained in step 2 and the distributions obtained in step 3, compute the model moments: average leverage, 90-50 wealth ratio, risky asset participation, risky assets, and net safe assets.
5. Continue steps 1–4 until the difference between model moments and corresponding data targets are less than a specified threshold.

C.2. Change to Transition Matrix

As documented in Section 2, the Great Recession featured an increase in downward earnings risk and a decrease in upward earnings risk. In the model, I assume a 3 percentage point increase in downward risk and a 3 percentage point decrease in upward risk, and construct a recession transition matrix $\widehat{\Gamma}$, using the following procedure:

1. let $\Delta = \min\{0.03, \Gamma\}$.
2. $1 < i < N$: set $\widehat{\Gamma}_{i,i+1} = \Gamma_{i,i+1} - \Delta_{i,i+1}$ and $\widehat{\Gamma}_{i,i-1} = \Gamma_{i,i-1} + \Delta_{i,i+1}$.
3. set $\widehat{\Gamma}_{12} = \Gamma_{12} - \Delta_{12}$ and $\widehat{\Gamma}_{11} = \Gamma_{11} + \Delta_{12}$.
4. set $\widehat{\Gamma}_{NN} = \Gamma_{NN} - \Delta_{NN}$ and $\widehat{\Gamma}_{N,N-1} = \Gamma_{N,N-1} + \Delta_{NN}$.

C.3. Algorithm for Solving Transition Path

Let the economy begin in steady state in $t = 0$, and let $t = 1$ be the period of the recession as defined in Section 5.1.

1. Guess the prior that prices recover $\theta \in [0, 1]$.
2. Compute the $t = 1$ value function $\widehat{v}(j, a, z, k)$ and policy functions $\widehat{c}(j, a, z, k)$, $\widehat{b}'(j, a, z, k)$, $\widehat{x}'(j, a, z, k)$ by solving the household problem in (4) for all j, a, z, k.
3. Using the policy functions obtained in step 2, the shock processes, and the steady states distributions, calculate the distributions $\widehat{\mu}(j, a, z, k)$ using the following law of motion: for all $(\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) \in \mathcal{B}$,

$$\widehat{\mu}(\mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) = \int_{J \times A \times Z \times K} \widehat{Q}((j, a, z.k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) d\mu(j, a, z, k)$$

   where

$$\widehat{Q}((j, a, z, k), \mathcal{J} \times \mathcal{A} \times \mathcal{Z} \times \mathcal{K}) =$$
$$\psi_j \mathbb{1}_{\{k'(j,z,k) \in \mathcal{K}\}} \mathbb{1}_{\{j+1 \in \mathcal{J}\}} \sum_{z' \in \mathcal{Z}} \sum_{\xi \in \Xi} \mathbb{1}_{\{a'(j,a,z,k;\xi) \in \mathcal{A}\}} \pi(\xi) \widehat{\Gamma}^j_{z,z'}$$

   where $\mathbb{1}$ is the indicator function, k' follows (1), $a'(j, a, z, k; \xi) = b'(j, a, z, k)(1 + r(b', x', j)) + \widehat{p}_x x'(j, a, z, k) e^{\xi}$, and $b'(j, a, z, k)$ and $x'(j, a, z, k)$ are the steady state policy functions.
4. Continue steps 1–3 until the quantity of $t = 1$ risky asset demand equals $t = 0$ risky asset demand.

C.4. Welfare Decomposition

Let $\delta^p(j,a,z,k)$ be the one-period consumption equivalent for the baseline exercise, described in Appendix B.3. Let $\delta_1^p(j,a,z,k)$, $\delta_2^p(j,a,z,k)$, and $\delta_3^p(j,a,z,k)$ denote the analogously defined one-period consumption equivalent for the counterfactual where there is no labor market changes, no risky asset market changes, and no interest rate changes, respectively. Then the welfare change attributable to labor market changes is

$$\frac{\delta^p(j,a,z,k)-\delta_1^p(j,a,z,k)}{-\left|\sum_{i=1}^{3}\left[\delta^p(j,a,z,k)-\delta_i^p(j,a,z,k)\right]\right|},$$

the welfare change attributable to changes in the risky asset market is

$$\frac{\delta^p(j,a,z,k)-\delta_2^p(j,a,z,k)}{-\left|\sum_{i=1}^{3}\left[\delta^p(j,a,z,k)-\delta_i^p(j,a,z,k)\right]\right|},$$

and the welfare change attributable to changes in interest rates is

$$\frac{\delta^p(j,a,z,k)-\delta_3^p(j,a,z,k)}{-\left|\sum_{i=1}^{3}\left[\delta^p(j,a,z,k)-\delta_i^p(j,a,z,k)\right]\right|}.$$

D. Additional Tables

Table 14: Increase in downward earnings risk for 2-year changes.

age	downward earnings risk (percent)		
	1996–2006	2006–2010	difference
21–29	15.6	17.1	1.6
30–38	18.8	20.2	1.3
39–47	17.7	19.5	1.8
48–63	14.8	18.9	4.1
21–63	16.7	18.9	2.1

Downward earnings risk is defined as the share of households that experience a reduction in disposable labor income (after controlling for quadratic age effects) that exceeds 25 percent in a 2-year window. The columns for 1996–2006 and 2006–2010 report the averages for multiple 2-year windows. Source: PSID.

Table 15: Decrease in upward earnings risk for 2-year changes.

age	upward earnings risk (percent)		
	1996–2006	2006–2010	difference
21–29	16.3	15.7	−0.6
30–38	17.1	14.4	−2.8
39–47	20.2	17.1	−3.1
48–63	19.8	15.5	−4.3
21–63	18.4	15.6	−2.8

Upward earnings risk is defined as the share of households that experience an increase in disposable labor income (after controlling for quadratic age effects) that exceeds 25 percent in a 2-year window. The columns for 1996–2006 and 2006–2010 report the average for multiple 2-year windows. Source: PSID.

Table 16: Downward earnings risk, model and data.

age	percent of households with income drops greater than 25 percent, controlling for quadratic age effects (percent)					
	model			data		
	steady state	recession	difference	1996–2006	2006–2010	difference
21–29	22.4	25.4	3.0	18.6	21.4	2.8
30–38	22.4	25.4	3.0	21.7	23.9	2.2
39–47	22.4	25.4	3.0	19.0	22.0	3.0
48–61	22.4	25.4	3.0	15.9	20.5	4.6
21–61	22.4	25.4	3.0	18.8	21.9	3.1

Source: PSID. Data reports averages for 2- and 4-year changes.

Table 17: Upward earnings risk, model and data.

age	percent of households with income gains greater than 25 percent, controlling for quadratic age effects (percent)					
	model			data		
	steady state	recession	difference	1996–2006	2006–2010	difference
21–29	22.4	19.4	−3.0	19.0	17.0	−2.0
30–38	22.4	19.4	−3.0	19.6	16.2	−3.4
39–47	22.4	19.4	−3.0	22.3	18.5	−3.7
48–61	22.4	19.4	−3.0	24.0	17.5	−6.6
21–61	22.4	19.4	−3.0	21.2	17.3	−4.0

Source: PSID. Data reports averages for 2- and 4-year changes.

Table 18: Detailed welfare gains.

age	remaining lifetime cons. equivalent (percent)	one-period cons. equivalent (percent)		
		all	risky participant	risky nonparticipant
21–23	−5.0	−15.3	−30.1	−11.1
24–26	−7.3	−20.3	−30.5	−13.5†
27–29	−10.0	−21.7	−37.2†	−8.8
30–32	−9.7	−23.5	−30.5	−11.1
33–35	−10.7†	−24.9†	−31.5	−10.3
36–38	−9.8	−22.6	−28.8	−4.8
39–41	−8.3	−21.6	−25.1	−5.3
42–44	−6.6	−20.8	−22.9	−6.4
45–47	−5.4	−20.9	−22.6	−6.1
48–50	−5.4	−21.0	−22.2	−5.2
51–53	−5.3	−21.0	−22.0	−3.5
54–56	−4.2	−18.6	−19.5	3.5
57–59	−4.0	−19.0	−19.7	6.1
60–62	−4.2	−20.1	−20.8	8.8*
63–65	−4.3	−20.7	−21.2	3.1
66–68	−4.6	−20.6	−21.4	4.5
69–71	−4.6	−19.7	−20.8	3.1
72–74	−4.7	−18.7	−20.1	4.8
75–77	−5.0	−18.2	−19.7	1.8
78–80	−4.8	−16.2	−18.1	1.1
81–83	−4.9	−14.6	−16.7	0.3
84–86	−5.4	−13.8	−16.7	−0.1
87–89	−5.3	−11.3	−15.4	−0.2
90–92	−3.3	−5.3	−10.9	0.0
all ages	−6.2	−19.8	−22.9	−7.9

† : largest welfare loss. * : largest welfare gain.

Table 19: Sensitivity analysis, recalibrated.

	$\sigma = 2.5^a$			$\sigma = 3.5^b$			$\lambda = 0.911^c$		
age	all	risky>0	risky$=0$	all	risky>0	risky$=0$	all	risky>0	risky$=0$
21–29	−18.3	$-30.3^\dagger$	$-11.5^\dagger$	−21.9	$-43.3^\dagger$	$-10.1^\dagger$	−26.1	$-53.7^\dagger$	$-10.7^\dagger$
30–38	$-21.6^\dagger$	−27.3	−8.9	$-25.9^\dagger$	−34.8	−8.1	$-33.0^\dagger$	−43.5	−9.1
39–47	−21.3	−23.4	−6.2	−20.5	−22.8	−7.1	−23.9	−26.7	−6.4
48–56	−20.3	−21.2	−2.9	−19.9	−20.9	−3.8	−20.0	−21.4	−0.5
57–65	−19.6	−20.2	6.2^*	−20.2	−20.9	5.6^*	−20.4	−21.4	8.1^*
66–74	−18.9	−20.0	2.7	−20.4	−21.5	3.1	−20.0	−21.7	4.2
75–83	−15.4	−17.3	0.2	−17.6	−19.4	0.7	−16.0	−18.7	0.8
84–92	−10.1	−14.5	−0.0	−11.8	−16.0	−0.1	−9.7	−15.7	−0.1
all	−19.1	−22.0	−8.0	−20.7	−24.2	−7.4	−22.7	−27.1	−7.0

^{a} : Model recalibrated with $\sigma = 2.5$ (baseline value in Glover et al. 2011). ^{b} : Model recalibrated with $\sigma = 3.5$.

^{c} : Model is recalibrated with the same targets as the baseline, but calibrated parameters are $\{\beta, f, \mu_\xi, \sigma_\xi, \lambda_x\}$ while r_s is set to an annualized 1.6 percent (10-year treasury rate minus CPI inflation from 2001 to 2007).

† : largest welfare loss. * : largest welfare gain.

Table 20: Sensitivity analysis, baseline calibration.

	Transitory labor shocks^a			No change to σ_ξ^2			Default^b		
age	all	risky>0	risky$=0$	all	risky>0	risky$=0$	all	risky>0	risky$=0$
21–29	−15.9	$-31.6^\dagger$	$-7.1^\dagger$	−18.1	$-31.5^\dagger$	$-10.7^\dagger$	−13.7	−18.5	$-11.1^\dagger$
30–38	$-21.1^\dagger$	−28.4	−4.8	$-19.7^\dagger$	−25.1	−7.9	−15.8	−18.9	−9.0
39–47	−18.6	−21.4	−1.5	−10.4	−11.4	−4.4	−17.0	−18.8	−5.8
48–56	−18.4	−19.5	1.5	−1.1	−1.1	−1.1	−18.8	−19.8	2.5
57–65	−19.6	−20.3	6.2^*	3.6^*	3.4^*	11.0^*	$-19.8^\dagger$	−20.5	6.3^*
66–74	−19.7	−20.8	4.2	−2.2	−2.8	10.0	−19.6	$-20.7^\dagger$	4.2
75–83	−16.5	−18.3	1.0	−5.1	−6.3	5.9	−16.4	−18.3	1.0
84–92	−11.1	−15.4	−0.1	−7.8	−10.9	0.2	−11.1	−15.3	−0.1
all	−18.2	−21.8	−4.3	−8.1	−8.5	−6.7	−17.0	−19.3	−7.8

^{a} : All of the drop in labor income ($t = 1$) is modeled as a temporary downward scaling of the income profile.

^{b} : Households are allowed to default on debt ex post in $t = 1$, in which case they are excluded from asset markets for one period and up to 25 percent of income can be garnished for one period (Chatterjee and Gordon 2012).

† : largest welfare loss. * : largest welfare gain.