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# Inflation, Debt, and Default<sup>1</sup>

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## Abstract

We show that the co-movement of inflation and domestic consumption growth affects real interest rates and the likelihood of debt crises. In particular, a positive co-movement of inflation and consumption lowers risk premia as it makes returns on nominal domestic government debt negatively correlated with domestic consumption. However, such procyclicality also generates default risk since the debt becomes more risky for the government when the economy deteriorates. We calibrate a model of sovereign default on domestic nominal debt, in the presence of exogenous inflation risk and domestic risk averse agents, to assess these joint equilibrium properties of nominal debt, default, and interest rates. Compared to the countercyclical inflation economy, the procyclical inflation economy enjoys a sizable “inflation procyclicality discount” as it features lower real interest rates despite higher default risk. However, in bad times, the procyclical economy faces higher real interest rates due to sharper default risk spikes. These findings are consistent with the evidence across advanced economies and have implications for the debate on the secular decline in real interest rates.

KEYWORDS: inflation risk, domestic nominal debt, interest rates, sovereign default  
JEL CLASSIFICATION CODES: E31, F34, G12, H63

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# 1 Introduction

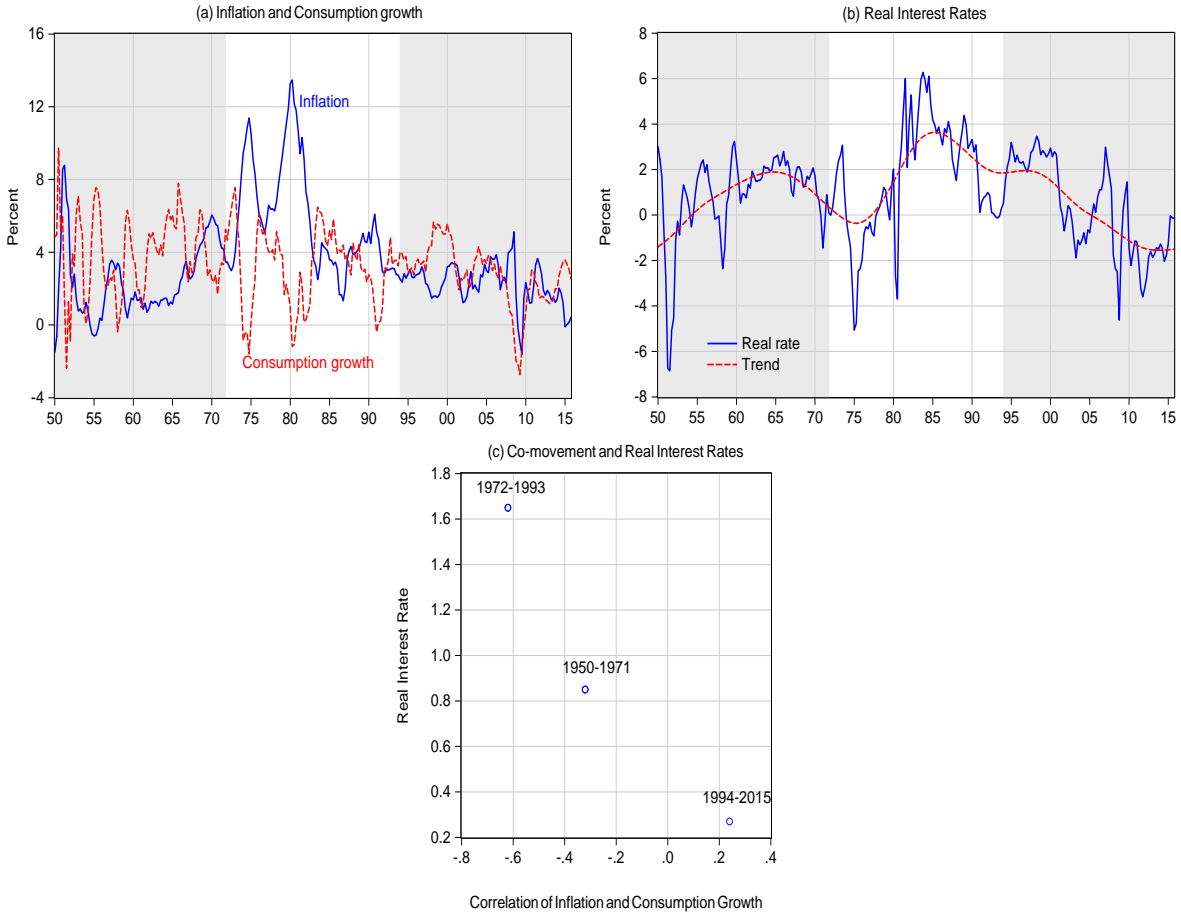
The conduct of monetary policy in advanced economies has experienced deep changes over time such as the Great Moderation, the creation of the European Monetary Union, the granting of operational independence, or the nature of macroeconomic shocks hitting the economy. These developments affect the dynamics of inflation, which in turn influences the pricing of nominal debt. In fact, in advanced economies, the vast majority of public debt is nominal and held domestically.<sup>2</sup> The goal of this paper is to study how changes in the co-movement between inflation and economic activity affect real sovereign yields, debt dynamics, and debt crises.

To be more concrete, in Figure 1 we provide some motivating evidence for the mechanism we want to highlight. In panel (a), we plot quarterly time series for year-on-year U.S. inflation and consumption growth from 1950 to 2015. The point of the panel is to highlight changes in the co-movement of two series over three equal length sub-samples. It shows how in the first sub-sample (1950–1971), the co-movement between inflation and consumption growth is mildly negative, turns to strongly negative in the second sub-sample (1972–1993), and finally becomes positive in the most recent sample (1994–2015). If inflation co-varies positively with domestic consumption growth, then returns on domestic nominal debt are high (low) when consumption growth is low (high). This feature makes domestic nominal bonds less risky from a domestic investor’s perspective, and thus — if government debt is mostly held domestically, as in most developed countries — they should trade, *ceteris paribus*, at a lower real interest rate. The second and third panels in Figure 1 show that this indeed is the case. Panel (b) plots the U.S. real interest rate (along with its trend depicted by the dashed line) over the same sample, while panel (c) plots the average real rate and the average co-movement between inflation and consumption growth in each of the three sub-samples. Notice how the middle sample, which displays the most negative inflation consumption growth co-movement, also is the one with the highest real rate. The most recent sample — where the co-movement has turned positive — displays the lowest real

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<sup>2</sup>For example, as of 2015, the share of public debt held by domestic creditors is 64 percent in the U.S., 69 percent in the United Kingdom, and 78 percent in Canada. [Aizenman and Marion \(2011\)](#) report that the share of U.S. public debt held in Treasury Inflation-Protected Securities (TIPS) is less than 8 percent, as of 2009.

Figure 1: Inflation and Consumption Growth in the US



Note: Inflation is the log difference between CPI in quarter  $t$  and CPI in quarter  $t-4$ . Consumption growth is the log difference in real personal consumption expenditures over the same interval. Real interest rates are nominal interest rates on government securities (from the IMF IFS database) minus expected inflation computed using a linear univariate forecasting model estimated on actual inflation.

rate, while the early sample has intermediate co-movement and an intermediate real rate.

Altogether, this evidence in Figure 1 suggests that the co-movement between inflation and consumption growth is closely connected with the real yield on government debt. The evidence, though striking, is obviously not conclusive as there might be a variety of other factors inducing this pattern. For this reason, in the first part of this paper, we establish this relationship in a more systematic fashion. In particular, we show that for a large sample of advanced economies, in countries and periods in which the co-movement of inflation with domestic consumption growth is high, real interest rates on government bonds tend to be low, even after controlling for a broad array of macroeconomic variables. This suggests that

this co-movement is systematically connected to real interest rates. In that sense, the paper also suggests a new channel through which monetary policy may have been a determinant of the secular decline in real interest rates.

Notice, though, that the same logic that makes nominal debt more attractive to lenders when inflation is procyclical, also suggests that nominal debt is less attractive to borrowers. Consider, for example, a recession which is also accompanied by deflation. In that state, the lenders are happy to receive a large payoff on their nominal assets at the time when their consumption is low (recession). For this reason, they will demand a lower interest rate when lending to the government as discussed earlier. Consider now though the point of view of the borrower (the government), which has to make larger payments at exactly the time when its income is low. Thus, inflation procyclicality would also tend to make debt crises and default by the borrower more likely, which in turn would abruptly push up interest rates in bad times. In fact, we show that the “procyclicality discount” is stronger in good times.

In the second part of the paper, we develop a simple model of debt and default with stochastic inflation that serves two purposes. The first is to show that the empirical connection between debt pricing and inflation dynamics can be understood using simple asset pricing logic. The second is to analyze how the original asset pricing logic for debt pricing and its dynamics is modified in the presence of sovereign default risk.<sup>3</sup>

Our model extends existing models of sovereign debt in two directions. First, we introduce domestic risk averse lenders, in contrast to the common assumption of foreign risk neutral lenders in the literature on sovereign debt crises in emerging economies. This distinction is important since a large amount of public debt is held domestically in advanced economies.<sup>4</sup> Second, we introduce exogenous stochastic inflation so that government bond rates endogenously reflect both inflation risk and default risk. These two features allow us to explicitly analyze the endogenous connection between stochastic discount factors of the domestic lenders, debt pricing, and default probabilities, and to analyze how this relation changes as the co-movement between inflation and consumption growth varies.

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<sup>3</sup>Sovereign default risk, reflected in credit default swap (CDS) spreads, was material even among developed economies during the European Debt Crisis. For instance, Belgium, Greece, Italy, Portugal, and Spain had CDS-implied default probabilities that exceeded 5 percent in 2011 and 2012.

<sup>4</sup>One could also consider the more intricate case where foreign risk averse lenders experience inflation that is correlated with domestic inflation through financial and trade linkages.

Consistent with the data, our model predicts that borrowing costs fall as the covariance of inflation and consumption growth increases. During normal times, relative to its countercyclical counterpart, the procyclical inflation economy sustains similar levels of debt, exhibits higher default risk, and yet enjoys lower borrowing costs. The procyclicality discount reflects the overall reduction in risk perceived by domestic lenders.

However, for a domestic government, debt becomes less attractive in bad times: deflation makes real government obligations larger in recessions, when the government values consumption more. In contrast, the government in the countercyclical economy benefits from the haircut induced by inflation in bad times: a form of partial default. Thus, the cyclical-ity of inflation affects the government’s incentives to borrow or to repay. In our calibrated model, default risk — and thereby the sovereign real rate — spikes markedly and by more during bad times in the the procyclical economy, compared to the countercyclical economy.

Our paper also has implications for the debate on the costs and benefits of joining or exiting a monetary union. Suppose that the union goes into a recession where some, but not all, members of the union get into fiscal trouble. Then the countries in fiscal trouble would like a more countercyclical monetary policy while the others don’t: the contrast over monetary policy increases in a recession. Our paper also suggests that monetary policy may have contributed to the secular decline in real interest rates, while also increasing the likelihood of debt crises during bad times.

**Related literature.** Our paper is related to several strands of literature. On the empirical side, our findings are related to studies on the importance of the inflation risk premium and its variation, as, for example, [Boudoukh \(1993\)](#) or [Ang et al. \(2008\)](#). [Campbell et al. \(2009\)](#), and [Du et al. \(2016\)](#) focus on the covariance of bond returns and other macroeconomic variables such as consumption growth and equity returns. [Song \(2016\)](#) sheds light on the nature of inflation risk in U.S. bonds markets by estimating a model with time variations in the stance of monetary policy as well as the co-movement of inflation and economic activity. On the theoretical side, the backbone of our set-up is a debt default model with incomplete markets as in [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#), or [Arellano \(2008\)](#). While these papers focus on foreign debt, [Reinhart and Rogoff \(2011\)](#) suggest that the connection

between default, domestic debt, and inflation is an important one. [D’Erasmus and Mendoza \(2012\)](#), [Pouzo and Presno \(2014\)](#), and [Arellano and Kocherlakota \(2014\)](#) tackle the issue of default on domestic debt but do not include inflation.<sup>5</sup> [Araujo et al. \(2013\)](#), [Sunder-Plassmann \(2016\)](#), [Mallucci \(2015\)](#) and [Fried \(2017\)](#) study how the currency composition of debt interacts with default crises in emerging economies while [Berriel and Bhattarai \(2013\)](#), [Faraglia et al. \(2013\)](#), and [Ottonello and Perez \(2016\)](#) study nominal debt with inflation, in the absence of default. [Kursat Onder and Sunel \(2016\)](#) and [Nuño and Thomas \(2016\)](#) consider the interaction of inflation and default on foreign investors.<sup>6</sup> Our paper is also related to [Lizarazo \(2013\)](#) who studies default in the context of risk averse international lenders. [Aguiar et al. \(2016\)](#) provide an excellent compendium on modeling risk averse competitive lenders in the sovereign default literature.

Our general question is also related to recent work that studies how joining a monetary union can affect the probability of a self-fulfilling crisis in a debt default model (see [Aguiar et al. 2013](#) and [Corsetti and Dedola 2016](#)). We complement these papers by highlighting how the cyclicity of inflation impacts fundamental-driven default crises, suggesting a promising extension of existing models of self-fulfilling debt crises.

The paper is structured as follows. In section 2, we discuss our empirical findings. In section 3, we develop a simple model of domestic nominal debt, where we discuss the main mechanisms at work. In section 4, we build a quantitative model of domestic debt and default, and section 5 presents our main results on the impact of inflation cyclicity. Section 6 concludes.

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<sup>5</sup>[Broner et al. \(2010\)](#) examine the role of secondary asset markets which make the distinction between foreign and domestic default less stark.

<sup>6</sup>Much of the existing literature has focused on strategic inflation, even hyperinflation, as a countercyclical policy option that governments with limited commitment can use when faced with a high debt burden in bad times. That focus is certainly legitimate for emerging economies but less warranted in the context of advanced economies mainly because of monetary policy independence. Central bank independence makes the cost of hyperinflation higher than the cost of a default, perhaps due to limited participation in bond markets in contrast to the economy-wide effects of hyperinflation.

## 2 Inflation and Real Interest Rates

In this section, we study the empirical relation between several conditional moments of inflation and real interest rates on government debt. The main novel finding of the section is that higher covariance of inflation with economic activity is robustly and significantly associated with lower real interest rates on government debt.

Our dataset includes quarterly observations on real consumption growth, inflation, interest rates on government bonds, and government debt-to-GDP ratios for an unbalanced panel of 21 OECD economies from 1985Q1 to 2015Q4. The countries in the dataset are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Korea, Mexico, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and the United States.

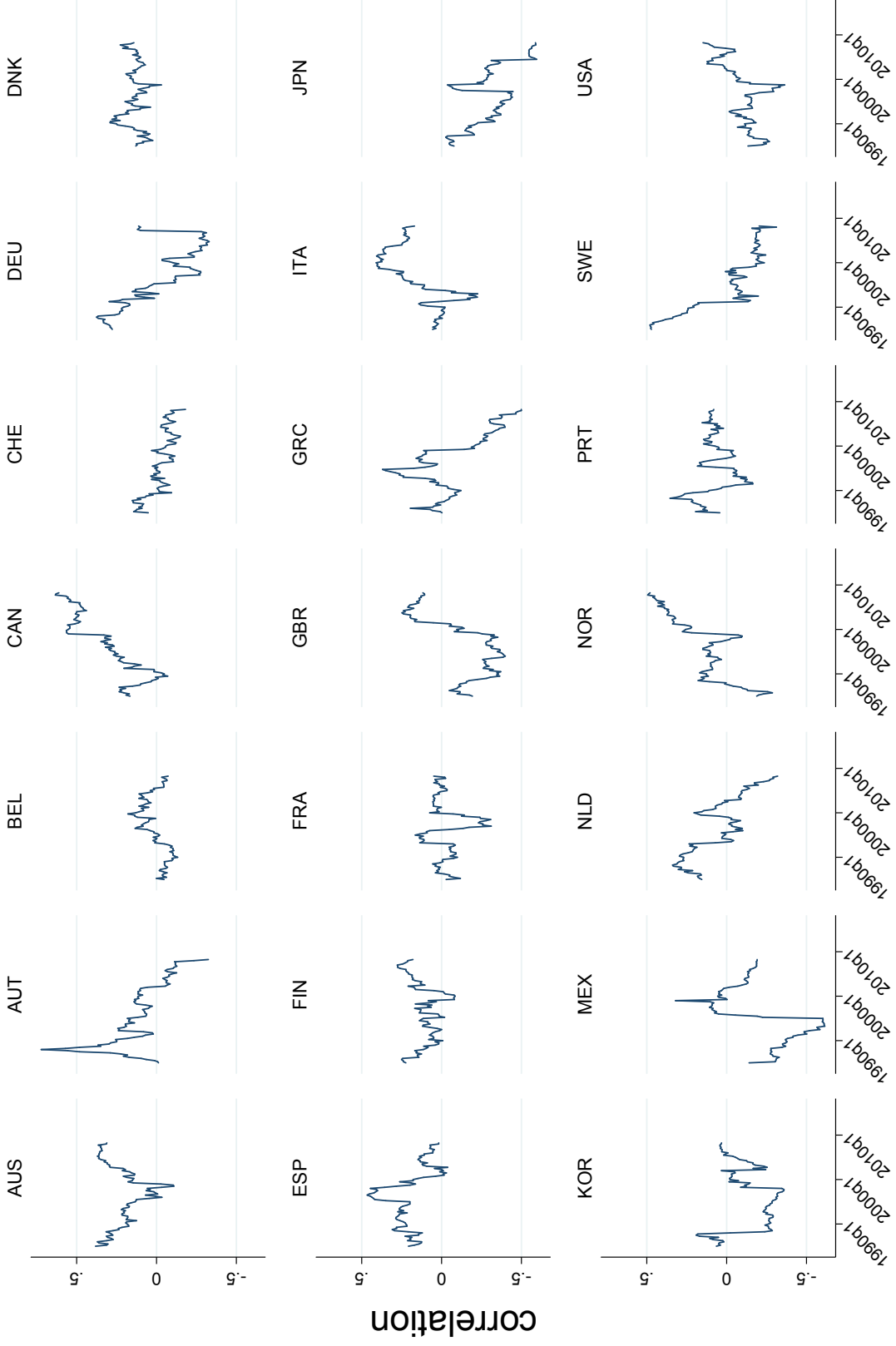
We mainly use quarterly data from the IMF and the OECD to document our empirical findings. We compute inflation as the change in log GDP deflator using data from the OECD. We use nominal interest rates on government bonds from the IMF International Financial Statistics (IFS). For government debt, we use quarterly series from Oxford Economics on gross government debt relative to GDP, extended with quarterly OECD data on central government debt relative to GDP. Quarterly real consumption is constructed as the sum of private and public real consumption using the data from the OECD.

Using this data, we construct real interest rates using expected inflation and the conditional co-movement between inflation and consumption growth. To do, so we follow [Boudoukh \(1993\)](#) and first formulate a vector auto-regression (VAR) model for inflation and consumption growth. The basic VAR is:

$$\begin{bmatrix} \pi_{it} \\ g_{it} \end{bmatrix} = \mathbf{A}_i \begin{bmatrix} \pi_{it-1} \\ g_{it-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\pi it} \\ \varepsilon_{g it} \end{bmatrix} \quad (1)$$

where  $\pi_{it}$  is inflation, and  $g_{it}$  is the change in log consumption in country  $i$  in period  $t$ ,  $A_i$  is a country-specific 2-by-2 matrix, and  $\varepsilon_{\pi it}$  and  $\varepsilon_{g it}$  are innovations in the two time series. We then estimate the VAR using standard OLS and construct time series for residuals  $\varepsilon_{\pi it}$  and  $\varepsilon_{g it}$  for each country.

Figure 2: Conditional correlation between inflation and consumption growth



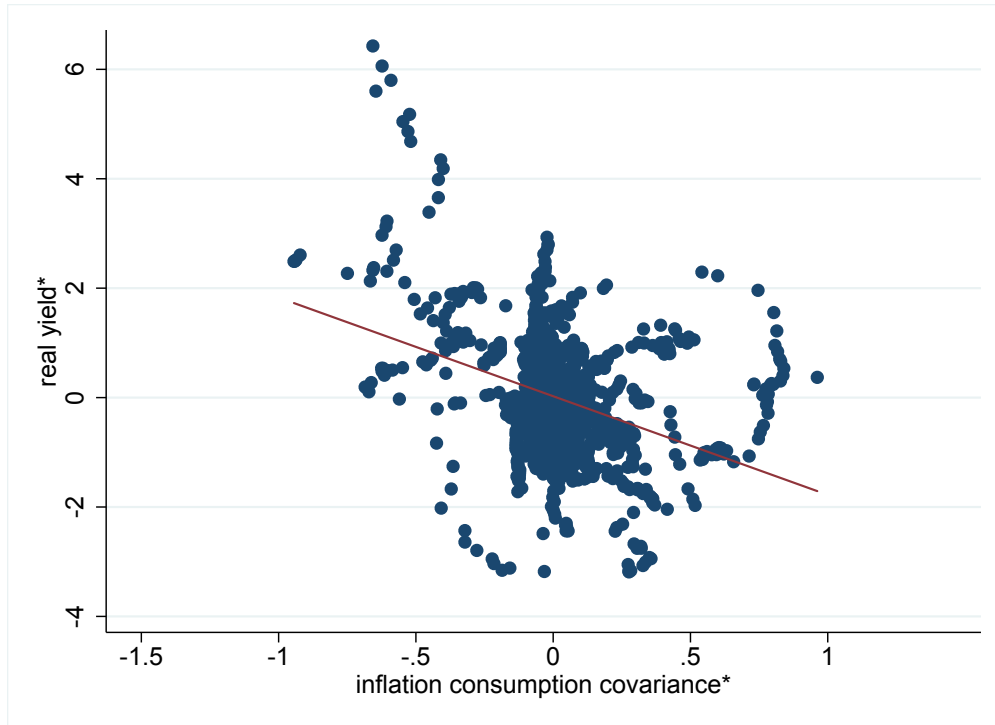


We measure the expected inflation as the forward-looking predicted inflation from the VAR, that is  $\mathbf{E}[\pi_{i,t+1}]$ . We then derive real rates on government debt as nominal rates less expected inflation. Finally, we measure the conditional co-movement between inflation and consumption growth by measuring the co-variance between the two innovations,  $\varepsilon_{\pi it}$  and  $\varepsilon_{git}$ , in overlapping country-windows, comprised of 40 quarters.

In Figure 2, we plot the path of the conditional correlation for the countries in our sample. It illustrates that the co-movement of inflation and consumption growth varies over time and across countries. In many countries, such as Canada, Italy, Norway, the U.S., or the U.K., the co-movement of inflation and consumption growth has clearly increased since the mid-1980s, while it has sharply decreased or fluctuated in other countries such as Germany.

With this dataset, we estimate how the conditional covariance of inflation and consumption growth relates to interest rates faced by governments. All specifications include a full set of country and time fixed effects.

Figure 3: Inflation consumption covariance and real interest rates



\*: conditional residual from a linear model with country and time fixed effects

In Table 1, we regress the real interest rate on the conditional covariance of inflation with consumption growth. The main result from Table 1 is that in periods with higher conditional covariance between inflation and consumption growth, governments face lower interest rates, which can also be seen in Figure 3. This finding is robust to the inclusion of the level of government debt and average inflation and consumption growth (column 2). This association is also robust to the inclusion of the variances of inflation and consumption growth as additional regressors (columns 3). In the appendix, we also show that the results are robust to using different yield measures and different debt measures.

Table 1: Inflation consumption growth co-movement and real interest rates

	Real yield on government debt			
		covariance		correlation
	(1)	(2)	(3)	(4)
Inflation consumption co-movement	-1.890*** (0.383)	-1.514*** (0.324)	-1.563*** (0.450)	-0.997** (0.389)
Lagged government debt	0.017*** (0.005)	0.015*** (0.005)	0.015*** (0.005)	0.015*** (0.005)
Average inflation residual l		2.765** (1.031)	2.748** (1.097)	2.508** (1.075)
Average cons. growth residual		-2.339* (1.148)	-2.337* (1.160)	-2.296* (1.231)
Variance of inflation residual			0.0327 (0.210)	0.167 (0.253)
Var. of cons. growth residual			-0.032 (0.105)	0.203 (0.118)
standard deviation of co-movement	0.200	0.200	0.200	0.211
adj. $R^2$	0.853	0.884	0.884	0.881
$N$	1841	1841	1841	1841

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data is a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, ITA, JAP, KOR, MEX, NLD, NOR, POR, SWE, USA. All variables are computed over a forward-looking ten-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window:  $\mathbf{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$ . Other regressors are averages and variances of those residuals in the window and lagged debt.

Overall, our results show that the co-movement of inflation and consumption growth are

associated with lower real interest rates that governments face. We call this the *inflation procyclicality discount*. The magnitude of this discount is economically significant. As an illustration of its magnitude, consider moving from a country/time period in which the inflation/consumption correlation is around -0.3 (for example, the U.S. in the 1980s) to a sample period in which the correlation is around 0.1 (for example, the U.S. in the 2000s). This roughly corresponds to a change in correlation equal to two times the standard deviation of correlation in our sample. Using the coefficient estimated in column (4) of Table 1, we can see that this increase in procyclicality is associated to a lowering of real rates by 40 basis points. Similarly, using the coefficient estimated in column (3) of Table 1, a fall in covariance that is twice as large as our sample standard deviation, is associated with a decrease in real rates by 62 basis points.

Table 2: Inflation procyclicality discount: Interaction with good times by region

Region	Real yield on government debt					
	(1) full sample	(2)	(3) excluding KOR,MEX,GRC	(4)	(5) Adv. and Euro excluding GRC	(6)
Inflation consumption covariance	-1.563*** (0.450)	-1.346** (0.572)	-1.987** (0.747)	-1.168 (0.794)	-1.150 (1.045)	1.187 (2.070)
Indicator(good times)		-0.215 (0.223)		-0.022 (0.172)		-0.171 (0.169)
Interaction term		-0.800 (1.018)		-1.527* (0.851)		-3.430** (1.493)
Other controls	Yes	Yes	Yes	Yes	Yes	Yes
adj. $R^2$	0.884	0.887	0.913	0.915	0.911	0.916
$N$	1841	1841	1644	1644	1102	1102

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data is a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, ITA, JAP, KOR, MEX, NLD, NOR, POR, SWE, USA. All variables are computed over a forward-looking ten-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window:  $\text{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$ . Other regressors are averages and variances of those residuals in the window and lagged debt.

Our second main finding is that there is a stronger procyclicality discount in good times. This can be seen in Table 2, which includes an indicator for good times, defined to be a (10-year) window in which the average residual consumption growth is positive, and its

interaction with the covariance of inflation and consumption growth. Column (2) shows that over the entire sample, the interaction term is negative but not statistically significant. Columns (3) and (4) show that the general procyclicality discount is robust to excluding Korea, Mexico, and Greece, and that the interaction term is negative and statistically significant, implying that the inflation cyclicality discount is stronger in good times. Finally, when we further limit the sample to advanced Euro-area economies and Great Britain, Japan, and the United States, the unconditional procyclicality discount is no longer statistically significant, but its interaction with the good-times indicator remains negative and statistically significant.

Overall, we find that procyclical inflation episodes are associated with a significant discount on real sovereign yields albeit such “inflation procyclicality discount” vanishes in bad times. The standard consumption-based asset pricing model suggests that the hedging benefits of procyclical inflation rationalize an inflation procyclicality discount. However, the state-dependent nature of the procyclicality discount suggests that bad times are associated with additional credit risk, possibly default risk. From the government’s perspective, inflation procyclicality is not desirable in bad times *ceteris paribus* and reduces the government’s willingness to pay. In the next section, we develop a simple theory to understand the relation between the inflation-consumption growth co-movement, interest rates, and default.

### 3 Simple Model

#### 3.1 Simple model without default

Consider a two-period model with a representative lender and a representative borrower. The borrower and the lender receive endowments of  $\tau$  and  $1 - \tau$  respectively in the first period, and  $\tau x$  and  $(1 - \tau)x$  in the second period where  $x$  is a random variable with c.d.f.  $F$  over  $X$ . The lender can sell bonds  $b$  at a price  $q$ . The borrower repays  $(1 - \gamma x)b$  in the second period where  $(1 - \gamma x)^{-1}$  is inflation and  $\gamma$  represents the cyclicality of inflation.

The borrower solves

$$\max_b u(\tau + qb) + \beta_b \int_X u(\tau x - b(1 - \gamma x)) dF(x), \quad (2)$$

and the lender solves

$$\max_b u(1 - \tau - qb) + \beta_\ell \int_X u((1 - \tau)x + b(1 - \gamma x)) dF(x), \quad (3)$$

where  $\beta_b$  and  $\beta_\ell > \beta_b$  are the discount factors of the borrower and lender, respectively.

Let  $u(c) = Ac - \frac{\phi}{2}c^2$ , with  $c < A/\phi$  and  $x \in X \equiv [x, \bar{x}]$ . Further, we assume that  $\mu_x = \int_X x dF(x) = 0$ . Then  $\sigma^2 = \int_X x^2 dF(x)$ . We also assume that  $A/\phi > 1$ .

Theorem 1 shows that, under certain conditions, we can demonstrate an inflation procyclicality discount arising from the hedging benefits of inflation procyclicality.

### Theorem 1. Inflation procyclicality discount

*Near  $\gamma = 0$ , there is an inflation procyclicality discount. That is,*

$$\frac{\partial q}{\partial \gamma} > 0. \quad (4)$$

*Proof:* See Appendix B.

The intuition for the result can be seen from the lender's Euler equation, which is given by

$$q(A - \phi(1 - \tau - qb_\ell)) = \beta_\ell(A + \phi(1 - \tau)\gamma\sigma_x^2) - \beta_\ell\phi(1 + \gamma^2\sigma_x^2)b_\ell. \quad (5)$$

Note that the first term on the right-hand side of equation (5) shows the covariance effect and the second term shows the variance effect. One can show that near  $\gamma = 0$ , the covariance effect dominates the variance effect, leading to a higher bond price, that is, lower interest rates, as  $\gamma$  increases.<sup>7</sup>

## 3.2 Simple model with default

Now consider the possibility that the borrower may choose to default on its bond payments. In the case of default, no payments are made, and the second period endowment becomes  $\tau x_{def}$  and  $(1 - \tau)x_{def}$  for the borrower and lender respectively. In this environment, the

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<sup>7</sup>For simplicity, here  $\gamma$  governs both the cyclicity and the volatility of inflation. As a result, when  $\gamma$  is large the inflation variance penalty can confound the inflation procyclicality discount.

default set  $\widehat{X}$  is given by

$$x < \widehat{x} \equiv \frac{\tau x_{def} + b}{\tau + b\gamma}. \quad (6)$$

It is straightforward to show that, *ceteris paribus* and for  $x_{def}$  low enough, the probability of default  $F(\widehat{x})$  increases with the level of debt  $b$  and the cyclicity of inflation  $\gamma$ .

The borrower solves

$$\max_b u(\tau + qb) + \beta_b \int_{\widehat{x}}^{\bar{x}} u(\tau x - (1 - \gamma x)b) dF(x) + \beta_b u(\tau x_{def}) F(\widehat{x}) \quad (7)$$

and the lender solves

$$\begin{aligned} \max_b u(1 - \tau - qb) + \beta_\ell \int_{\widehat{x}}^{\bar{x}} u((1 - \tau)x + (1 - \gamma x)b) dF(x) \\ + \beta_\ell u((1 - \tau)x_{def}) F(\widehat{x}). \end{aligned} \quad (8)$$

The lender's Euler equation is given by

$$\begin{aligned} q(A - \phi(1 - \tau - qb)) = \underbrace{(1 - F(\widehat{x}))}_{\text{default premium}} \beta_\ell \left[ A + \underbrace{\phi(1 - \tau)\gamma\sigma_R^2}_{\text{conditional covariance term}} - \underbrace{\phi(1 + \gamma^2\sigma_R^2)}_{\text{conditional variance term}} b \right] \\ - (1 - F(\widehat{x})) \beta_\ell [A\gamma + \phi(1 - \tau) - 2\phi\gamma b] \mu_R \end{aligned} \quad (9)$$

where the conditional moments are

$$\mu_R \equiv \frac{\int_{\widehat{x}}^{\bar{x}} x dF(x)}{1 - F(\widehat{x})} \quad (10)$$

$$\sigma_R^2 \equiv \frac{\int_{\widehat{x}}^{\bar{x}} x^2 dF(x)}{1 - F(\widehat{x})}. \quad (11)$$

Compared to equation (5), we can see that equation (9) features a default premium  $F(\widehat{x})$  in addition to the channels highlighted earlier: a conditional inflation cyclicity discount and a conditional variance premium. On one hand, high inflation cyclicity leads to lower real rates. On the other hand, high inflation cyclicity leads to higher default probabilities,

which results in higher real rates.<sup>8</sup> This simple model motivates our quantitative model, in which the cyclical nature of inflation is a key driver of real sovereign yields, nominal debt dynamics, and default risk.

## 4 Quantitative Model

We extend the standard sovereign default model of [Eaton and Gersovitz \(1981\)](#) and [Arellano \(2008\)](#) in two dimensions: exogenous *inflation* and risk averse *domestic lenders*.

### 4.1 Environment

We consider a closed economy that is populated by a continuum of households who lend to the government. Both the representative household and the government have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta_i^t u_i(c_{it}) \quad (12)$$

where  $0 < \beta_g < \beta_\ell < 1$  and  $c_{gt}, c_{\ell t}$  are the discount factors and consumption at time  $t$  of the government and the household respectively. Both agents' period utility function is given by

$$u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i} \quad (13)$$

where  $\gamma_g$  and  $\gamma_\ell$  represent the risk aversion of the government and households respectively. The households receive a stochastic stream of non-storable consumption good  $y$ , which is taxed by the government at rate  $0 < \tau < 1$ .

### 4.2 Government

The government has access to debt markets in which it issues one-period non-contingent bonds to the domestic lenders. Bonds are risky because debt contracts are not enforceable, which may lead to government default, and also because they may lose value due to exogenous

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<sup>8</sup>The last term is related to the conditional mean,  $\mu_R$ , and does not have a counterpart in the model without default since we assume that  $\mu_x = 0$ .

inflation. Stochastic endowments  $y$  and inflation  $\pi$  follow a joint Markov Process. Let us denote  $s = (y, \pi)$ . Given the option to default,  $V^o(B, s)$  satisfies

$$V^o(B, s) = \max_{c,d} \{V^c(B, s), V^d(B, s)\} \quad (14)$$

where  $B$  is incoming government assets,  $V^c$  is the value of not defaulting, and  $V^d$  is the value of default.

When the government defaults, the economy is in financial autarky for a stochastic number of periods and income may fall. Upon reentry, after  $k$  periods, the government's debt obligation is  $-\lambda^k B$  where  $1 - \lambda$  is the rate at which the government's debt obligation decays each period. This tractable way of modeling partial default is also consistent with the fact that longer default episodes are associated with lower recovery rates, as documented by [Benjamin and Wright \(2009\)](#). Setting  $\lambda = 0$  corresponds to the model with full default.

The government's value of default is then given by

$$V^d(B, s) = u_g(\tau(y - \phi^d(y))) + \beta_g \mathbf{E}_{s'|s} \left[ \theta V^o\left(\frac{\lambda B}{1 + \pi'}, s'\right) + (1 - \theta) V^d\left(\frac{\lambda B}{1 + \pi'}, s'\right) \right] \quad (15)$$

where  $0 < \theta < 1$  is the probability that the government will regain access to credit markets, and  $\phi^d(y)$  is the loss in income during default. In particular, we assume a quadratic function

$$\phi^d(y) = \max \left\{ 0, \frac{d_1}{d_0} y + \left( d_1 - \frac{d_1}{d_0} \right) y^2 \right\}, \quad (16)$$

similar to [Chatterjee and Eyigungor \(2013\)](#), except that the expression has been written such that  $d_1$  is the default cost at mean output ( $y = 1$ ) and  $d_0$  is the deviation from mean output above which the default costs are positive (that is,  $y > 1 + d_0$ ).

The value of not defaulting, is given by

$$V^c(B, s) = \max_{B' \leq 0} \left\{ u_g(\tau y - q(B, s, B')B' + B) + \beta_g \mathbf{E}_{s'|s} \left[ V^o\left(\frac{B'}{1 + \pi'}, s'\right) \right] \right\} \quad (17)$$

where  $q(B, s, B')$  is the bond price schedule the government faces. Note that the real return on government debt is stochastic, even in the absence of default, due to inflation risk.



### 4.3 Households

Households take as given the price of bonds,  $q^*(B, s)$ , and the policy functions for government assets,  $B^*(B, s)$ , and default,  $d^*(B, s)$ . If the government does not have access to credit markets, the lender's value function is given by

$$W^d(b; B, s) = \max_{b'} u_\ell((1 - \tau)(y - \phi^d(y)) - q^d(B, s)(b' - b)) \quad (18)$$

$$+ \beta_\ell \mathbf{E}_{s'|s} \left\{ \begin{array}{l} \theta (1 - d^*(\frac{\lambda B}{1+\pi'}, s')) W^c(\frac{\lambda b'}{1+\pi'}; \frac{\lambda B}{1+\pi'}, s') \\ + (1 - \theta + \theta d^*(\frac{\lambda B}{1+\pi'}, s')) W^d(\frac{\lambda b'}{1+\pi'}; \frac{\lambda B}{1+\pi'}, s') \end{array} \right\}$$

where  $q^d$  is the price of a bond in default and  $W^c$  is the household's value function when the government has access to credit markets, which is given by

$$W^c(b; B, s) = \max_{b'} u_\ell((1 - \tau)y + b - q^*(B, s)b') \quad (19)$$

$$+ \beta_\ell \mathbf{E}_{s'|s} \left\{ \begin{array}{l} \left(1 - d^*\left(\frac{B^*(B, s)}{1+\pi'}, s'\right)\right) W^c\left(\frac{b'}{1+\pi'}; \frac{B^*(B, s)}{1+\pi'}, s'\right) \\ + d^*\left(\frac{B^*(B, s)}{1+\pi'}, s'\right) W^d\left(\frac{b'}{1+\pi'}; \frac{B^*(B, s)}{1+\pi'}, s'\right) \end{array} \right\}$$

where  $q^*(B, s) = q(B, s, B^*(B, s))$ . Note that the first term inside the expectation operator in (18) represents the continuation value in the case that government regains access to credit next period and does not immediately default. The second term represents the continuation value in the case that either the government does not regain access to credit markets or immediately defaults upon regaining access to credit. In both cases, the lender's real assets decay by  $1 - \lambda$ .

### 4.4 A political economy interpretation

The current model setup, in which the government maximizes the expected utility derived from its own consumption, is equivalent to a political economy model, in which the government borrows from a mass  $\mu$  of domestic risk averse households who are rich and patient, and maximizes the welfare of a median voter group composed by a unit mass of hand-to-mouth risk averse households who are  $\alpha$ -times poorer and impatient. When  $\tau = (1 + \alpha\mu)^{-1}$ , the

two models are equivalent.

## 4.5 Recursive equilibrium

**Definition.** The *recursive equilibrium* for this economy is defined as a set of (i) policy functions for household assets  $b^*(b; B, s)$ , (ii) policy functions for government assets  $B^*(B, s)$  and default  $d^*(B, s)$ , and (iii) price functions  $q(B, s, B')$  and  $q^d(B, s)$  such that:

1. Taking as given government policies and bond prices, the representative household's policy functions solve the optimization problem in (18) and (19).
2. Taking as given the bond price schedule, the government's policy functions solve the optimization problem in (14), (15), and (17).
3. The bond market clears,

$$b^*(B; B, s) + B^*(B, s) = 0. \quad (20)$$

4. No bond trading in default,

$$b^*(b; B, s) = b. \quad (21)$$

## 4.6 Characterization

In this environment, the bond price schedule satisfies

$$\begin{aligned} q(B, s, B') &= \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{1 - d^*\left(\frac{B'}{1+\pi'}, s'\right) u'_\ell\left(c_\ell^*\left(\frac{B'}{1+\pi'}, s'\right)\right)}{1 + \pi'} \frac{u'_\ell\left(c_\ell(B, s, B')\right)}{u'_\ell\left(c_\ell(B, s, B')\right)} \right] \\ &+ \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{d^*\left(\frac{B'}{1+\pi'}, s'\right)}{1 + \pi'} q^d\left(\frac{B'}{1 + \pi'}, s'\right) \frac{u'_\ell\left((1 - \tau)(y' - \phi^d(y'))\right)}{u'_\ell\left(c_\ell(B, s, B')\right)} \right] \end{aligned} \quad (22)$$

where the state-contingent consumption of the lender is

$$c_\ell(B, s, B') = (1 - \tau)y - B + q(B, s, B')B' \quad (23)$$

$$c_\ell^*(B, s) = (1 - \tau)y - B + q^*(B, s)B^*(B, s) \quad (24)$$

The bond price schedule  $q(B, s, B')$  reflects a penalty (discount) for future inflation (deflation)  $(1 + \pi)$ , a default premium for future default  $d^* \left( \frac{B'}{1 + \pi'}, s' \right)$ , and the lender's stochastic discount factor (SDF), which is also endogenous since it varies with the debt policy function  $B^*(\cdot)$ .

Similarly, the default bond price is

$$q^d(B, s) = \beta_\ell \lambda \theta \mathbf{E}_{s'|s} \left[ \frac{1 - d^* \left( \frac{\lambda B}{1 + \pi'}, s' \right)}{1 + \pi'} \frac{u'_\ell \left( c_\ell^* \left( \frac{\lambda B}{1 + \pi'}, s' \right) \right)}{u'_\ell \left( (1 - \tau)(y - \phi^d(y)) \right)} \right] \quad (25)$$

$$+ \beta_\ell \lambda \mathbf{E}_{s'|s} \left[ \left( 1 - \theta + \theta d^* \left( \frac{\lambda B}{1 + \pi'}, s' \right) \right) \frac{q^d \left( \frac{\lambda B}{1 + \pi'}, s' \right)}{1 + \pi'} \frac{u'_\ell \left( (1 - \tau)(y' - \phi^d(y')) \right)}{u'_\ell \left( (1 - \tau)(y - \phi^d(y)) \right)} \right].$$

It is convenient to further decompose the bond pricing formula in order to highlight the different channels in the bond price. By using the definition of covariance, we can expand the expectation of the product of future deflation, repayment probabilities, and the lender's SDF to obtain

$$q(B, s, B') = \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{1}{1 + \pi'} \right] \mathbf{E}_{s'|s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] \mathbf{E}_{s'|s} \left[ \frac{u'_\ell \left( c_\ell^* \left( \frac{B'}{1 + \pi'}, s' \right) \right)}{u'_\ell \left( c_\ell(B, s, B') \right)} \right] \quad (26)$$

$$+ \beta_\ell \mathbf{E}_{s'|s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right] \mathbf{cov}_{s'|s} \left[ \frac{1}{1 + \pi'}, \frac{u'_\ell \left( c_\ell^* \left( \frac{B'}{1 + \pi'}, s' \right) \right)}{u'_\ell \left( c_\ell(B, s, B') \right)} \right]$$

$$+ \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{1}{1 + \pi'} \right] \mathbf{cov}_{s'|s} \left[ 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right), \frac{u'_\ell \left( c_\ell^* \left( \frac{B'}{1 + \pi'}, s' \right) \right)}{u'_\ell \left( c_\ell(B, s, B') \right)} \right]$$

$$+ \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{u'_\ell \left( c_\ell^* \left( \frac{B'}{1 + \pi'}, s' \right) \right)}{u'_\ell \left( c_\ell(B, s, B') \right)} \right] \mathbf{cov}_{s'|s} \left[ \frac{1}{1 + \pi'}, 1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right) \right]$$

$$+ \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{d^* \left( \frac{B'}{1 + \pi'}, s' \right)}{1 + \pi'} q^d \left( \frac{B'}{1 + \pi'}, s' \right) \frac{u'_\ell \left( (1 - \tau)(y' - \phi^d(y')) \right)}{u'_\ell \left( c_\ell(B, s, B') \right)} \right].$$

The first line in (26) shows that the conditional probability of default and expected inflation increase borrowing costs — effects which are standard but are now endogenous to the cyclicity of inflation.

The second line shows the standard direct effect of inflation cyclicity: inflation procycli-

cality *reduces* borrowing costs through the hedging benefit of the co-movement of inflation and consumption growth.

The third line shows that the countercyclicality of default also increases borrowing costs since the lenders are domestic and value consumption more in the bad states that coincide with default. The fourth line shows how the co-movement of default and inflation affects borrowing costs: in the procyclical economy, defaults tend to coincide with surprise deflation, eliminating what would have been high returns for the lender.

Overall, equation (26) elicits the intuition from equation (9) in the simple model: the cyclicity of inflation in a model with domestic default entails various endogenous channels including, but not limited to, an endogenous default risk and the standard hedging argument. The interplay between these channels also vary over the cycle: inflation procyclicality is likely to be associated with a discount when default risk is low, but not in bad times as default motives are increased with inflation procyclicality. We turn to a quantitative analysis of these forces in the next section and use the model to assess the implications of the “inflation procyclicality discount” we documented.

## 5 Quantitative analysis

In this section, we use a calibrated version of the model to investigate the role of the inflation process on the dynamics of interest rates, debt, and default crises. In particular, we calibrate the model with zero covariance to match conditional default frequencies and the spread in advanced economies. We then use the covariance estimates from section 2 to assess the impact of different inflation processes on interest rates, debt dynamics, and default crises.

### 5.1 Functional forms and calibration

Endowments  $y$  and inflation  $\pi$  follow a joint process:

$$\begin{bmatrix} \log y' \\ \pi' \end{bmatrix} = \begin{bmatrix} \rho_{y,y} & \rho_{\pi,y} \\ \rho_{y,\pi} & \rho_{\pi,\pi} \end{bmatrix} \begin{bmatrix} \log y \\ \pi \end{bmatrix} + \begin{bmatrix} \epsilon_y \\ \epsilon_\pi \end{bmatrix} \quad (27)$$

where

$$\begin{bmatrix} \epsilon_y \\ \epsilon_\pi \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \sigma_{\pi,y} \\ \sigma_{\pi,y} & \sigma_\pi^2 \end{bmatrix} \right).$$

Since we consider a closed economy environment, output in our model is equal to consumption. This allows us to use our VAR results from (1) to guide our parameter values. Specifically, we set the persistence of output  $\rho_{y,y}$  to 0.8, the persistence of inflation  $\rho_{\pi,\pi}$  to 0.8, the spillover terms  $\rho_{y,\pi}$  and  $\rho_{\pi,y}$  to zero, and both variance terms  $\sigma_y$  and  $\sigma_\pi$  to 0.010 based on the parameters estimated for the cross section of OECD economies in our dataset. Table 9 in appendix A contains the detailed estimates by country. We consider two values for the covariance of inflation and output  $\sigma_{\pi,y}$ : +0.00003 and -0.00003. These values represent a 1.5 standard deviation increase and decrease from the median of the covariance of inflation and consumption residuals computed at 10 year windows, which is close to zero.

We choose the government's discount factor  $\beta_g$  and income loss parameters  $d_0$  and  $d_1$  to jointly match default probabilities of 1.11 percent in good times and 2.71 percent in bad times, and an average interest rate spread, which is 0.74 percent. The default probabilities are computed using CDS-implied default probabilities for OECD countries between 2001 and 2015 with a bad-times threshold of 1 standard deviation below trend consumption. Sovereign spreads are computed by subtracting the German nominal rate from the nominal rates of Eurozone economies, excluding Greece, over the same period.

We set the discount factor  $\beta_\ell$  of the lender to be 0.99 to match an implied risk-free rate of 1 percent. We set the government's risk aversion  $\gamma_g$  to be 2, as is standard in the macro and sovereign debt literature. We set the lender's risk aversion  $\gamma_\ell$  to be 8, following [Storesletten et al. \(2007\)](#). This higher level of risk aversion of the lender is also common in the finance and equity premium puzzle literature (for example, see [Bansal and Yaron 2004](#) and [Mehra and Prescott 1985](#)).

The probability of re-entry  $\theta$  is set to match the average exclusion of 10 quarters as documented by [Richmond and Dias \(2008\)](#) and the recovery parameter  $\lambda$  is set to be consistent with the average recovery rate of 50 percent reported by [Benjamin and Wright \(2009\)](#). To compute the average recovery rate, we take the following steps. First, we consider a default to be over when the government regains access to credit, which on average lasts 10 quarters.

Table 3: Parameters

parameters	values	targets
gov't discount factor $\beta_h$	0.758	default prob. in good times: 1.11 percent*
income loss cutoff $d_0$	-0.033	average spread: 0.74 percent**
default penalty at mean $d_1$	0.044	default prob. in bad times: 2.71 percent*
$\rho_{y,y} = \rho_{\pi,\pi}$	0.80	VAR estimates (OECD cross section)
$\rho_{\pi,y} = \rho_{y,\pi}$	0.00	VAR estimates
$\sigma_y = \sigma_\pi$	0.01	VAR estimates
$\sigma_{\pi,y}$	0.00	acyclical baseline $\pm 0.00003$ (1.5 s.d.)
Lender discount factor $\beta_\ell$	0.99	risk-free rate: 1 percent
Lender risk aversion $\gamma_\ell$	8	<a href="#">Storesletten et al. (2007)</a>
Gov't risk aversion $\gamma_g$	2	
Probability of re-entry $\theta$	0.10	average exclusion: 10 quarters <sup>†</sup>
Recovery parameter $\lambda$	0.96	average recovery rate, discounted to period of default: 50 percent <sup>‡</sup>
Tax rate $\tau$	0.19	gov't consumption (percent GDP)

\* : CDS-implied default probabilities 2001-2015 (threshold: 1 s.d. below trend consumption), \*\* : Eurozone (ex. Greece) nominal rates less German rates 2001-2015, † : [Richmond and Dias \(2008\)](#), ‡ : [Benjamin and Wright \(2009\)](#)

Second, we discount the payment back to the period of default by an annualized interest rate of 10 percent as in [Benjamin and Wright \(2009\)](#).

Finally, we set the tax rates to be 19 percent to match the government consumption share of GDP in OECD economies between 1985 and 2015. A summary of our parameters can be found in Table 3.<sup>9</sup>

## 5.2 Results

Using the calibrated model, we contrast two inflation regimes: a procyclical economy with an inflation-output covariance of +0.00003 and a countercyclical economy with a covariance of -0.00003, which corresponds to 1.5 standard deviations above and below zero. We report the long-run business cycle statistics for both regimes in Table 4.

<sup>9</sup>A tax rate  $\tau = .18$  is equivalent to the political economy version calibrated to match U.S. income inequality between those with an annual income above \$40,000 a year ( $\sim$  top 42 percent) and those below ( $\sim$  bottom 58 percent). Specifically, this is achieved by choosing  $\mu = 42.1/57.9 = .73$  is the relative size of the two groups and  $\alpha = 110.5K/17.5K = 6.31$  is the ratio of their average incomes.

Table 4: Business cycle statistics

	Positive co-movement	Negative co-movement
Default rate (percent)	1.44	1.25
Spreads (percent)	0.76	0.79
Debt (percent of borrower income)	5.60	5.61

We find that overall, relative to the countercyclical economy, the procyclical inflation regime (i) enjoys lower borrowing costs, (ii) accumulates almost the same debt level, and (iii) defaults more frequently. The calibrated model features low levels of debt for both regimes. This is partly due to our targeted default probabilities, which are somewhat higher than usual since the periods for which we have the CDS-implied default probabilities include the Great Recession and the European Debt crisis.

More importantly, the fact that the procyclical regime features lower rates despite higher default likelihood highlights the endogenous “inflation procyclicality discount” in the model. In fact, from the perspective of a hypothetical risk-neutral lender, the procyclical economy would face interest rates that are about 20 basis points higher than the countercyclical economy since its default probability is about higher by .20 percentage point higher. Instead, the endogenous effects of inflation cyclicity affect real rates, default risk and debt dynamics.

A further decomposition of the bond price, as shown in Table 5, reveals that the standard direct ‘hedging’ effect of the higher covariance of inflation and output is a discount of 13 basis points, which accounts for 14 percent of the difference of 94 basis points across the two regimes that is implied by the empirical estimate from Table 1. Taking into account the general equilibrium effects of the change in cyclicity, we also report the overall effect, defined as the difference in bond price controlling for default, which is 25 basis points, accounts for 27 percent of the difference in the data. The overall effect of inflation cyclicity on interest rates is therefore amplified through the endogenous effect on default risk and the implied co-movement of default with inflation and with consumption growth (see equation (26) for a decomposition of the pricing of bonds).

However, procyclicality does not always have an overall positive effect of lower rates on interest rates relative to its countercyclical counterpart. When times are good, the procycli-

Table 5: Bond price decomposition

	Positive co-movement	Negative co-movement	Difference (annual b.p.)
price $q$	0.9902	0.9902	2
no default $\mathbf{E}[1 - d]$	0.9962	0.9968	-21
direct effect: $\mathbf{E}[\text{no def.}] \mathbf{cov}(\text{irms, defl.})$	0.0141	-0.0184	13
overall effect: $q/\mathbf{E}[1 - d]$	0.9940	0.9934	25

cal economy enjoys lower borrowing costs despite a higher likelihood of default. When the economy begins to deteriorate, for example with a successive sequence of low output shocks, then the procyclical economy is more likely to face low inflation, or possibly deflation, resulting in an appreciation of the government's real debt obligations, leading to a debt crisis. In bad times, the procyclical economy faces higher borrowing costs, reflecting the significantly higher probability of default. We classify a period as a bad (good) time in the model if output is below (above) a threshold of 1 standard deviation below mean output. This can be seen in Table 6 where we report average spreads and conditional default probabilities. The table shows that the procyclical economy effectively faces typically lower, but also more volatile, interest rates. In bad times, default risk spikes more sharply in the procyclical economy compared to the countercyclical economy as shown in Table 6. The countercyclical faces smaller surges in default risk partly because inflation innovations shave off some of the debt burden precisely when the default risk is heightened.

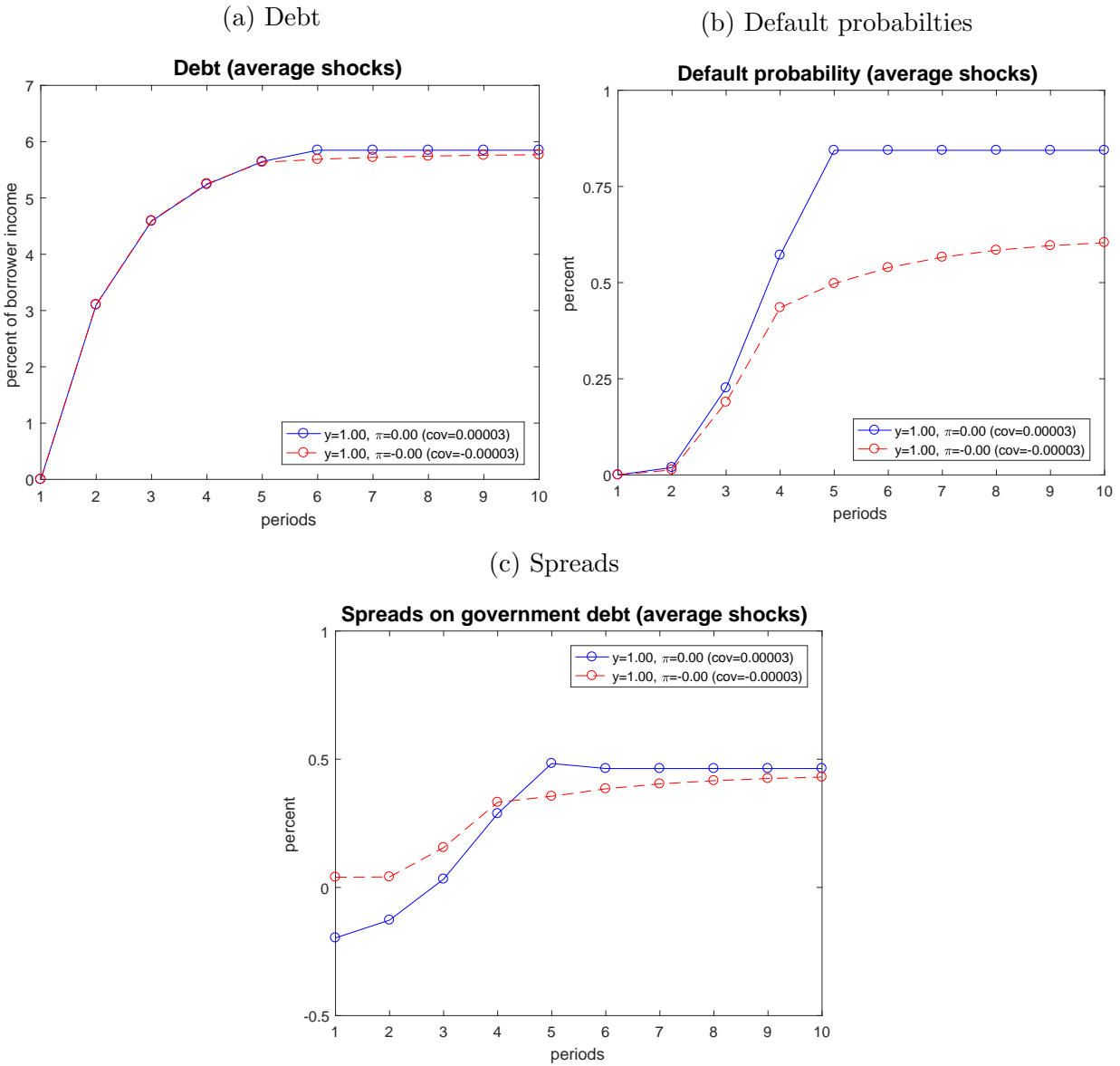
Table 6: Spreads

	Positive co-movement	Negative co-movement	Difference
Overall spreads (percent)	0.76	0.79	-0.03
Spreads in good times (percent)	0.55	0.63	-0.08
Spreads in bad times (percent)	1.86	1.66	0.20
Def. prob. in good times (percent)	1.08	0.97	0.11
Def. prob. in bad times (percent)	3.44	2.73	0.71

We can further investigate these channels by considering a simulation in which we begin both economies with zero debt and simulate a sequence of average output and inflation



Figure 4: Simulation of average shocks



shocks. In this simulation, we observe that the procyclical economy accumulates more debt, initially at a lower cost, despite having higher probabilities of default, as can be seen in Figure 4. Notice that until period 4, the procyclical economy faces significantly lower borrowing costs, despite having higher default probabilities than the countercyclical economy. However, from period 5 onward, the procyclical economy becomes much more likely to default, leading to higher borrowing costs than the countercyclical economy. This simulation illustrates

the state-dependent nature of the inflation procyclicality discount we document as well as potential vulnerabilities to debt crises that are associated with such procyclicality-driven discount.

## 6 Conclusion

The goal of this paper was to investigate how inflation cyclicality affects borrowing costs, and debt and default dynamics. Empirically, we documented that the co-movement of inflation innovations and consumption growth innovations fluctuates over time across a large number of advanced countries. Moreover, we find that increased co-movement of inflation and consumption growth is associated with lower borrowing costs, especially in good times. Theoretically, we showed that the inflation processes—especially inflation cyclicality—can be important in explaining interest rates and the dynamics of default. In particular, the procyclical inflation economy faces lower borrowing costs, even while default is more likely. However, when the economy deteriorates, the procyclical economy faces a much higher likelihood of facing a debt crisis, because it is more likely to face lower inflation, possibly even deflation, and thus an increasing real debt burden. Our findings have implications for the debate on the costs and benefits of joining or exiting monetary unions. Our findings also suggests a new channel, the interaction of monetary policy and interest rates in the presence of sovereign credit risk, that can help understand the secular decline in real rates.

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# Appendix

## A Additional Tables

Table 7: Sensitivity to yield measure

Yield Source	Real yield on government debt		
	(1) IFS	(2) Fame 5-year	(3) Fame 10-year
Inflation consumption covariance	-1.563*** (0.450)	-2.205** (0.986)	-2.582*** (0.831)
Other controls	Yes	Yes	Yes
adj. $R^2$	0.884	0.857	0.896
$N$	1841	1216	1465

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data is a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, ITA, JAP, KOR, MEX, NLD, NOR, POR, SWE, USA. All variables are computed over a forward-looking ten-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window:  $\text{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$ . Other regressors are averages and variances of those residuals in the window and lagged debt.

Table 8: Sensitivity to debt measure

Debt Source	Real yield on government debt			
	(1) Oxford+OECD	(2) OECD	(3) Oxford	(4) OECD+Oxford
Inflation consumption co-movement	-1.563*** (0.450)	-1.347 (1.579)	-1.596*** (0.427)	-1.240** (0.464)
Other controls	Yes	Yes	Yes	Yes
adj. $R^2$	0.884	0.816	0.892	0.902
$N$	1841	933	1663	1785

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data is a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, ITA, JAP, KOR, MEX, NLD, NOR, POR, SWE, USA. All variables are computed over a forward-looking ten-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window:  $\text{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$ . Other regressors are averages and variances of those residuals in the window and lagged debt.

Table 9: VAR results

country	$\rho_{\pi\pi}$	$\rho_{c\pi}$	$\rho_{\pi c}$	$\rho_{cc}$	$\sigma_c$	$\sigma_\pi$	$\sigma_{\pi,c}$
USA	0.93	0.06	-0.10	0.86	0.17	0.34	0.00
AUS	0.82	0.10	-0.02	0.67	0.67	0.54	0.07
AUT	0.82	0.04	-0.10	0.65	0.27	0.43	0.00
BEL	0.85	0.02	-0.04	0.77	0.33	0.33	0.00
CAN	0.75	0.18	-0.02	0.72	0.63	0.42	0.06
CHE	0.90	0.09	-0.02	0.83	0.27	0.29	0.01
DEU	0.85	0.10	-0.15	0.49	0.32	0.53	0.02
DNK	0.56	-0.05	-0.25	0.71	0.56	0.66	0.02
ESP	0.87	0.01	-0.04	0.91	0.34	0.59	0.01
FIN	0.67	0.12	-0.01	0.87	0.65	0.73	0.05
FRA	0.89	0.10	-0.18	0.67	0.22	0.32	-0.01
GBR	0.83	0.09	-0.11	0.83	0.56	0.51	-0.06
GRC	0.77	0.07	-0.10	0.90	0.83	0.87	-0.17
ITA	0.67	-0.03	-0.01	0.88	0.61	0.44	-0.01
JPN	0.92	0.10	-0.26	0.48	0.37	0.70	-0.11
KOR	0.69	0.10	-0.30	0.81	0.97	1.24	-0.32
MEX	0.91	-0.35	0.02	0.86	2.70	1.27	-0.98
NLD	0.67	0.04	-0.05	0.85	0.53	0.44	0.00
NOR	0.81	0.14	-0.02	0.68	1.79	0.80	-0.02
PRT	0.88	-0.04	0.02	0.89	0.68	0.71	-0.02
SWE	0.75	-0.12	-0.02	0.75	0.72	0.52	0.09
average	0.80	0.04	-0.08	0.76	0.68	0.60	-0.07
median	0.82	0.07	-0.04	0.81	0.56	0.53	0.00
min	0.56	-0.35	-0.30	0.48	0.17	0.29	-0.98
max	0.93	0.18	0.02	0.91	2.70	1.27	0.09

The data is a quarterly panel from 1985Q1 to 2015Q4.

## B Proofs

### Theorem 1. Inflation procyclicality discount

Near  $\gamma = 0$ , there is an inflation procyclicality discount. That is,

$$\frac{dq(\gamma)}{d\gamma} > 0.$$

*Proof:* The borrower's first order condition is given by

$$q[A - \phi(\tau + qb)] = \beta_b [A + \phi\tau\gamma\sigma^2 + \phi b(1 + \gamma^2\sigma^2)] \quad (28)$$

Hence, debt demand is given by

$$b_b(q; \gamma) = \frac{\left[\frac{A}{\phi} - \tau\right] q - \beta_b \left[\frac{A}{\phi} + \tau\gamma\sigma^2\right]}{q^2 + \beta_b [1 + \gamma^2\sigma^2]} \quad (29)$$

Consider the derivative with respect to  $\gamma$ :

$$\frac{\partial b_b(q; \gamma)}{\partial \gamma} = -\beta_b \sigma^2 \frac{\tau + 2\gamma b_b(q; \gamma)}{q^2 + \beta_b (1 + \gamma^2\sigma^2)}$$

At  $\gamma = 0$ ,

$$\frac{\partial b_b(q; 0)}{\partial \gamma} = -\frac{\tau\beta_b\sigma^2}{q^2 + \beta_b} < 0 \quad (30)$$

which implies that the demand for debt decreases as  $\gamma$  increases, locally around  $\gamma = 0$ . This is the *risky debt effect*: low endowment shocks are associated with higher debt obligations.

Also, the derivative with respect to  $q$  is:

$$\frac{\partial b_b(q; \gamma)}{\partial q} = \frac{\left(\frac{A}{\phi} - \tau\right) - 2qb_b(q; \gamma)}{q^2 + \beta_b (1 + \gamma^2\sigma^2)} \quad (31)$$

The lender's first order condition is given by

$$q(A - \phi(1 - \tau - qb)) = \beta_\ell [A + \phi(1 - \tau)\gamma\sigma^2 - \phi b (1 + \gamma^2\sigma^2)] \quad (32)$$

Hence debt supply is given by

$$b_\ell(q; \gamma) = \frac{-\left[\frac{A}{\phi} - (1 - \tau)\right] q + \beta_\ell \left[\frac{A}{\phi} + (1 - \tau)\gamma\sigma^2\right]}{q^2 + \beta_\ell [1 + \gamma^2\sigma^2]} \quad (33)$$

First, consider the derivative with respect to  $q$ :

$$\frac{\partial b_\ell(q; \gamma)}{\partial q} = -\frac{\frac{A}{\phi} - (1 - \tau) + 2qb_\ell(q; \gamma)}{q^2 + \beta_\ell (1 + \gamma^2\sigma^2)} \quad (34)$$

$$< 0 \quad (35)$$



if  $b_\ell(q; \gamma) \geq 0$  since  $A/\phi > (1 - \tau)$ .

Second, consider the derivative with respect to  $\gamma$ :

$$\frac{\partial b_\ell(q; \gamma)}{\partial \gamma} = \beta_\ell \sigma^2 \frac{(1 - \tau) - 2\gamma b_\ell(q; \gamma)}{q^2 + \beta_\ell (1 + \gamma^2 \sigma^2)} \quad (36)$$

At  $\gamma = 0$ ,

$$\frac{\partial b_\ell(q; 0)}{\partial \gamma} = \frac{(1 - \tau)\beta_\ell \sigma^2}{q^2 + \beta_\ell} > 0 \quad (37)$$

which implies that the supply of debt increases as  $\gamma$  increases, locally around  $\gamma = 0$ . This is the *hedging effect*: low endowment shocks are associated with higher real returns on debt.

The market clearing condition is

$$b_\ell(q; \gamma) = b_b(q; \gamma) \quad (38)$$

Using the implicit function theorem, we obtain

$$\frac{\partial b_\ell(q; \gamma)}{\partial q} \frac{dq(\gamma)}{d\gamma} + \frac{\partial b_\ell(q; \gamma)}{\partial \gamma} = \frac{\partial b_b(q; \gamma)}{\partial q} \frac{dq(\gamma)}{d\gamma} + \frac{\partial b_b(q; \gamma)}{\partial \gamma}$$

At  $\gamma = 0$ ,

$$\frac{dq(0)}{d\gamma} = - \frac{\frac{\partial b_\ell(q; 0)}{\partial \gamma} - \frac{\partial b_b(q; 0)}{\partial \gamma}}{\frac{\partial b_\ell(q; 0)}{\partial q} - \frac{\partial b_b(q; 0)}{\partial q}}. \quad (39)$$

It suffices to show that  $\partial b_\ell(q; 0)/\partial q < \partial b_b(q; 0)/\partial q$  since we have shown that  $\partial b_b(q; 0)/\partial \gamma < 0$  in (30) and  $\partial b_\ell(q; 0)/\partial \gamma > 0$  in (37). Furthermore, we have shown that  $\partial b_\ell(q; 0)/\partial q < 0$  in (35). Thus it suffices to show  $\partial b_\ell(q; 0)/\partial q < \partial b_b(q; 0)/\partial q$ .

This holds if and only if

$$\begin{aligned} -\frac{\partial b_\ell(q; 0)}{\partial q} &> -\frac{\partial b_b(q; 0)}{\partial q} \\ &\Leftrightarrow \\ \frac{\frac{A}{\phi} - (1 - \tau) + 2qb_\ell(q; 0)}{q^2 + \beta_\ell} &> \frac{-\frac{A}{\phi} + \tau + 2qb_b(q; 0)}{q^2 + \beta_b}. \end{aligned}$$

Using the market clearing conditions and  $\beta_\ell > \beta_b$ , it suffices to show that

$$\frac{A}{\phi} - (1 - \tau) > -\frac{A}{\phi} + \tau,$$

which holds if

$$\frac{A}{\phi} > \frac{1}{2},$$

which holds by assumption.  $\square$