A Theory of Rollover Risk, Sudden Stops, and Foreign Reserves☆

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Abstract

Emerging economies have accumulated very large foreign reserve holdings since the turn of the century. We argue that this policy is an optimal response to an increase in foreign debt rollover risk. In our model, reserves play a key role in endogenously reducing debt rollover crises ("sudden stops") by allowing governments to be solvent in more states of the world. Using a dynamic multi-country environment with learning, we find that a relatively small unanticipated increase in rollover risk jointly accounts for (i) the outburst of sudden stops in the late 1990s, (ii) the subsequent increase in foreign reserves holdings, and (iii) the salient resilience of emerging economies to sudden stops ever since. We also show that a policy of pooling reserves may substantially reduce reserves because mutual insurance across countries dampens rollover risk.

Keywords: rollover risk, optimal reserves, endogenous sudden stops, debt crises, learning
JEL: F42, F34, H63

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1. Introduction

Since the turn of the century, emerging economies have accumulated massive amounts of international reserves. Summers (2006) considered this dramatic rise in reserves to be "the most surprising development in the international financial system over the last half dozen years," a buildup that was "was neither predictable nor predicted ... far in excess of any previously enunciated criterion of reserve need for financial protection." According to Bernanke (2005), this global "savings glut" has been the most important force behind the widening of the U.S. current account deficit. At the time of Bernanke's "savings glut" speech, China's foreign reserve holdings alone amounted to nearly one trillion U.S. dollars and represented approximately 45 percent of the (negative) net foreign asset position of the United States. While massive from an absolute perspective, China's reserves as a percentage of GDP, which averaged 30 percent from 2002 to 2006, are comparable to those of other emerging economies, such as Korea (25 percent), Malaysia (45 percent), Thailand (30 percent), and Russia (21 percent).

This raises the question of why emerging economies have accumulated such large amounts of reserves. In the existing literature, reserves are typically held to prevent the adverse effects of a sudden stop in capital inflows (Alfaro and Kanczuk 2009, Bianchi et al. 2012, Caballero and Panageas 2005, Jeanne and Ranciere 2011). Motivated by the stylized fact that emerging economies have accumulated and maintained large foreign reserves while crises have been much less frequent since the outburst of crises in the late 1990s, our paper complements this literature by allowing reserve accumulation to endogenously reduce the probability of crisis. This endogenous channel is then used to explain the joint evolution of crises and reserves in the data. In a related paper, Gourinchas and Obstfeld (2012) also show that reserves are negatively associated with default crises, banking crises, and currency crises. In fact, reserves managers and central banks in emerging markets indicate that reserves are held mainly to stave off liquidity crises.[1]

To explain the joint evolution of reserves and the occurrences of crises, we develop a theory in which reserves endogenously prevent crises.[2] In particular, we focus on sudden stops (of external capital inflows) because they are a common symptom of financial crises such as currency crises, banking crises, and default crises in emerging economies.[3] In this theory, sudden stops occur

[1]See International Monetary Fund (2011).

[2]While acknowledging other potential motives for holding reserves, such as foreign exchange management (see, for example, Dooley et al. 2004), we focus on the role of reserves as a buffer (and preventive measure) against crises. This is consistent with the view of policymakers. For example, Bernanke (2005) stated that "foreign reserves have been used as a buffer against potential capital outflows," and a recent IMF survey of reserve managers found that building a "buffer for liquidity needs" was the foremost reason for building reserves (International Monetary Fund 2011). In an excellent review, Chang (2007) highlights this liquidity motive across central banks in Latin America. For instance, the stated goal of Colombia's Banco de la Republica is to "maintain an adequate level of international reserves that reduce the vulnerability of the economy to foreign shocks."

[3]Sudden stops are defined as unusually large reversals of external capital inflows along with a severe contraction in

when foreign lenders choose not to roll over a country's external liabilities. We derive closed-form solutions for the optimal reserves and the induced probability of a sudden stop. The analytical expressions reveal how reserves are optimally set to balance the reduction in sudden stop probability, the induced fall in interest rates, and the reduction in final output due to lower investment.

Specifically, we consider the problem of a small open economy that borrows short-term from foreign lenders to finance long-term investments. This maturity mismatch gives rise to rollover risk: in the interim, a random fraction of creditors can choose to roll over while the other creditors cannot. Rollover risk in this environment is endogenous because the actual amount of debt that is rolled over depends on the debt arrangement. Faced with stochastic interim liquidity needs, the government may pay with the reserves it had set aside or liquidate its investment. For small liquidity shocks, interim payments are optimally paid with reserves, and no sudden stop occurs. For large shocks, the government cannot finance its debt obligations without liquidation, resulting in a sudden stop as all lenders refuse to roll over. Reserves therefore reduce the probability of sudden stops by inducing lenders to roll over in more states of the world. We also discuss the scope for reducing reserves holdings under mutual insurance across countries facing idiosyncratic and correlated rollover risk.

We extend the model to a dynamic multi-country extension in which countries learn from each other to form beliefs about the true rollover risk they face.[4] Countries have incentives to learn about the true rollover risk, as it is a critical determinant of the allocation of reserves and the likelihood of sudden stops. In particular, a change in liquidity risk will affect the evolution of sudden stops and reserves. Indeed, using the *de jure* measure of financial openness introduced by Chinn and Ito (2006), we observe that capital openness suddenly entered a new phase around the mid-1990s (see Figure 1 for an illustration of this surge). We use this evidence to posit a regime change in the liquidity risk faced by these countries.[5] In our theory, an unexpected increase in rollover risk temporarily causes an underinvestment in reserve holdings, which increases the probability of a sudden stop. After observing the global increase in aggregate liquidity shocks and sudden stops, agents rationally update their common belief about the prevailing debt rollover risk. When agents have fully learned the new regime, reserves are permanently higher and sudden stops subside.

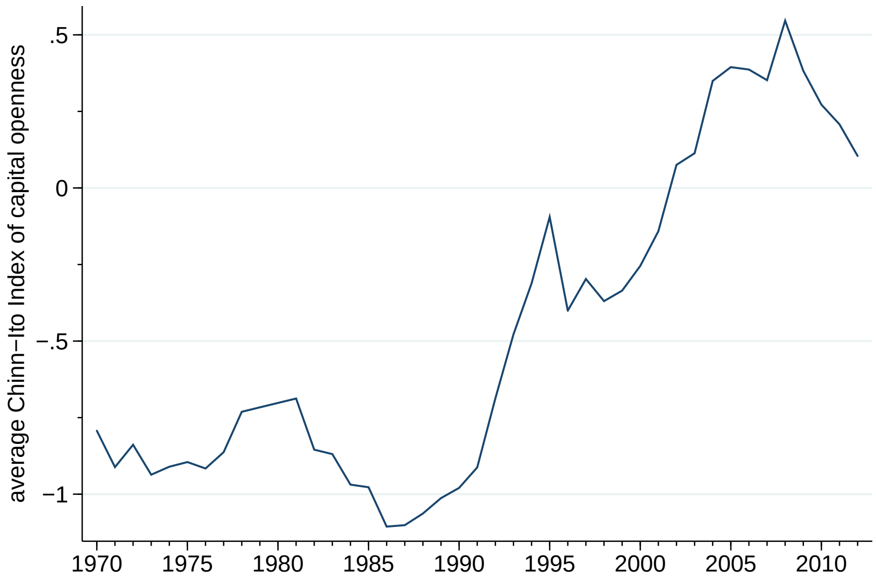
The model is then calibrated for two quantitative applications. First, we show that an unanticipated and permanent increase in rollover risk can account for both the short-lived outburst of

economic activity. See also Gourinchas and Obstfeld (2012) for a discussion on how sudden stops can lead to currency crises and financial crises.

[4]See Buera et al. (2011), who suggested that learning from peer countries is an important driver in the adoption of liberal and market-oriented policies over time and across countries.

[5]Our view is that as emerging economies moved towards capital liberalization in the early 1990s and experimented with external borrowing, many countries may have underestimated the volatility of external capital flows. Increased volatility can, for example, be a result of the increasing ease with which investors can reallocate funds across countries.

Figure 1: Evolution of Capital Openness in Emerging Economies



sudden stops in the late 1990s and the large accumulation of foreign reserves ever since. A model in which reserves do not reduce the probability of a sudden stop cannot jointly match these facts: higher reserves and fewer crises cannot coexist. Introducing learning in the model is essential for explaining the short-lived outburst in sudden stops in the late 1990s: countries learned from one another and updated their beliefs after being caught off-guard. An empirical prediction of the model is that countries might hold large stocks of foreign reserves, even in the absence of sudden stops, which is consistent with the dynamics of reserves and sudden stops in the data. Quantitatively, an extension in which governments learn only from events in their own region fits the joint evolution of reserves and crises particularly well. Second, using the calibrated liquidity risk, we find that mutual insurance across emerging economies may reduce the reserves needed by as much as three-fifths: pooling or swapping reserves lowers the rollover risk when liquidity shocks are not perfectly correlated across countries.[6]

This paper builds on a large body of literature on reserves, sudden stops, and debt crises. In particular, it relates to other papers on reserves (Aizenman and Lee 2007, Calvo et al. 2012, Frenkel and Jovanovic 1981, Heller 1966, Obstfeld et al. 2010)[7] and on sudden stops (Calvo et al. 2004, Durdu et al. 2009, Forbes and Warnock 2012, Kehoe and Ruhl 2009, Mendoza 2010).[8] Our

[6]This corresponds to an upper bound on the reduction of reserves, since there may be limits to mutual insurance such as moral hazard, private information, or aggregate uncertainty. We analytically characterize an extension of the model in which there is aggregate uncertainty arising from correlated shocks across countries.

[7]See also Gourinchas and Jeanne (2013) for the literature on the capital allocation puzzle and reserves holdings.

[8]Benigno and Fornaro (2012) provide an insightful model in which reserves stimulate trade through real exchange rate depreciation, which in turn generates growth externalities especially during recessions. In that sense, their model

work departs from the literature by explicitly modeling the rollover decision of foreign lenders, thereby crucially endogenizing the probability of a sudden stop. The endogenous relationship between reserves, rollover risk, and sudden stops is precisely what allows our model with learning to generate both a *temporary* outburst of sudden stops and a *secular* rise of reserves in response to a *permanent* increase in rollover risk.

Our model is related to the banking models in the vein of Bryant (1980), Diamond and Dybvig (1983), and Chang and Velasco (2001), from which it borrows the modeling of the maturity mismatch and liquidity shocks. Foreign lenders who provide short-term loans are subject to liquidity shocks, similar to the time-preference shocks in Diamond and Dybvig (1983). Aggregate uncertainty in our model is generated by the random fraction of lenders who receive these shocks, as in Chari (1989). This randomness is intended to capture the volatile nature of international capital flows in emerging economies. For instance, Blustein (1998) reports that during the 1997 financial crisis in Korea, "about $1 billion a day was flowing out of the country as foreign banks pulled their lines of credit to South Korean banks and companies." Our work departs from this literature in some important ways. Most importantly, in our model, the government is not a Bryant-Diamond-Dybvig *bank*. First, the government is not providing *insurance* to the lenders. In fact, all agents are risk-neutral in our setup.[9] Second, the government does not maximize the *lender's welfare*. It offers debt contracts that make the lenders break even in expectation; the net proceeds from the government's debt and investment operations are consumed domestically. Moreover, the debt contracts in our environment should not be interpreted as demand deposit contracts, as in Diamond and Dybvig (1983), but rather short-term loans that are subject to rollover risk.[10] Further, we do not focus on "bank runs" in the presence of *multiple equilibria*; instead, we focus on sudden stops that the government chooses to let occur.[11] This focus on "fundamental" runs closely follows the equilibrium selection argument used by Allen and Gale (1998) in their banking model of optimal financial crises under aggregate uncertainty. Conceptually, the government can avoid costly sudden stops when it has the resources to credibly convince its patient lenders to roll over.

Finally, we have modeled reserves, borrowing, and investment decisions as being made by the government, when in fact some of these decisions may be made by private agents. We view

is also a model of endogenous reserves and crises. A vast related literature discusses the growth effects of private capital flows. See, for example, Alfaro et al. (2004), Buera and Shin (2009), Carroll and Jeanne (2009), Song et al. (2011), Benhima (2013), Sandri (2014), Alfaro et al. (2014).

[9] Given the key role of insurance and precautionary motives in other models, we deliberately focus on risk-neutral agents in order to clearly show the key drivers of reserves and crises in our setup.

[10] Radelet and Sachs (2000) argues that the refusal of foreign lenders to roll over short-term loans triggered the East Asian Crisis. Arellano and Ramanarayanan (2012) and Broner et al. (2013) explore models of debt maturity and why emerging economies issue short-term debt.

[11] See Calomiris and Gorton (1991) for an extensive discussion of debates surrounding the foundations of bank panics, especially the sequential-service constraint used to generate bank runs in Diamond and Dybvig (1983).

this as a simplification that is reasonable in light of the implicit (or explicit) guarantees made by governments in emerging economies on private loans. For example, Pearlstein (1998) reports that during the crisis in Korea, the international debt was renegotiated under an agreement in which the Korean government assumed "responsibility for loans originally made to the country's private banks, in effect guaranteeing their repayment."

This paper is structured as follows. In Section 2, we empirically analyze foreign reserves and sudden stops in emerging economies from 1992 to 2011. In Section 3, we present a model of rollover risk, sudden stops, and reserves, and we characterize optimal reserves and endogenous sudden stop probabilities. In Section 4, we calibrate a multi-country dynamic extension of the model with learning applied to emerging economies. We also discuss the experience of Baltic economies and the euro area periphery economies. Section 5 concludes.

2. Reserves and Sudden Stops in Emerging Economies

In this section, we document a set of stylized facts regarding foreign reserves, external debt liabilities, and sudden stops in 23 emerging economies from 1992 to 2011. We use the International Financial Statistics (IFS) dataset in conjunction with the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007).[12] The list of emerging economies used in this paper is Argentina, Brazil, Chile, China, Colombia, the Czech Republic, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, South Korea, Thailand, and Turkey. This list includes countries appearing in most classifications of emerging countries.[13]

2.1. Sudden Stops in Emerging Economies

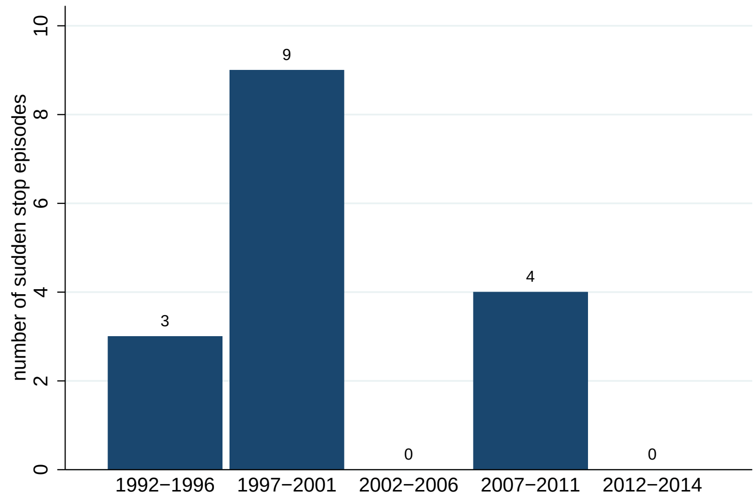
Following Calvo et al. (2004), we define a sudden stop episode as a spell with exceptionally large current account reversals and a recession. We find 16 sudden stop experiences during 1992–2011 across the 23 emerging economies, with a salient outburst of sudden stops during 1997–2001.[14]

[12]The updated series by Lane and Milesi-Ferretti (2007) stop in 2011. For robustness, we extend the data to 2014 using data from the IFS. International portfolio data are not consistently available before 1990. We report statistics starting in 1992 so as to have 5-year windows through 2011. Starting in 1990 does not change our results.

[13]The set of economies that we classify as emerging is based on the existing emerging markets classifications used by the *Financial Times* and the London Stock Exchange (FTSE), Morgan Stanley Capital International (MSCI), the *Economist*, Standard & Poor's (S&P), and Dow Jones Indexes. We focus on the countries consistently listed in two or more classifications, except for Hong Kong, Singapore, Taiwan, and the United Arab Emirates.

[14]Our methodology is explained in the data appendix. Our sudden stop episodes are: Turkey (1994); Mexico (1995); Hungary (1996); Thailand (1997); Czech Republic, Indonesia, Philippines, and South Korea (1998); Chile, Peru, and Russia (1999); Argentina and Turkey (2001); and Hungary, Romania, South Africa, and Turkey (2009). Durdu et al. (2009) report additional episodes: Argentina (1995); Malaysia (1997); and Brazil, Colombia, and Pakistan (1999). In any case, there was an outburst in sudden stops between 1997 and 2001. Forbes and Warnock (2012), for the purpose

Figure 2: Sudden Stops in Emerging Economies



In Figure 2, we highlight this outburst of sudden stops during the late 1990s across our set of emerging economies. With 9 sudden stop episodes, 1997–2001 stands out as a period of high-frequency sudden stops. In contrast, the other eras had much fewer or no sudden stops: the early 1990s (1992–1996) had 3 occurrences, the early 2000s (2002–2006) had no occurrence, and the latest era (2007–2011) had 4 sudden stops in the wake of the 2008–2009 Global Financial Crisis (GFC).[15] Finally, we find no sudden stop in the sample extended to 2014.

2.2. *Foreign Reserves*

In the IFS dataset, foreign reserves are defined as "all official public sector foreign assets, except gold, that are readily available to and controlled by the monetary authorities." We highlight two notable facts regarding foreign reserve holdings. The first fact is that foreign reserves in emerging economies, both as a percent of GDP and as a percent of external debt liabilities, are significantly higher than those in advanced economies.[16] The second fact is that these ratios have substantially increased in emerging economies in the wake of the crises of the late 1990s.

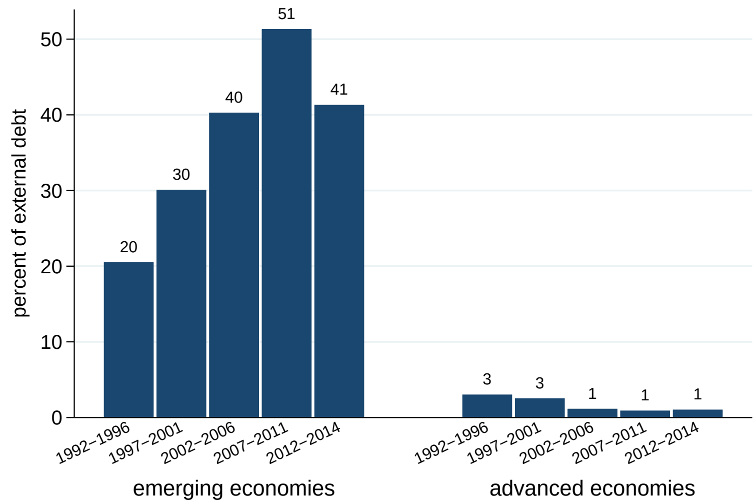
These facts are summarized in Figure 3: from the early 1990s to the early 2000s, the median reserves as a percentage of external debt liabilities doubled from 20 percent in the early 1990s to

of a more nuanced taxonomy of capital flows, do not require sudden stops to feature a recession. Their approach naturally generates more sudden stops than the historical episodes documented in the literature.

[15] While the inclusion of the GFC does not change the stylized fact of an outburst of sudden stops in the late 1990s among the countries we consider, many other countries experienced sudden stops during the GFC. The Baltic economies who regained independence in the early 1990s and aggressively liberalized in the early 2000s are a particularly prominent case. We document and discuss their unique experience in Section 4.5.

[16] Advanced economies here include the major reserve currencies: France, Germany, U.K., and U.S. We view advanced economies as facing fundamentally different frictions than emerging economies. Chang and Velasco (2001) also argue that emerging economies face *illiquidity* because their access to world capital markets is limited.

Figure 3: Foreign Reserves in Emerging Economies



Note: The value for each period and each bloc is the median across economies of the period-average for each economy.

40 percent by the mid-2000s. Although reserves subsequently reached record levels during the Global Financial Crisis (GFC) of 2008–2009, they have since returned to pre-GFC levels.

It is worth noting that this phenomenon of increasing reserves is not limited to just a few countries. In fact, foreign reserves are increasing in almost all emerging economies, with only Chile and Hungary decreasing in both reserves measures. This robust observation is shown in Tables A.4 and A.5 in Appendix A for each measure of foreign reserves by country.

2.3. Reserves and Sudden Stop Probabilities

Gourinchas and Obstfeld (2012) use a panel discrete-choice model to document that foreign reserves are associated with a reduced probability of a banking crisis, currency crisis, or sovereign default. As explained above, we focus on *sudden stops* because they are a common symptom of financial crises such as default crises, banking crises, and currency crises in emerging countries. To further motivate our emphasis on sudden stops and reserves, we document that higher foreign reserves are associated with reduced sudden stop likelihood using annual data from 1990 to 2011.

As in Gourinchas and Obstfeld (2012), we use a panel logit model with country fixed effects:

$$\Pr\left(S_k^i = 1 \,|\, x_i\right) = \frac{\exp\left(\alpha_i + \beta x_i\right)}{1 + \exp\left(\alpha_i + \beta x_i\right)}$$

where S_k^i denotes whether country i is in a sudden stop episode in the next k years, and x_i denotes foreign reserves and net foreign assets in country i during a year that is not 0 to 3 years after a sudden stop episode (that is, "tranquil" times using the terminology of Gourinchas and Obstfeld 2012). The sample is restricted to "tranquil" times to avoid post-crisis bias.

Table 1: Panel Logit Estimation across Emerging Economies

	S.D.	Crisis in 1–2 years δp	Crisis in 1–2 years $\frac{\partial p}{\partial x}$	Crisis in 1–3 years δp	Crisis in 1–3 years $\frac{\partial p}{\partial x}$
Panel A: Sudden Stops (baseline sample: country FE and years 1990–2011)					
Reserves	24.90	-6.81***	-0.40***	-9.65***	-0.52***
over External Debt		(1.40)	(0.11)	(2.04)	(0.15)
Net Foreign Assets	12.01	-2.99	-0.28	-5.63**	-0.54*
over GDP		(2.34)	(0.25)	(2.82)	(0.32)
Probability in percent ($\overline{p}$)		11.56		18.07	
N=14 ; $N \times T$=204					
Panel B: Sudden Stops (baseline sample + time trend)					
Reserves	19.21	-4.60***	-0.30**	-4.21*	-0.25
over External Debt		(1.65)	(0.14)	(2.51)	(0.17)
Net Foreign Assets	12.00	-3.53*	-0.35	-6.84***	-0.70***
over GDP		(2.06)	(0.24)	(2.28)	(0.28)
Probability in percent ($\overline{p}$)		11.34		17.20	
N=14 ; $N \times T$=204					
Panel C: Sudden Stops (baseline sample + dummies for late 1990s and GFC)					
Reserves	24.00	-6.16***	-0.37***	-8.87***	-0.48***
over External Debt		(1.40)	(0.12)	(2.02)	(0.15)
Net Foreign Assets	11.84	-2.79	-0.27	-5.49**	-0.54*
over GDP		(2.21)	(0.24)	(2.77)	(0.31)
Probability in percent ($\overline{p}$)		10.77		17.87	
N=14 ; $N \times T$=204					

Note: *, **, and *** denote significance at the 10, 5, and 1 percent level. $\partial p/\partial x$ is the marginal effect in percentage at "tranquil" sample mean. δp is the effect in percentage for an increase of one standard deviation in x at the "tranquil" sample mean. $s.d.(x)$ is the unconditional standard deviation of x over "tranquil" times. $\overline{p}$ is the probability of a crisis at the sample mean. Robust standard errors in parentheses are computed using the delta-method. The estimation sample is an unbalanced panel that spans 14 emerging countries between 1990 and 2011. The data stops in 2011 as the updated series by Lane and Milesi-Ferretti (2007) stop in 2011. Due to the use of country fixed effects, countries with no sudden stops are not in the logit estimation sample.

Foreign reserves are significantly associated with a reduced probability of sudden stops, as shown in Table 1. For instance, in panel A, an increase of one standard deviation in the ratio of foreign reserves to external debt liabilities (around 25 percent) is associated with a fall of nearly 7 percent in the probability of a sudden stop over the next two years. Also, unlike foreign reserves, net foreign assets are not commonly associated with sudden stops. These results are robust to the inclusion of a time trend (see panel B) and the addition of indicators for crisis-prone periods of the late 1990s and the GFC (see panel C).

These findings are consistent with the original results of Gourinchas and Obstfeld (2012) for default, banking, and currency crises (see Table A.6 in Appendix A).[17] These results extend the key message of Gourinchas and Obstfeld (2012) on the importance of reserves for financial stability and justify our interest in endogenizing both reserves and sudden stops.

Overall, we documented that reserves increased dramatically in emerging markets while sudden stops have been much less frequent since the short-lived outbursts of the late 1990s. Moreover, reserves are associated with a reduced likelihood of sudden stops. In the next section, we characterize a model in which reserves endogenously reduce the probability of a sudden stop. We then use the model to offer a parsimonious learning-based account of the joint evolution of reserves and crises in emerging markets since the early 1990s.

3. Model

3.1. Environment

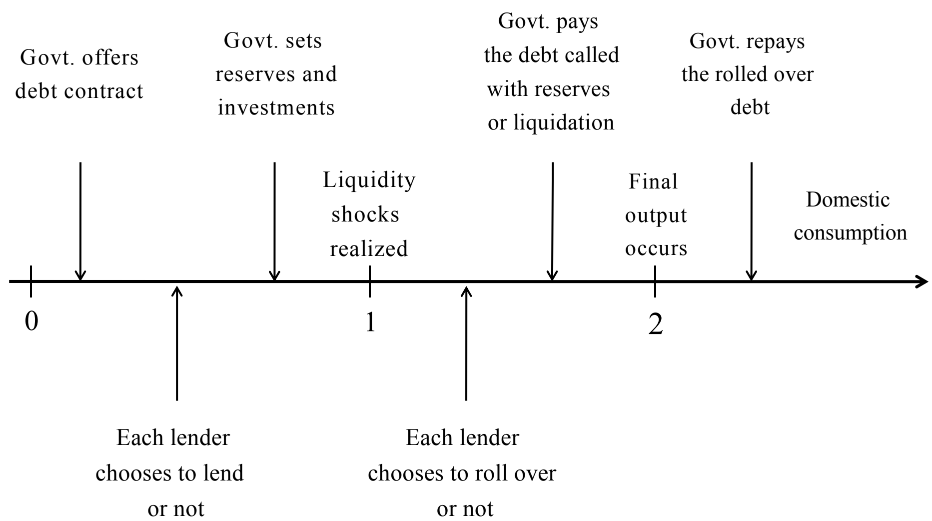
We consider a small open economy model with three stages: $s = 0$ (initial), 1 (interim), and 2 (final). There is a unit measure of risk-neutral foreign lenders who can lend to the domestic country.[18] The domestic country has a representative agent who has linear preferences $u(C) = C$ over final stage consumption C. The government chooses allocations and debt arrangements to maximize the expected utility of the domestic agent. An overview of the sequence of actions taken by the government and the lenders is presented in Figure 4.

The domestic country has access to two technologies ï¿œ la Diamond and Dybvig (1983). The first technology transforms the investment K made in the initial stage into AK units in the final stage if production is uninterrupted. However, if production is interrupted in the interim through the liquidation of $L \in [0, K]$ units of investment, the technology yields λL in the interim and $A(K - L)$ in the final stage. We assume that liquidation is costly: $\lambda < 1$.

[17] We obtain similar results when we separately estimate the model for each variable. The findings are also similar using alternative measures of reserves, such as the reserves-to-GDP ratio. We prefer reserves as a fraction of external debt liabilities since it is a measure consistent with our theory. See also Bussière et al. (2015) for similar evidence on reserves holdings and the impact of the Global Financial Crisis of 2008–2009.

[18] We assume that the foreigners' endowment is finite and large enough to ensure an exogenous world interest rate.

Figure 4: Timeline



Further, we impose that there is no partial interim liquidation:

$$L \in \{0, K\}. \tag{1}$$

This assumption of full liquidation is made for analytical tractability and is relaxed in the next section. The second technology stores resources (reserves) across stages without depreciation. These technologies are summarized by the following table:

Technologies	$s=0$	$s=1$	$s=2$
Production and liquidation	$-K$ investment	λL liquidation	$A(K-L)$ final output
Reserves	$-R_1$ initial reserves	R_1	
		$-R_2$ interim reserves	R_2

In the initial stage, the domestic government borrows a fixed amount D from foreign lenders to finance its initial stage investments:

$$R_1 + K \leq D. \tag{2}$$

In the interim, a random fraction φ of the foreign lenders receive liquidity shocks denoted by $\varphi^i = 1$, meaning that they must call the loan and be paid back. The remaining fraction $(1-\varphi)$

of lenders with $\varphi^i = 0$ can call or roll over their loans. The random aggregate liquidity shock $\varphi \in [0,1]$ has a cumulative distribution function that follows the bounded Pareto distribution given by $F_\sigma(\varphi) = 1-(1-\varphi)^{1/\sigma}$, with $\sigma > 0$.

We denote $\psi^i = 0$ if lender i rolls over the loan and $\psi^i = 1$ otherwise. The fraction of lenders calling the loan is $\psi \equiv \int \psi^i di$. We call it a *sudden stop* when no lender accepts rolling over in the interim ($\psi = 1$). We distinguish between self-fulfilling "panic" runs in which all lenders panic despite the government's capacity to sustain a non-run outcome and "fundamental" sudden stops that occur as part of the optimal contract depending on the state of the economy. As in Allen and Gale (1998), we focus on the latter by letting the government select the welfare-maximizing outcome: sudden stops are costly and the government credibly avoids them whenever it is resource-feasible to do so by *ex ante* coordinating and recommending a rollover policy, which lenders rationally commit to follow.[19]

We allow the debt repayment of the debt D to be contingent on whether or not the economy is facing a sudden stop. During normal times, foreign lenders receive $P_1 = D$ if they call the loan in the interim and $P_2 = (1+r_N)D$ in the final stage if they roll over the loan.[20] During a sudden stop, however, all the lenders call the debt and receive $P_1 = (1+r_S)D$ in the interim. The debt repayment schedule can be summarized as

	Interim payment P_1	Final payment P_2
Normal times ($\psi < 1$)	D	$(1+r_N)D$
Sudden stop ($\psi = 1$)	$(1+r_S)D$	0

Because the interest rate can be different when the economy is in sudden stop, the government can choose to partially default during sudden stop episodes by setting $r_S < 0$. However, there is a limit to the haircut the lenders can suffer because they can collectively bargain and extract a fraction $\theta \leq 1$ of the interim resources available $(R_1 + \lambda K)$.[21] The constraint arising from this

[19] In this approach, a sudden stop occurs only if the fundamentals — not a panic among lenders — justify it. More generally, one can allow for both "panic" crises ï¿œ la Diamond-Dybvig and our "fundamental" sudden stops using a sunspot variable, or use a global games approach to self-fulfilling equilibria (see, for example, Morris and Shin 1998, Cole and Kehoe 2000, Goldstein and Pauzner 2005, Kim 2008). This is without loss of generality to the qualitative results if the domestic crisis payoff is zero, as is the case in this section.

[20] The assumption that lenders receive zero net return on debt called in the interim is not essential to the qualitative results. The interim return may be set to any arbitrary number less than the world interest rate. In contrast, in the Bryant-Diamond-Dybvig framework, the payouts in both periods are jointly determined to smooth the consumption of risk-averse depositors. In our small open economy environment, the participation constraint of risk-neutral lenders and the sudden stop risk pin down the borrowing costs.

[21] The recent case of Argentina illustrates the limited safety of foreign assets. The U.S. Supreme Court ruled that holdout creditors can use U.S. subpoenas to hunt for Argentine assets abroad (see Republic of Argentina v. NML Capital, Ltd. 2014). This eventually forced the Argentine government to negotiate a settlement. Panizza et al. (2009) document cases in which creditors received close to full payment based on the threat of disrupting the debtor's inter-

collective bargaining outcome is given by

$$(1+r_S)D \geq \min\{D, \theta(R_1+\lambda K)\}. \tag{3}$$

In this section, we impose $\theta = 1$. This assumption is relaxed in the next section, where θ is calibrated to match the average rate of haircuts in default episodes.

3.2. Feasible Debt Contracts

We now define the feasibility constraints that the debt contract offered by the government must satisfy in this environment. First, we define a debt contract as a list of

- four scalars $\{R_1,\ K,\ r_N,\ r_S\}$ representing the initial reserves, the invested capital, the normal interest rate, and the sudden stop interest rate; and
- four state-contingent functions $\left\{C(\varphi),\ R_2(\varphi),\ L(\varphi),\ \psi^i\left(\varphi,\varphi^i\right)\right\}$, which respectively denote the final consumption, the interim reserves, the interim liquidation, and the individual rollover policies for each aggregate liquidity shock $\varphi \in [0,1]$ and individual liquidity shock $\varphi^i \in \{0,1\}$.

Resource feasibility. A debt contract is *resource feasible* if it satisfies equations (1) and (2) as well as the following constraints:

$$R_2(\varphi)+\psi(\varphi)P_1(\psi(\varphi)) \leq R_1+\lambda L(\varphi) \quad \forall\varphi \tag{4}$$

$$C(\varphi)+(1-\psi(\varphi))P_2(\psi(\varphi)) \leq R_2(\varphi)+A(K-L(\varphi)) \quad \forall\varphi \tag{5}$$

$$0 \leq R_1,\ R_2(\varphi),\ C(\varphi) \quad \forall\varphi. \tag{6}$$

Equation (4) requires that interim reserves and interim debt payments cannot exceed initial reserves and interim liquidation, while equation (5) requires that consumption and final debt payments cannot exceed interim reserves and final output.

Interim individual rationality. A debt contract is *interim individually rational* if, for each aggregate liquidity shock φ and individual liquidity shock φ^i,

national financial transactions and seizing the debtor's assets. In fact, reserves increase a country's credit worthiness and reduce borrowing costs in the data (see Yeyati 2008). The sovereign debt literature also documents the use of collective action during debt renegotiation (see Yue 2010).

$$V\left(\psi^i\,|\,\varphi,\varphi^i\right) \geq V\left(1-\psi^i\,|\,\varphi,\varphi^i\right) \tag{7}$$

$$\text{where} \quad V\left(\psi^i\,|\,\varphi,\varphi^i\right) = \begin{cases} P_1\left(\psi\left(\varphi\right)\right) & \text{if } \psi^i = 1 \\ P_2\left(\psi\left(\varphi\right)\right) & \text{if } \psi^i = 0 \text{ and } \varphi_i = 0. \\ 0 & \text{otherwise} \end{cases}$$

This condition requires that the rollover policy yield a payoff at least as high as that from deviating. The lender payoff is given by $P_1\left(\psi\left(\varphi\right)\right)$ when calling and $P_2\left(\psi\left(\varphi\right)\right)$ when rolling over, if the lender did not receive a liquidity shock.

Ex ante participation constraint. A debt contract satisfies the *ex ante participation constraint* if ex ante the debt contract is as profitable as investing at the world interest rate r_W:

$$\mathbf{E}\left[V\left(\psi^i\,|\,\varphi,\varphi^i\right)\right] \geq (1+r_W)D. \tag{8}$$

Ex post renegotiation proofness. Finally, a debt contract is *ex post renegotiation-proof* if it satisfies the collective bargaining outcome in equation (3). This condition limits the haircut suffered by lenders in a sudden stop.

3.3. Optimal Debt Contract

An *optimal debt contract* is a tuple,

$$B^* = \left\{R_1^*, K^*, r_N^*, r_S^*, C^*\left(\varphi\right), R_2^*\left(\varphi\right), L^*\left(\varphi\right), \psi^{i*}\left(\varphi,\varphi^i\right)\right\},$$

which maximizes the expected utility of the domestic agent subject to (i) resource feasibility, (ii) interim individual rationality, (iii) the ex ante participation constraint, and (iv) ex post renegotiation-proofness. In other words, the government solves

$$\begin{aligned} \max_B \quad & \mathbf{E}_\varphi\left[C\left(\varphi\right)\right] \\ \text{subject to} \quad & (1)-(8). \end{aligned}$$

3.4. Characterization

We now characterize the solution to the optimal debt contract problem.

Proposition 1. Optimal Debt Contract

An optimal debt contract B^* satisfies the following:

(i) Interim payments are paid exclusively with reserves until they are depleted:

$$\exists \quad \varphi_R^* \in [0,1] \quad \text{s.t.} \quad L^*(\varphi) = 0 \iff \varphi \leq \varphi_R^*.$$

(ii) All lenders call their loans whenever reserves are depleted:

$$\psi(\varphi) = \begin{cases} \varphi & \forall \varphi \leq \varphi_R^* \\ 1 & \forall \varphi > \varphi_R^* \end{cases}.$$

(iii) Finally, if the productivity (A) and the aggregate risk (σ) are high enough, then the optimal reserves ratio is

$$\frac{R_1^*}{D} = \varphi_R^* = 1 - \left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}.$$

Proof: See Appendix B.

Proposition 1(i) and 1(ii) establish that there are cutoff rules for reserves, liquidation, and sudden stops. In Proposition 1(i), φ_R^* is the liquidity shock at which reserves are depleted and the government must liquidate the invested capital to meet the promised payments. Because $\lambda < 1$, the government uses existing reserves to meet payments before eventually liquidating the invested capital.

In Proposition 1(ii), φ_R^* is also the liquidity shock above which all lenders exit. We identify this debt rollover crisis as a *sudden stop*. The sudden stop cutoff is equal to the reserves cutoff φ_R^* because we assumed there is no partial liquidation. A sudden stop therefore occurs as soon as the normal interim payments cannot be met using reserves. We later relax the full liquidation assumption. With partial liquidation, the sudden stop cutoff and the reserves cutoff no longer coincide.

Proposition 1(iii) derives analytically the optimal reserves-to-liabilities ratio φ_R^*. In fact, we can denote expected consumption (with $D = 1$) as[22]

$$J(\varphi_R) \equiv \int_0^{\varphi_R} \left[\underbrace{A(1-\varphi_R) + \varphi_R}_{\text{production + reserves}} - \underbrace{(\varphi + (1-\varphi)(1 + r_N(\varphi_R)))}_{\text{interim and final payments}} \right] dF_\sigma(\varphi)$$

where the interest rate is:

$$r_N(\varphi_R) = \frac{1 + r_W - F_\sigma(\varphi_R) - [1 - F_\sigma(\varphi_R)](\lambda + (1-\lambda)\varphi_R)}{\int_0^{\varphi_R}(1-\varphi)dF_\sigma(\varphi)}.$$

[22] The productivity (A) needs to be high enough to ensure that the government has incentives to let everyone rollover, especially when φ is small. $1 \leq (\sigma+1)(1-\lambda+r_W)$ is a sufficient, but not necessary, condition for the objective function to be concave and the constraint set to be convex. See Appendix B for the detailed derivation.

Then,

$$J'(\varphi_R) = -\underbrace{(A-1)F_\sigma(\varphi_R)}_{\text{less productive capital}} \underbrace{-r_N'(\varphi_R)\int_0^{\varphi_R}(1-\varphi)dF_\sigma(\varphi)}_{\text{lower interest rate}} + \underbrace{[A-(1+r_N(\varphi_R))](1-\varphi_R)f_\sigma(\varphi_R)}_{\text{lower probability of sudden stop}}$$

This expression clearly shows that the optimal reserves-to-debt ratio φ_R^* is chosen to balance: (i) the opportunity cost of idle reserves due to reduced capital investment, (ii) the reduced interest rate, and (iii) the lower likelihood of a crisis. These last two forces reveal novel theoretical insights.[23]

The following corollary establishes the endogenous relation between the optimal reserves and the probability of sudden stops. The optimal reserves-to-debt ratio depends endogenously on the process $F_\sigma(\cdot)$ governing the exogenous liquidity risk φ.

Corollary 1. Endogenous Sudden Stop Probability

The optimal contract B^* induces a positive probability that a sudden stop occurs. Furthermore,

$$\Pr(\psi = 1) = 1 - F_\sigma(\varphi_R^*) = \frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right).$$

Proof: This follows immediately from Proposition 1.

3.5. Comparative Statics

In this subsection, we discuss how reserves and sudden stop probabilities are affected by changes in the underlying liquidity risk; that is, changes in σ.

Proposition 2. Reserves, Sudden Stop Probability, and Debt Rollover Risk

(i) The optimal reserves ratio is increasing in the aggregate liquidity risk. That is,

$$\frac{\partial \varphi_R^*}{\partial \sigma} > 0.$$

(ii) The sudden stop probability is increasing in the aggregate liquidity risk. That is,

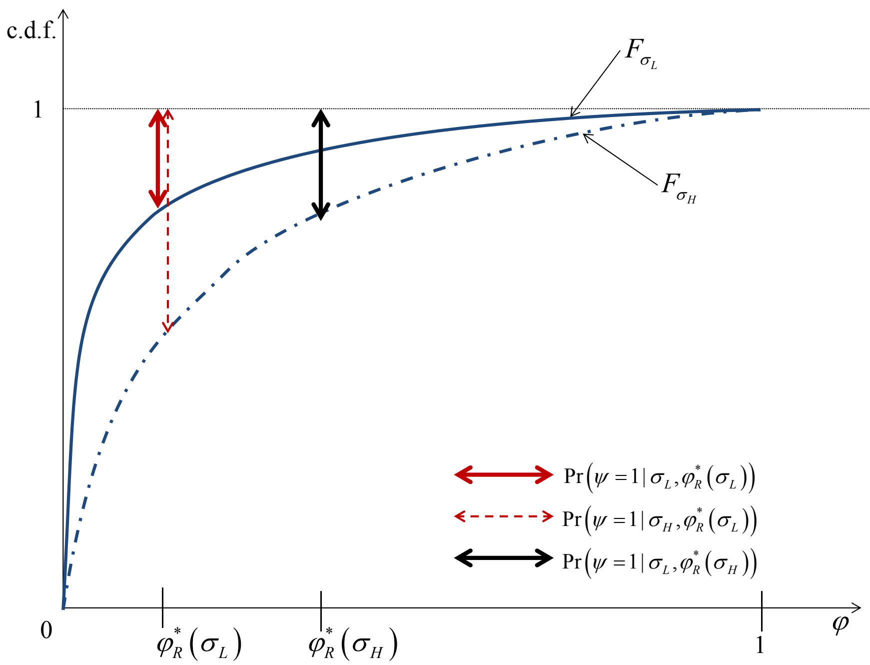
$$\frac{\partial \Pr(\psi = 1)}{\partial \sigma} > 0.$$

[23] The literature has emphasized the opportunity cost channel (see, for example, Frenkel and Jovanovic 1981, Rodrik 2006, Jeanne and Ranciere 2011). Yeyati (2008) rightly highlights the mitigating effect of the interest channel in the data, in contrast with existing theories. The crisis prevention channel is consistent with the findings of Gourinchas and Obstfeld (2012). Jeanne and Ranciere (2011) also discuss the crisis prevention channel, albeit in reduced form.

Proof: See Appendix B.

Proposition 2 establishes that both the optimal reserves and the implied sudden stop probability are increasing in the liquidity risk. A larger liquidity risk σ induces larger interim shocks and prompts the domestic government to invest in additional reserves. However, the increase in reserves does not completely offset the higher probability of larger shocks, thus leading to an increase in the debt rollover risk. Based on this proposition, we simply refer to the aggregate liquidity risk parameter σ as "rollover risk" throughout the paper.

Figure 5: Sudden Stop and Debt Rollover Risk



A central question that we address using Proposition 2 is, what happens during an unexpected increase in rollover risk, say in the wake of globalization? In Figure 5, we illustrate how an unexpected increase in σ from σ_L to σ_H can lead to a large sudden stop probability. Specifically, as the liquidity parameter increases from σ_L to σ_H, the cumulative distribution function of liquidity shocks shifts rightward from the solid blue curve to the dash-dotted blue curve. From Corollary 1, we know that the sudden stop probability is the mass of shocks above the reserves held, as represented by the solid vertical lines above the c.d.f. at the optimal reserve ratios. When there is an unexpected increase in rollover risk, the government does not hold enough reserves, leading to a large increase in the sudden stop probability, as represented by the dashed vertical line. Moreover, the model exhibits nonlinear effects: the levels of the rollover risk matter for the magnitude of outbursts, the induced change in reserves, and the corresponding sudden stop probabilities. We later show quantitatively that a relatively small, but unanticipated increase in rollover risk leads to a short-lived outburst of sudden stops and a dramatic rise in reserves as seen in the data.

3.6. Self-Insurance versus Mutual Insurance

In the self-insurance setup above, a government faces aggregate uncertainty stemming from its debt rollover risk. Therefore, an individual government accumulates more reserves compared to a world in which governments can pool reserves and mutually insure against their idiosyncratic rollover risk.

For simplicity, consider the problem of a planner who can swap resources across a continuum of countries facing i.i.d. idiosyncratic liquidity shocks φ^j with c.d.f. F_σ. By the law of large numbers, the total measure of lenders who must call the debt is $\mathbf{E}[\varphi] = \sigma/(\sigma+1)$. In that sense, there is no aggregate uncertainty across countries as they insure each other. For instance, the planner could set reserves to $\mathbf{E}[\varphi]$ and thereby prevent any sudden stop from occurring in any country. This policy is indeed optimal when the liquidity risk is sufficiently low.[24]

Proposition 3. Self-Insurance versus Mutual Insurance

Consider a continuum of countries subject to i.i.d. liquidity shocks. Each country individually accumulates more reserves compared to the mutual insurance outcome φ_R^C. That is:

$$\varphi_R^* > \mathbf{E}[\varphi] \geq \varphi_R^C \quad \forall \sigma \in (0,1).$$

Moreover, if $\sigma \leq (1-\lambda)/A$, then $\varphi_{\mathbf{R}}^{\mathbf{C}} = \mathbf{E}[\varphi]$.

Proof: See Appendix B.

Proposition 3 establishes that countries hold more reserves than would be needed if they could mutually insure against idiosyncratic liquidity shocks. The mutual insurance problem (see Appendix B.3) has close similarities with the liquidity provision problem studied by Holmstrom and Tirole (1998). Under mutual insurance, economies that face large liquidity shocks in the interim can access the reserves of economies with small liquidity needs, thereby reducing the overall debt rollover risk and the reserves required to manage it. In the next section, we quantify the extent of reserves over-accumulation after calibrating the liquidity risk faced by emerging economies.

[24] Since liquidity shocks can be correlated across countries, the i.i.d. case overstates the scope for mutual insurance. See Appendix B.5 for the case of correlated liquidity shocks. Akıncı (2013) finds that global factors account for 20 percent of movements in aggregate activity in emerging economies.

4. A Multi-Country Dynamic Application

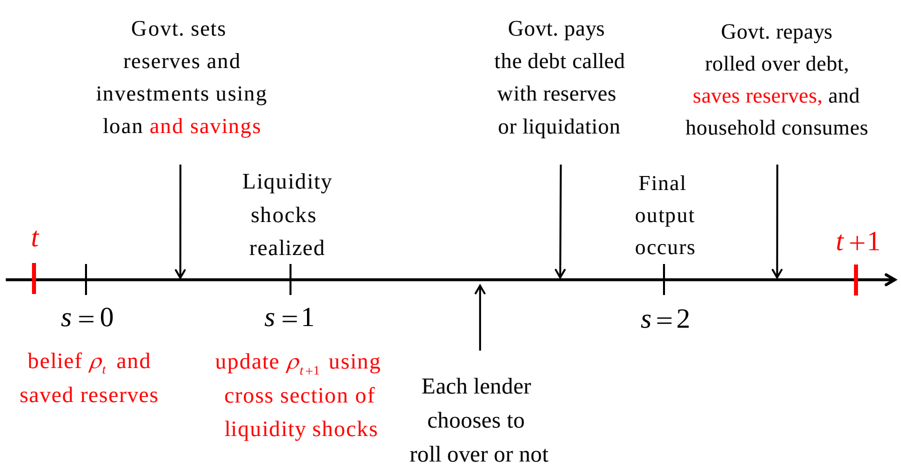
We now extend the simple model to address the puzzling coevolution of reserves and sudden stop dynamics. In particular, we aim to jointly account for the large accumulation of reserves and the evolution of sudden stops across emerging economies.

4.1. Extended Environment

First, the model is extended to an infinite horizon environment in which each period t embeds the three stages of the basic model. At the end of each period t, the government chooses how much resources to save. Second, the model is extended to a multi-country environment in which agents learn about the common liquidity risk process using the cross-section of shocks.

We consider N ex ante identical small open economies indexed by $j = 1, \ldots, N$. Time is infinite, discrete, and indexed by $t = 0, 1, \ldots, \infty$. Each country is populated by an infinitely-lived representative agent and a welfare-maximizing government. The agents in country j order consumption sequences according to $\mathbf{E}_0 \left[\sum_{t=0}^{\infty} \beta^t C_t^j\right]$ where β is the discount factor. There is a continuum of infinitely lived risk-neutral foreign lenders indexed by $i \in [0,1]$. An overview of the timeline of this extended model is shown in Figure 6.

Figure 6: Extended Timeline



Each time period t is divided into three stages ($s = 0, 1, 2$) and encapsulates the three stages of the previous model: (i) $s = 0$ is the initial contracting stage, (ii) $s = 1$ is the interim stage when liquidity shocks and rollover decisions occur, and (iii) $s = 2$ is the final production and consumption stage.

Within each period t, the technologies available at a stage s are identical to those in the previous section.[25] We now allow for partial liquidation in the interim: $L_t^j \in \left[0,\, K_t^j\right]$. This implies that sudden stops may not occur as soon as reserves are depleted.

The aggregate interim liquidity shock in country j at time t is denoted by $\varphi_t^j \in [0,1]$. As in the simple model, this means that a fraction φ_t^j of country j's creditors must call the debt in the interim while the others can roll over or call the debt. The aggregate shocks $\left\{\varphi_t^j \,:\, j = 1\ldots N\right\}_{t=0}^{\infty}$ are independent and identically distributed across countries and time, with cumulative distribution function $F_{\sigma_t}(\varphi) = 1-(1-\varphi)^{1/\sigma_t}$. We assume $\sigma_t \in \{\sigma_L, \sigma_H\}$ with $\sigma_L < \sigma_H$.

4.2. *Bayesian Learning*

The rollover risk parameter σ_t is unobserved and unknown to the agents, even though they know that $\sigma_t \in \{\sigma_L, \sigma_H\}$. All agents share a common belief ρ_t at time t: $\rho_t \equiv \Pr(\sigma_t = \sigma_L)$. In the interim stage of each period t, agents observe the cross-section of liquidity shocks in the N countries. The vector of liquidity shocks, denoted by $\vec{\varphi}_t = \left\{\varphi_t^j \mid j = 1,..,N\right\}$, has a joint probability density function

$$f_\sigma^N(\vec{\varphi}_t) = \prod_{j=1}^{N} f_\sigma\left(\varphi^j\right).$$

By Bayes' rule, given a realization of $\vec{\varphi}_t$, the posterior is given by

$$\rho_{t+1}(\rho_t, \vec{\varphi}_t) = \frac{\rho_t f_{\sigma_L}^N(\vec{\varphi}_t)}{\rho_t f_{\sigma_L}^N(\vec{\varphi}_t) + (1-\rho_t) f_{\sigma_H}^N(\vec{\varphi}_t)}. \tag{9}$$

We later restrict countries to learn from a subset of countries in order to capture economic and geographic linkages in learning dynamics and to highlight the role of learning in our model.[26]

4.3. *Optimal Recursive Debt Contracts*

We represent the government's infinite horizon problem as a recursive dynamic programming problem. The problem has one endogenous state, the level of incoming saved reserves, $R_{0,t}^j$, and one exogenous state, the common belief, ρ_t. The state of economy j at time t is then given by $(R_0;\rho) = \left(R_{0,t}^j;\rho_t\right)$.

The optimal recursive debt contract, $B^*(R_0;\rho)$, is a set of policy functions for initial reserves, $R_1(R_0;\rho)$; invested capital, $K(R_0;\rho)$; normal interest rates, $r_N(R_0;\rho)$; sudden stop interest rates,

[25] The superscript j and the subscript t are added to the variables from the original model to denote the country and the period. We keep the subscripts indicating the stage s when necessary, for example, as in the case of reserves $R_{s,t}^j$.

[26] Here, σ_t is not a time-varying process. It is possible to expand the setup to allow for a Markov process for σ_t (see, for example, Boz and Mendoza 2014). Maćkowiak and Wiederholt (2015) also study learning and rational inattention when the unknown underlying state is itself randomly distributed. Their setup features insightful differences between learning from the cross-section and learning from the time-series.

$r_S(R_0;\rho)$; consumption, $C(R_0,\vec{\varphi};\rho)$; interim reserves, $R_2(R_0,\vec{\varphi};\rho)$; liquidation, $L(R_0,\vec{\varphi};\rho)$; saved reserves, $R_0'(R_0,\vec{\varphi};\rho)$; and rollover policies, $\psi^i\left(R_0,\vec{\varphi},\varphi^i;\rho\right)$, which satisfy

$$W(R_0;\rho) = \max_{B\in\Gamma(R_0;\rho)} \mathbf{E}_{\vec{\varphi}|\rho}\left[C(R_0,\vec{\varphi};\rho) + \beta W\left(R_0'(R_0,\vec{\varphi};\rho);\rho'(\rho,\vec{\varphi})\right)\right]. \tag{10}$$

The interim policy functions depend on the domestic shock $\varphi \equiv \vec{\varphi}(j)$ as well as the entire global vector of shocks $\vec{\varphi}$ since the posterior belief $\rho'(\rho,\vec{\varphi})$ affects the reserves-savings decision.

As in the previous section, a debt contract is feasible — that is $B \in \Gamma(R_0;\rho)$ — if it satisfies resource feasibility, interim individual rationality, the *ex ante* participation constraint, and *ex post* renegotiation proofness. Resource feasibility is modified to allow for saved reserves and partial liquidation. The initial $(s=0)$ resource constraint, which incorporates incoming reserves saved (R_0), is now

$$R_1 + K \leq D + R_0. \tag{11}$$

The final $(s=2)$ resource constraint, modified by inter-period savings $(R_0'(\varphi))$, is now

$$C(\varphi) + (1-\psi(\varphi))P_2(\psi(\varphi)) + R_0'(\varphi) \leq R_2(\varphi) + A(K - L(\varphi)) \quad \forall\varphi. \tag{12}$$

Also, saved reserves and liquidation must satisfy

$$L(\varphi) \in [0\,,\,K] \quad \forall\varphi \tag{13}$$

$$R_0'(\varphi) \in [0\,,\,A(K-L(\varphi)) + R_2(\varphi)] \quad \forall\varphi. \tag{14}$$

As in the simple model, we assume that $K \leq D$. Since the government, in principle, could save resources across periods and invest more than it borrows, we are implicitly assuming that the long-term technology depends on foreign investment. This assumption also makes the problem linear in debt, allowing us to normalize $D = 1$.[27]

4.4. Characterization

In this section, we simplify the problem described in Section 4.3 in two steps. The first involves reducing the dimension of the state space. As shown in the Bayesian learning formula in (9), the posterior belief depends on the joint draw of shocks across all N economies. Needless to say, this is a significant computational burden. We derive novel results that allow us to replace the

[27] We deliberately focus on the case of linear technologies and preferences in order to highlight how the rollover risk channel operates. Alternatively, we could endogenize the level of debt by adding curvature to the utility or production functions. This would greatly add to the computational burden without adding insight to the question of how reserves interact with rollover risk in determining sudden stop probabilities. Such a model of endogenous debt would be important to study fluctuations of reserves, debt, and crises over the business cycle. In fact, both Jeanne and Ranciere (2011) and Bianchi et al. (2012) acknowledge that the preventive role of reserves is indeed a promising mechanism. See also (Durdu et al., 2009) for an insightful discussion on models of precautionary savings and sudden stops.

interim state $\vec{\varphi} \in [0,1]^N$ with the sufficient representation $\{\varphi^j, \rho'(\rho, \vec{\varphi})\} \in [0,1]^2$. The main step is to derive an analytical expression for the conditional distribution g of posterior beliefs ρ' (see Appendix B.6).[28]

The second step involves reducing the dimension of the optimal contract. The main step is to characterize the thresholds for liquidation and sudden stops. We do this in Proposition 4, which provides the analogue to Proposition 1 of the previous section.

Proposition 4. Optimal Recursive Debt Contract

For any $(R_0;\rho)$, an optimal recursive debt contract B^* satisfies

(i) Interim payments are paid exclusively with reserves until they are depleted:

$$\exists \quad (\varphi_N^*, \varphi_R^*) \in [0,1]^2 \text{ s.t. } \begin{cases} L^*(\varphi) = 0,\ R_2^*(\varphi) \geq 0 & \forall \varphi \in (\varphi_N^*, \varphi_R^*) \\ L^*(\varphi) \geq 0,\ R_2^*(\varphi) = 0 & \forall \varphi \notin (\varphi_N^*, \varphi_R^*) \end{cases}.$$

(ii) Rollover policies satisfy

$$\exists \quad \varphi_S^* \in [0,1] \text{ s.t. } \begin{cases} \psi(\varphi) = \varphi & \forall \varphi \in [\varphi_N^*, \varphi_S^*) \\ \psi(\varphi) = 1 & \forall \varphi \notin [\varphi_N^*, \varphi_S^*) \end{cases}.$$

Proof: See Appendix B.

In Proposition 4(i), φ_R^* is the liquidity shock at which reserves are depleted and the government must begin to liquidate the invested capital to meet the promised payments. Because $\lambda < 1$, the government uses existing reserves to meet payments before eventually liquidating the invested capital. In Proposition 4(ii), φ_S^* is the liquidity shock at which the government is indifferent about whether a sudden stop occurs or not: any liquidity shock above φ_S^* results in a sudden stop. This proposition also greatly simplifies our computational strategy. In principle, if the liquidity shock is sufficiently small and too many lenders roll over the debt, the government may prefer a sudden stop over paying the normal interest rate: any liquidity shock below φ_N^* results in a sudden stop. However, for the set of parameters we consider, $\varphi_N^* = 0$.

4.5. Quantitative Analysis

We now use the calibrated model to establish how a small but unexpected increase in debt rollover risk can explain the sharp increase in reserves and the temporary outburst in sudden stops documented in Section 2. In our experiment, we assume that after 1996, there was an unexpected increase from a σ_L-regime to a σ_H-regime. This is motivated by the idea that globalization and

[28] In essence, what really matters is the induced posterior, not the entire cross-section of shocks. Indeed, only the belief is a state for the functional equation, while foreign shocks do not affect resource constraints domestically.

Table 2: Calibration values

Name	Symbol	Value	Target
Discount factor	β	0.931	non-transition sudden stop probability (0.33 percent)
Bargaining parameter	θ	0.815	volume-weighted average haircut (30 percent)
Low rollover risk	σ_L	0.060	median reserves-to-debt, 1992–1996 (20 percent)
High rollover risk	σ_H	0.175	median reserves-to-debt, 2002–2006 (40 percent)
Number of economies	N	23	emerging countries in sample
World interest rate	r_W	0.01	risk-free rate
Divestment parameter	λ	0.6	see discussion
Productivity	A	1.2	see discussion

widespread financial liberalization led to an unprecedented increase in capital mobility and debt rollover risk (see Figure 1). We view this as a simple way to capture increased capital mobility in emerging economies. Moreover, we model this change in liquidity risk as being unexpected, since governments may have underestimated the consequences of opening their capital markets. This view is consistent with the fact that market participants, policymakers, and the IMF were surprised by the sudden stops in 1997–1999.[29]

Based on our theory, an unexpected increase in the rollover risk will temporarily cause an underinvestment in reserve holdings, which increases the probability of sudden stops. Governments and investors, seeing the increase in aggregate liquidity shocks and sudden stops, rationally update their common belief about the prevailing debt rollover risk. Once agents have fully learned the new regime, reserves are higher and sudden stops subside.[30]

Calibration. A period in the model is assumed to be a quarter. We choose $N = 23$, as we have 23 emerging economies in our dataset. We assume the aggregate liquidity shock distributions $(F_{\sigma_L}, F_{\sigma_H})$ belong to the class of Pareto distributions on $[0,1]$: $F_\sigma(\varphi) \equiv 1-(1-\varphi)^{1/\sigma}$. An increase in σ shifts the cumulative distribution function F_σ to the right, as illustrated in Figure 5. An increase from σ_L to σ_H therefore represents an increase in the underlying debt rollover risk.

The discount factor β, the bargaining parameter θ, and the debt rollover risk parameters σ_L and σ_H are jointly calibrated to match the sudden stop frequency in the non-transition era (1992–1996, 2002–2006) of 0.33 percent, the volume-weighted average haircut of 30 percent (Cruces and

[29]For example, a Federal Reserve report mentions that "the Asian Financial Crisis generally caught market participants and policymakers by surprise" (Carson and Clark 2013) while the *Washington Post* reports that "nasty developments (in Asia) have repeatedly taken IMF officials by surprise" (Blustein 1998; parentheses are ours). In the words of Durdu et al. (2009), "financial globalization had a rocky start in emerging economies hit by sudden stops." Using various measures of capital openness, Steiner (2013) also argues empirically that reserves rose due to increased "fear of capital mobility."

[30]Boz and Mendoza (2014) also provide an interesting application of learning to the recent U.S. financial crisis using a model in which agents gradually learn about a one-time increase in the riskiness of assets.

Table 3: Summary of Results

	1992–1996	1997–2001	2002–2006
Data			
Reserves-to-External Debt Liabilities	0.20	0.30	0.40
Sudden Stops	3	9	0
Model (1 region of 23 countries)			
Reserves-to External Debt Liabilities	0.20	0.39	0.40
Sudden Stops	1.35	5.83	1.69
Sudden Stop Probabilities (percent)	0.29	1.27	0.37
Model (3 regions of 8 countries each)			
Reserves-to External Debt Liabilities	0.20	0.37	0.40
Sudden Stops	1.41	11.21	1.73
Sudden Stop Probabilities (percent)	0.29	2.34	0.36

Trebesch 2013), and the median reserves-to-debt ratios in the emerging economies for the periods of 1992–1996 and 2002–2006, respectively.[31] We target the sudden stop frequency excluding the transition era of 1997–2001 and use the out-of-sample transition era as a test of the calibrated model. The world interest rate r_W is set to match a risk-free rate of 1 percent. We set the liquidation cost $1-\lambda$ to be 40 percent, which is in the range of estimates used in the literature.[32] The long-term technology productivity A is set to 1.2.[33] The parameters are summarized in Table 2. See the Online Appendix for details on the computation and calibration strategy.

Quantitative Results. The N ex ante identical economies experience different aggregate liquidity shock paths $\left\{\varphi_t^j\right\}_{j,t}$. As a result, their reserves holdings and sudden stops paths also evolve differently. The results shown are the averages across a large number of simulated paths for these N countries.

Table 3 summarizes our key results. The calibration reveals that the debt rollover risk σ increased from 0.060 to 0.175. This is a relatively small increase, in the sense that the implied sudden

[31] Ideally, one might want to calibrate the rollover risk parameters to match statistics on either the magnitude or volatility of gross external debt flows. However, the data on external debt flows is only available in net terms. We therefore rely on other testable implications of the model, such as the sudden stop frequency in the transition era to validate the model and calibration.

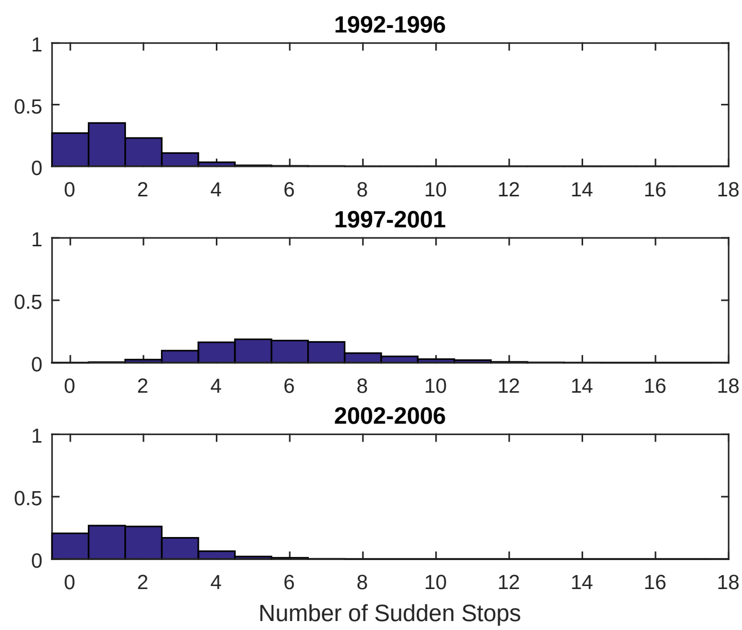
[32] For example, estimates for liquidation costs include 30.5 percent (James 1991) and 49.9 percent (Brown and Epstein 1992) in bank failures, and 37 percent (Alderson and Betker 1996) in chapter 7 liquidations, while Ennis and Keister (2003) use liquidation costs of 60 percent and 70 percent in their analysis. We also run sensitivity analysis with different values of λ and find little variation in the main results. See Table A.7 in Appendix A for details.

[33] The results hold as long as the productivity is sufficiently higher than the world interest rate. See Table A.7 in Appendix A for a sensitivity analysis.

stop probabilities only rise from 0.29 percent to 0.37 percent, compared with a 1.27 percent probability during the transition. Despite this *small* increase in debt rollover risk, there is an outburst of sudden stops, with the mode across simulations reaching 5 before subsiding (see Figure 7). In the meantime, the optimal reserves-to-debt ratios climbed from 20 percent to 40 percent. The temporary surge in sudden stops is consistent with our discussion of Proposition 2 in the simple model (see Figure 5): as governments learn the higher rollover risk, they choose to hold a higher level of reserves, thus returning sudden stop probabilities to lower levels.[34]

The calibration establishes quantitatively how a relatively small increase in rollover risk can explain the surge we observed in the data. For robustness, we also report the results for different values of σ_H in Table A.7 in Appendix A. We consistently find that higher values of σ_H result in more sudden stops in the transition before the economy reaches higher reserves, along with a low probability of sudden stops.

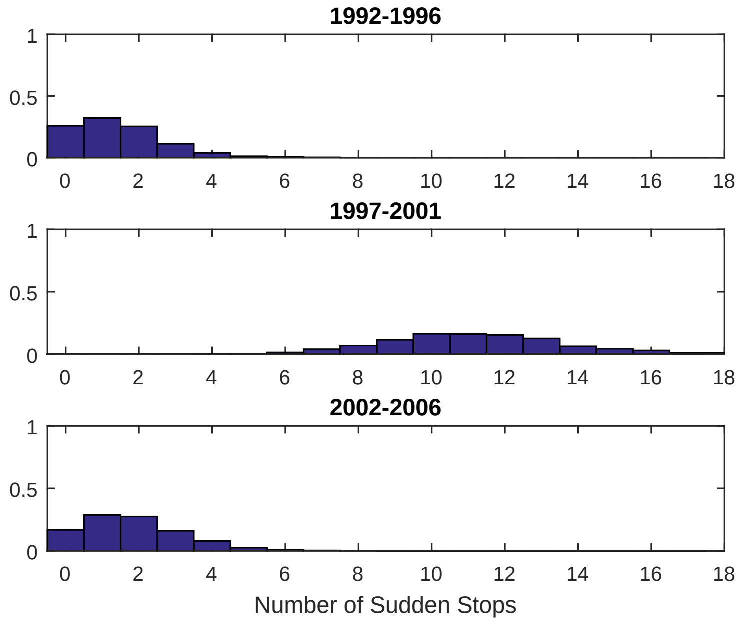
Figure 7: Histogram of Sudden Stops by Era with a Single Region



Regional Learning. The magnitude of the temporary outburst of sudden stops generated depends crucially on the ability to learn: if the change in rollover risk was perfectly observable, there would be an immediate rise in reserves and no outburst of sudden stops, and if countries could not learn about the rollover risk, reserves would not increase and sudden stops would be permanently higher.

[34]See also Bussière et al. (2015), who show that emerging countries promptly rebuilt their reserves holdings back to the levels they had before the depletion caused by the Global Financial Crisis of 2008–2009. This evidence suggests that these countries did not perceive the GFC as a permanent regime change.

Figure 8: Histogram of Sudden Stops by Era with Three Regions



We explore this issue by modeling regional learning: what if governments learn from only a subset of countries due to economic and geographic linkages? We consider a world with three regions of eight *ex ante* identical countries each ($N = 3 \times 8$), in which governments learn only from the cross-section of regional shocks.[35] In this environment, governments within a region still share a common belief, but these beliefs may be different across regions. Although the regions are subject to the same random process of liquidity shocks, they are assumed to only learn from the idiosyncratic shocks experienced in their region. Because governments are learning from a smaller sample of countries, learning is slower and sudden stops are more frequent in the transition. Outside of the transition era (1997–2001), governments eventually learn the true regime, making reserves and sudden stops similar to the single-region model.

Indeed, Table 3 shows that while the level of non-transition reserves and sudden stop probabilities are similar, countries now accumulate fewer reserves in the transition and thus experience a more severe outburst. Interestingly, the transition sudden stop frequency in the three-region model (2.34 percent) is remarkably similar to that in the data (2.17 percent). As can be seen in Figure 8, the same increase in rollover risk now leads to an outburst of sudden stops with a mode of 10 before subsiding. These results are strikingly close to the dynamics observed in the data.[36] These

[35] We use $N = 24$ countries, as it allows richer combinations of regional learning than the prime number $N = 23$ used in the single region case to match the data we have. As one would expect, the single region case with $N = 24$ is very close quantitatively to the one with $N = 23$.

[36] We also run the Gourinchas and Obstfeld (2012) exercise on our model simulated data. In the model, reserves are

results also highlight that learning can generate dynamics akin to *contagion*.

International Mutual Insurance and Reserves. Proposition 3 showed that countries may over-accumulate reserves compared to an allocation with mutual insurance with other countries. We now use the calibrated parameters to quantify the magnitude of the over-accumulation of reserves due to self-insurance.

Given the calibrated parameter values and using Proposition 3, the international planner facing no aggregate uncertainty will optimally set reserves to the mean liquidity shock.[37] Therefore, in the higher rollover risk (σ_H) regime, the international planner optimally sets the reserves-to-debt ratio at $\sigma_H/(1+\sigma_H) = 15.61$ percent. This amounts to nearly two-fifths of the level of 40 percent in reserves-to-debt that emerging economies held from 2002 to 2006. Certainly, this corresponds to an upper bound on the over-accumulation of reserves due to the correlation of shocks across the finite number of countries and moral hazard concerns.[38]

Nonetheless, this result clearly underscores the importance of mutual insurance or international coordination across governments facing uninsurable idiosyncratic debt rollover risk. In fact, during the 2008–2009 Global Financial Crisis, reserves swap agreements such as the ASEAN+3 Chiang Mai Initiative were expanded. The U.S. and Japan also extended swap lines to emerging economies such as Korea. The IMF could in principle assume the role of an international planner for rollover risk insurance. However, many economists and policymakers argue (see Ito 2012) that emerging economies still bear the scar and the stigma from the inadequate liquidity assistance provided by the IMF during the crises of the late 1990s.

Reserves and Crises in other Regions. Interestingly, before 1999, the euro area periphery economies (Greece, Ireland, Italy, Portugal, and Spain) held similar levels of reserves as the 23 emerging economies we consider. However, upon joining the euro area, these economies slashed their reserves holdings, as illustrated in Figure 9.

The common currency certainly explains part of the reduction in foreign reserves. However, to the extent that these economies still faced debt rollover risk, they may have under-invested in reserves. For instance, they may have mistakenly believed that they would no longer face rollover risk after joining the euro. Alternatively, the periphery economies *ex ante* may have counted on a mutual insurance policy against liquidity needs that showed its limits during the euro crisis.

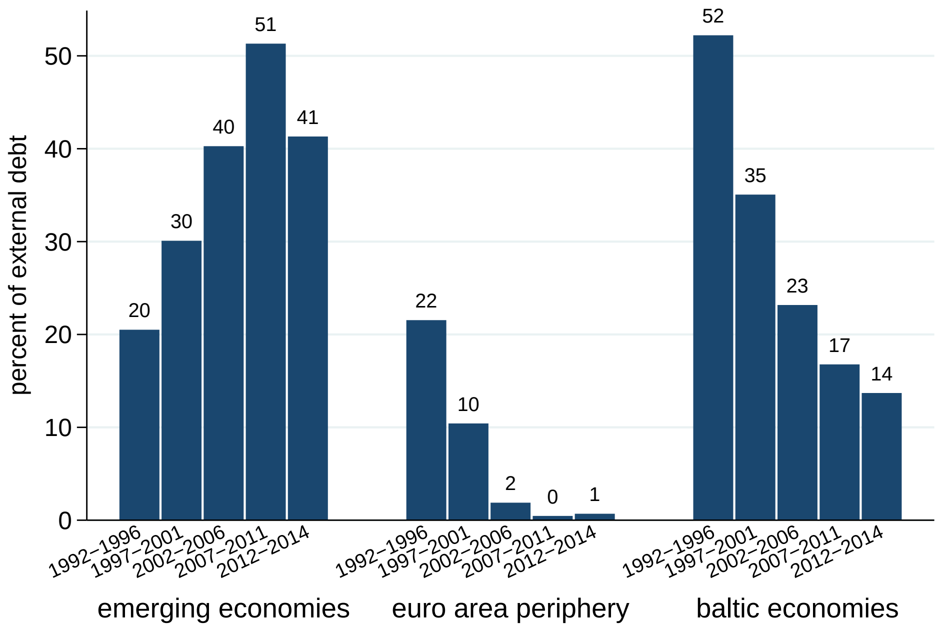
Baltic economies (Estonia, Latvia, and Lithuania) represent perhaps a more relevant case for our model. Soon after regaining independence in the 1990s, the Baltic "tigers" grew by an average of 8 percent per year from 2000 to 2007. They were all severely hit during the global financial

associated with a lower probability of a sudden stop. See Table A.8 in Appendix A.

[37]Note that this is planner's allocation since the condition $\sigma_H < (1-\lambda)/A$ from Proposition 3 holds.

[38]See in Appendix B.5 the formula for optimal reserves insurance with correlated shocks.

Figure 9: Foreign Reserves in the Baltic Economies and Other Blocs



crisis, experienced sudden stops in 2008–2009, and contracted by about 14 percent in 2009. Can our model shed light on the severe Baltic sudden stop experience?

Figure 9 shows that the reserves-to-external-debt ratio in the Baltic cluster sharply declined from 52 percent in the early 1990s to 23 percent in the early 2000s. As noted in our empirical analysis, emerging economies fared relatively well during the global financial crisis, with only four sudden stops across the entire sample. Yet, all Baltic states experienced a sudden stop.

Unlike the euro area periphery countries, the Baltic countries were not part of the common currency and could not have counted on an implicit insurance policy from the euro zone. Assuming the Baltic bloc is subject to the same global rollover risks as the emerging economies, the Baltic bloc's low reserves in 2008 made it as vulnerable in 2008 as emerging countries were in the late 1990s.[39] Moreover, the Baltic cluster may still be vulnerable, as its ratio of reserves to external debt has further dwindled to a historic low, even as they recovered from the financial crisis.

In the meantime, self-insurance through reserves helped emerging economies weather the global financial crisis, as noted by Dominguez et al. (2012), Gourinchas and Obstfeld (2012), and Bussière et al. (2015). This is also consistent with our findings on the preventive role of reserves. In emerging economies, reserves increased leading up to the global financial crisis — exceeding

[39]Interestingly, the reserves-to-GDP ratio in the Baltic economies gradually rose from 12 percent to 17 percent between 1997 and 2006, and remained high during the GFC, at around 17 percent (see Figure A.10 in Appendix A for reserves-to-GDP ratios). In the meantime, disproportionately large inflows of credit led to a decline in the ratio of reserves to external debt liabilities. Our model indicates that reserves-to-liabilities are the relevant metric.

50 percent of external debt. Since then, their reserves have returned to pre-crisis levels, suggesting that the global financial crisis was not perceived as a permanent but rather a temporary change in rollover risk.

Finally, the fact that a majority of sudden stops in the late 1990s were clustered in Asia and Latin America, while those occurring after 2008 took place mostly in Eastern and Southern Europe as well as the Baltic economies, lends support to our model with geographically localized learning.

5. Conclusion

In this paper, we developed a theory of rollover risk, sudden stops, and reserves that can jointly account for the puzzling coevolution of foreign reserves and sudden stops in emerging economies. In our theory, governments choose reserves to prevent "patient" foreign creditors from refusing to roll over their claims and inducing a sudden stop. Optimally, reserves are chosen to balance (i) the opportunity cost of idle reserves due to reduced capital investment, (ii) the endogenous change in interest rate, and (iii) the overall lower likelihood of a crisis.

We calibrate a dynamic multi-country extension of the model with Bayesian learning to emerging economies. A relatively small, unexpected, but permanent change in rollover risk leads to the surge in sudden stops in the late 1990s, the subsequent rise in reserves, and the salient fall in sudden stops ever since. We find that a policy of international mutual insurance may substantially reduce the reserves held by emerging economies. We also contrast these findings with the prominent sudden stop experiences in Baltic countries and the euro area periphery during the Global Financial Crisis.

Several caveats are in order. Our model ignores the decision to issue reserve assets. In particular, U.S. Treasuries, the most popular reserve asset, are being increasingly held by foreign officials as they accumulate reserves. Can the U.S. sustainably issue large amounts of reserve assets abroad? Moreover, our model does not consider the maturity composition of debt. We leave these considerations for future research.

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A. Additional Tables and Figures

Table A.4: Foreign Reserves as percent of GDP

	1992–1996	1997–2001	2002–2006	2007–2011	2012–2014
Argentina	5.6	7.9	12.8	13.9	6.6
Brazil	6.6	5.7	7.6	13.4	16.0
Chile	19.9	19.8	17.0	13.4	15.4
China	8.0	14.9	30.5	45.5	40.1
Colombia	9.1	8.9	10.6	9.9	10.9
Czech Republic	16.9	20.2	25.3	19.1	25.5
Egypt	26.3	17.8	19.1	16.4	4.4
Hungary	18.4	20.9	16.5	28.9	33.6
India	4.4	7.4	16.7	19.1	14.8
Indonesia	7.3	16.9	13.4	12.0	12.1
Korea	5.7	14.6	24.5	27.0	28.0
Malaysia	31.5	31.4	45.0	45.4	41.0
Mexico	4.2	6.0	8.1	10.5	14.1
Morocco	12.4	14.6	27.9	25.3	17.3
Pakistan	2.2	2.4	10.5	7.2	2.6
Peru	13.5	16.8	17.0	25.7	29.3
Philippines	8.5	14.1	15.8	24.0	27.5
Poland	7.4	14.7	14.1	16.1	19.6
Romania	5.1	7.6	17.8	22.9	22.6
Russia	3.0	6.1	20.6	29.6	21.3
South Africa	1.1	4.2	6.3	11.0	12.5
Thailand	20.2	25.0	29.8	45.3	43.2
Turkey	4.4	8.4	10.8	10.7	12.8
median	7.4	14.6	16.7	19.1	17.3

Table A.5: Foreign Reserves as percent of External Debt Liabilities

	1992–1996	1997–2001	2002–2006	2007–2011	2012–2014
Argentina	16.2	15.7	17.3	41.6	25.4
Brazil	25.0	17.8	28.2	78.2	74.5
Chile	57.0	52.4	40.2	38.3	41.3
China	50.8	108.0	245.5	438.0	290.9
Colombia	37.3	24.9	33.1	44.6	42.8
Czech Republic	57.2	63.0	82.4	51.3	54.1
Egypt	44.4	51.3	56.9	79.1	
Hungary	32.2	39.6	25.7	27.6	37.6
India	15.4	33.9	94.1	104.0	70.3
Indonesia	11.8	16.5	26.4	44.0	
Korea	27.9	49.0	103.1	74.6	83.6
Malaysia	75.9	57.2	98.4	101.4	72.5
Mexico	12.3	21.1	37.2	42.2	40.0
Morocco	17.4	30.1	93.8	94.6	
Pakistan	5.3	5.5	27.6	22.1	9.9
Peru	20.5	32.5	42.2	90.0	100.1
Philippines	16.1	20.0	24.8	64.1	
Poland	19.3	45.6	36.2	34.4	36.0
Romania	23.5	27.6	53.9	45.0	40.7
Russia	7.3	11.8	60.1	99.7	
South Africa	4.8	16.4	32.5	48.9	40.2
Thailand	39.8	37.8	93.0	178.9	
Turkey	13.7	19.7	25.0	26.2	25.5
median	20.5	30.1	40.2	51.3	41.3

Table A.6: Panel Logit Estimation for Other Crises

	S.D.	Crisis in 1–2 years δp	$\frac{\partial p}{\partial x}$	Crisis 1–3 years δp	$\frac{\partial p}{\partial x}$
Panel A: Default Crises (baseline sample: country FE and years 1990-2011)					
Reserves	20.45	-0.07	-0.02	-0.11	-0.03
over External Debt		(0.16)	(0.04)	(0.29)	(0.07)
Net Foreign Assets	8.91	-0.07	-0.02	-0.10	-0.03
over GDP		(0.15)	(0.04)	(0.27)	(0.07)
Probability in percent ($\overline{p}$)		0.07		0.11	
N=6 ; $N\times T$=98					
Panel B: Banking Crises (baseline sample: country FE and years 1990–2011)					
Reserves	51.28	-5.73***	-0.22***	-8.34***	-0.36***
over External Debt		(1.45)	(0.07)	(1.99)	(0.08)
Net Foreign Assets	12.49	-2.53	-0.25	-2.88	-0.27
over GDP		(1.81)	(0.20)	(1.98)	(0.21)
Probability in percent ($\overline{p}$)		7.16		9.64	
N=15 ; $N\times T$=249					
Panel C: Currency Crises (baseline sample: country FE and years 1990–2011)					
Reserves	46.63	-4.66***	-0.27***	-8.90***	-0.41***
over External Debt		(1.88)	(0.05)	(1.56)	(0.07)
Net Foreign Assets	11.45	-0.65	0.06	-0.84	-0.08
over GDP		(1.10)	(0.11)	(1.76)	(0.17)
Probability in percent ($\overline{p}$)		5.05		10.35	
N=18 ; $N\times T$=294					

Note: *, **, and *** denote significance at the 10, 5, and 1 percent level. $\partial p/\partial x$ is the marginal effect in percentage at "tranquil" sample mean. δp is the effect in percentage for an increase of one standard deviation in x at the "tranquil" sample mean. $s.d.(x)$ is the unconditional standard deviation of x over "tranquil" times. $\overline{p}$ is the probability of a crisis at the sample mean. Robust standard errors in parentheses are computed using the delta-method. The estimation sample is an unbalanced panel that spans 23 emerging countries between 1990 and 2011. The data stops in 2011 as the updated series by Lane and Milesi-Ferretti (2007) stop in 2011. Due to the use of country fixed effects, countries without a given crisis are not in the logit estimation sample for that type of crisis. Currency, banking, and default crises dates follow Gourinchas and Obstfeld (2012) and are listed in the data appendix. Sample means are higher in Panels B and C as China is part of the estimation sample. The results are similar without China as the current estimation features country fixed effects.

Table A.7: Sensitivity Analysis

(baseline values in parentheses)	1992–1996	1997–2002	2002–2006
$\lambda = 0.5$ (0.6)			
Reserves-to External Debt Liabilities	0.21	0.41	0.43
Sudden Stop Probabilities (percent)	0.45	1.67	0.69
$\lambda = 0.7$ (0.6)			
Reserves-to External Debt Liabilities	0.17	0.33	0.35
Sudden Stop Probabilities (percent)	0.31	1.21	0.30
$A = 1.1$ (1.2)			
Reserves-to External Debt Liabilities	0.24	0.39	0.40
Sudden Stop Probabilities (percent)	0.33	1.60	0.61
$A = 1.3$ (1.2)			
Reserves-to External Debt Liabilities	0.17	0.38	0.40
Sudden Stop Probabilities (percent)	0.27	1.08	0.26
$\sigma_H = 0.152$ (0.175)			
Reserves-to External Debt Liabilities	0.20	0.35	0.36
Sudden Stop Probabilities (percent)	0.29	1.25	0.47
$\sigma_H = 0.199$ (0.175)			
Reserves-to External Debt Liabilities	0.20	0.41	0.43
Sudden Stop Probabilities (percent)	0.29	1.41	0.36
baseline calibration			
Reserves-to External Debt Liabilities	0.20	0.39	0.40
Sudden Stop Probabilities (percent)	0.29	1.27	0.37

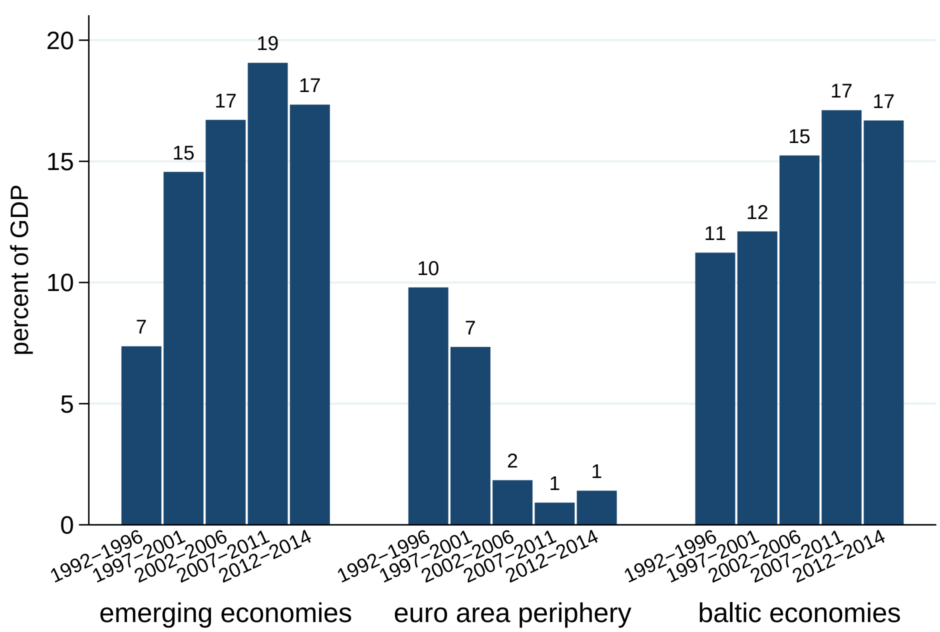
Note: The sensitivity analysis reports data generated by the model in which the parameters of the model have been set to the baseline calibration except for the changes to λ, A, and σ_H respectively.

Table A.8: Panel Logit Estimation on Simulated Data

	S.D.	Crisis in 1–2 years δp	$\frac{\partial p}{\partial x}$	Crisis 1-3 years δp	$\frac{\partial p}{\partial x}$
Panel A: Sudden Stops (one region model - simulated data with country FE)					
Reserves	9.41	-1.43***	-0.18***	-2.43	-0.31***
over External Debt		(0.31)	(0.05)	(7.52)	(0.06)
Probability in percent ($\overline{p}$)		4.86		7.29	
N=23 ; $N \times T \times S$=2738					
Panel B: Sudden Stops (three region model - simulated data with country FE)					
Reserves	8.96	-2.36***	-0.31***	-3.41***	-0.44***
over External Debt		(0.04)	(0.06)	(7.52)	(0.08)
Probability in percent ($\overline{p}$)		7.80		11.83	
N=24 ; $N \times T \times S$=2740					

Note: *, **, and *** denote significance at the 10, 5, and 1 percent level. $\partial p/\partial x$ is the marginal effect in percentage at "tranquil" sample mean. δp is the effect in percentage for an increase of one standard deviation in x at the "tranquil" sample mean. $s.d.(x)$ is the unconditional standard deviation of x over "tranquil" times. $\overline{p}$ is the probability of a crisis at the sample mean. Robust standard errors in parentheses are computed using the delta-method. The estimation sample is an unbalanced panel that spans 23 countries between 1992 and 2006 in 10 simulations. For the three region model, we use 24 countries. The estimation also include simulation fixed effects. The results are similar with more simulations. Naturally, the model features much less dispersion in reserves - and hence fewer crises - compared to the data.

Figure A.10: Foreign Reserves relative to GDP



B. Proofs

B.1. Proof of Proposition 1

We proceed in nine steps.

Step 1: *Interest rates satisfy*

$$r_S^* < 0 < r_N^* \tag{B.1}$$

$1 + r_N^* \geq 1$ follows from equation (7). Equation (2) and $\lambda < 1$ imply that $(R_1 + \lambda K)/D < 1$. Since $\theta = 1$, equation (3) implies $1 + r_S^* = (R_1 + \lambda K)/D$. Hence $r_S^* < 0$.

Step 2: *If* $\psi^*(\varphi) = 1$, *then*

$$L^*(\varphi) = K^* \tag{B.2}$$

$$R_2^*(\varphi) = 0 \tag{B.3}$$

$$C^*(\varphi) = 0 \tag{B.4}$$

By definition, if $\psi^*(\varphi) = 1$, then $P_1^*(\varphi) = \left(1 + r_S^*\right) D$. From step 1, we have that $r_S^* = (R_1 + \lambda K)/D$. Equations (4) and (6) imply equations (B.2) and (B.3). Then equation (B.4) follows from equations (5) and (6) .

Step 3: *If* $\psi^*(\varphi) = \varphi$, *then*

$$L^*(\varphi) = 0 \tag{B.5}$$

$$R_2^*(\varphi) = R_1^* - \varphi D \tag{B.6}$$

$$C^*(\varphi) = AK^* + R_2^*(\varphi) - (1 - \varphi)(1 + r_N^*) D \tag{B.7}$$

By definition, if $\psi^*(\varphi) = \varphi$, then $P_1^*(\varphi) = D$ and $P_2^*(\varphi) = (1 + r_N^*) D$. Suppose for contradiction that $L^*(\varphi) = K$. Then equation (4) implies $R_2^*(\varphi) = R_1^* + \lambda K^* - \varphi D$. Then we have that

$$\begin{aligned} C^*(\varphi) &= R_1^* + \lambda K^* - \varphi D - (1 - \varphi)(1 + r_N^*) D \\ &\leq R_1^* + \lambda K^* - D \\ &< 0 \end{aligned}$$

where the first equality comes from equation (5), the second inequality comes from $1 + r_N^* \geq 1$, and the third inequality comes from (2) and $\lambda < 1$. This violates equation (6). Hence equation (B.5) holds. Then equation (B.6) follows from equation (4), and equation (B.7) follows from (5).

Step 4: *If* $\psi^*(\varphi_1) = \varphi_1 < \varphi_2 = \psi^*(\varphi_2)$, *then*

$$R_2^*(\varphi_1) \quad > \quad R_2^*(\varphi_2) \tag{B.8}$$
$$C^*(\varphi_1) \quad < \quad C^*(\varphi_2) \tag{B.9}$$

Equation (B.6) implies that $R_2^*(\varphi_1) = R_1^* - \varphi_1 D > R_1^* - \varphi_2 D = R_2^*(\varphi_2)$. Similarly, step 3 implies that

$$\begin{aligned} C^*(\varphi_1) &= AK^* + R_1^* - \varphi_1 D - (1-\varphi_1)(1+r_N^*)D \\ &< AK^* + R_1^* - \varphi_2 D - (1-\varphi_2)(1+r_N^*)D \\ &= C^*(\varphi_2). \end{aligned}$$

Step 5: *Sudden stop policy satisfies*

$$\exists \varphi_S^* \in [0,1] \text{ s.t. } \begin{cases} \psi^*(\varphi) = \varphi & \forall \varphi \in [0, \varphi_S] \\ \psi^*(\varphi) = 1 & \forall \varphi \in (\varphi_S, 1] \end{cases}$$

First, note that $\psi^*(\varphi) \in \{\varphi, 1\}$, which follows from symmetry. Then, suppose, without loss of generality, that the optimal debt contract B^* has $\varphi_1^* < \varphi_2^* < \varphi_3^*$ such that

$$\psi^*(\varphi) = \begin{cases} 1 & \forall \varphi \in (\varphi_1, \varphi_2] \\ \varphi & \forall \varphi \in (\varphi_2, \varphi_3] \end{cases}$$

Then consider an alternative debt contract $\hat{B}$ that is identical to B^* except that $\hat{\psi}(\varphi) = \varphi \; \forall \varphi \in [\varphi_2 - \varepsilon, \varphi_2]$ for some $\varepsilon > 0$.

From equations (6) and (B.9), we know that $C^*(\varphi_2) > C^*(0) \geq 0$. By continuity, $\hat{C}(\varphi) > 0 \; \forall \varphi \in [\varphi_2 - \varepsilon, \varphi_2]$ for ε small enough.

In contrast, from step 2, $C^*(\varphi) = 0 \; \forall \varphi \in [\varphi_2 - \varepsilon, \varphi_2]$. Similarly, from equations (6) and (B.8), we know that $R_2^*(\varphi_2) > R_2^*(\varphi_3 - \varepsilon) \geq 0$. By continuity, $\hat{R}_2(\varphi) > 0 \; \forall \varphi \in [\varphi_2 - \varepsilon, \varphi_2]$ for ε small enough.

It remains to show that interim individual rationality, equation (7), holds for any lender with $\varphi^i = 0$ when $\forall \varphi \in [\varphi_2 - \varepsilon, \varphi_2]$. This is obvious since $\hat{P}_1(\varphi) = D < (1+r_N^*)D = \hat{P}_2(\varphi) \; \forall \varphi \in [\varphi_2 - \varepsilon, \varphi_2]$.

The participation constraint, equation (8), obviously holds since $r_N > 0 > r_S$.

Hence $\hat{B}$ is feasible, yet has strictly higher consumption than B^*, which is a contradiction.

Step 6: *Reserves and Liquidation policies satisfy*

$$\exists \varphi_R^* \in [0,1] \text{ s.t. } \begin{cases} R_2^*(\varphi) > 0 & \iff \varphi \in [0, \varphi_R) \\ L^*(\varphi) = 0 & \iff \varphi \in [0, \varphi_R] \end{cases}$$

From step 5, we know that

$$\exists \varphi_S^* \in [0,1] \text{ s.t. } \begin{cases} \psi^*(\varphi) = \varphi & \forall \varphi \in [0, \varphi_S] \\ \psi^*(\varphi) = 1 & \forall \varphi \in (\varphi_S, 1] \end{cases}$$

Let $\varphi_R^* = \varphi_S^*$. Then the result follows from steps 2 and 3. It also follows that $\varphi_R^* = R_1^*/D$.

Step 7: *The Optimal Reserves-to-Debt ratio satisfies*

$$\varphi_R^* = 1 - \left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}$$

760 The cutoff conditions imply that the state-contingent policy and payment functions can be written as:

$$\begin{aligned} L^*(\varphi) &= \begin{cases} 0 & \text{if } \varphi \leq \varphi_R^* \\ K^* & \text{otherwise} \end{cases} \\ R_2^*(\varphi) &= \begin{cases} R_1^* - \varphi D & \text{if } \varphi \leq \varphi_R^* \\ 0 & \text{otherwise} \end{cases} \\ \psi_i^*(\varphi, \varphi_i) &= \begin{cases} 0 & \text{if } \varphi \leq \varphi_R^* \text{ and } \varphi_i = 0 \\ 1 & \text{otherwise} \end{cases} \\ P_1^*(\varphi) &= \begin{cases} D & \text{if } \varphi \leq \varphi_R^* \\ R_1^* + \lambda K^* & \text{otherwise} \end{cases} \\ P_2^*(\varphi) &= \begin{cases} (1+r_N^*)D & \text{if } \varphi \leq \varphi_R^* \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

The participation constraint, holding with equality, can be written as

$$(1+r_W) = G(\varphi_R^*) + (1+r_N^*)\left(F(\varphi_R^*) - G(\varphi_R^*)\right) + \left(1 - F(\varphi_R^*)\right)(1+r_S^*) \tag{B.10}$$

where

$$G(x) \equiv \int_0^x \varphi dF(\varphi).$$

Substituting the resource constraints and the condition $\varphi_R = R_1/D$, the problem becomes:

$$\max_{\varphi_R} J(\varphi_R)$$

where

$$J(\varphi_R)\,D\int_0^{\varphi_R}\left[A(1-\varphi_R)+\varphi_R-\varphi+(1-\varphi)\,\frac{G(\varphi_R)+(1-F(\varphi_R))\,(\lambda+(1-\lambda)\varphi_R)-(1+r_W)}{F(\varphi_R)-G(\varphi_R)}\right]dF(\varphi).$$

The first order condition is given by $J'(\varphi_R)=0$:

$$[A-(1+r_N(\varphi_R^*))]\,(1-\varphi_R^*)f(\varphi_R^*)-(A-1)F(\varphi_R^*)-r_N'(\varphi_R^*)\,(F(\varphi_R^*)-G(\varphi_R^*))=0$$

Substituting (B.10) and its derivative, we get

$$(1-\varphi_R^*)\,f(\varphi_R^*)+1-F(\varphi_R^*)=\frac{A-1}{A-\lambda}$$

Using the bounded Pareto distribution, we get:

$$\varphi_R^*=1-\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}.$$

Step 8: *To verify the equilibrium is feasible, it suffices to show that*

$$C^*(\varphi)\geq 0\;\forall\varphi\in[0,\varphi_R^*).$$

Since $C^*(\varphi)$ is strictly increasing in φ, it suffices to show $C^*(0)\geq 0$.

$$\begin{aligned}
C^*(0) &= (A\,(1-\varphi_R^*)+\varphi_R^*)\,D+\frac{G(\varphi_R^*)+(1-F(\varphi_R^*))\,(\lambda+(1-\lambda)\varphi_R^*)-(1+r_W)}{F(\varphi_R^*)-G(\varphi_R^*)}D\\
&= (A-1)\,(1-\varphi_R^*)\,D-\frac{(1-\lambda)\,(1-\varphi_R^*)\,(1-F(\varphi_R^*))+r_W}{F(\varphi_R^*)-G(\varphi_R^*)}D\\
&= (A-1)\,(1-\varphi_R^*)\,D-(\sigma+1)\,\frac{(1-\lambda)\,(1-\varphi_R^*)\,(1-\varphi_R^*)^{\frac{1}{\sigma}}+r_W}{1-(1-\varphi_R^*)^{\frac{1}{\sigma}+1}}D\\
&= (A-1)\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}D-(\sigma+1)\,\frac{(1-\lambda)\left(\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right)^{\sigma+1}+r_W}{1-\left(\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right)^{\sigma+1}}D
\end{aligned}$$

Note that $\lim_{A\to\infty}C^*(0)=+\infty$. Hence $\exists A^*(\lambda,\sigma,r_W)$ such that $\forall A\geq A^*$, $C^*(0)\geq 0$.

Step 9: *Sufficiency condition*

Having derived the optimal reserves by solving $J'(\varphi_R)=0$, we now verify that $J''(\varphi_R)<0$. We do so using the following three lemmas.

Lemma 1.1: Monotonicity of Interest Rate in Reserves

If $1<(\sigma+1)(1-\lambda+r_W)$*, then* $r_N'(\varphi_R)<0$.

Proof. . See Online Appendix.

Lemma 1.2: Convexity of Interest Rates

If $1 < (\sigma+1)(1-\lambda+r_W)$*, then* $r_N''(\varphi_R) > 0$.

Proof. See Online Appendix.

Lemma 1.3: Sufficiency of FOC.

$$J''(\varphi_R) < 0$$

Proof. See Online Appendix.

This concludes the proof of proposition 1. □

B.2. Proof of Proposition 2

(i) From Proposition 1, we know that

$$\varphi_R^* = 1 - \left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}.$$

Then,

$$\begin{aligned} \frac{\partial \varphi_R^*}{\partial \sigma} &> 0 \\ &\Leftrightarrow \\ -\left\{\log\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right] + \frac{1}{\sigma+1}\right\}\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma} &> 0 \\ &\Leftrightarrow \\ \log\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right] + \frac{1}{\sigma+1} &< 0 \end{aligned}$$

Since $\lambda < 1 < A$, it suffices to show

$$h(\sigma) \equiv \log\left(\frac{\sigma}{\sigma+1}\right) + \frac{1}{\sigma+1} \leq 0$$

, which is true since $h(\sigma)$ is increasing in σ, $\lim_{\sigma \to +\infty} h(\sigma) = 0^+$, and $\lim_{\sigma \to 0^+} h(\sigma) = -\infty$, which implies that $h(\sigma) < 0$ for all $\sigma > 0$.

(ii) From Corollary 1, we know that

$$\Pr(\psi = 1) = 1 - F(\varphi_R^*).$$

Substituting for φ_R^*, we get

$$\Pr(\psi = 1) = \frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right),$$

which is increasing in σ. $\square$

B.3. Proof of Proposition 3

Reserves Shortfall. Before writing the planner's problem, it is useful to derive how many countries have to suffer a crisis for a given level of reserves shortfall. Suppose all countries coordinate to set $\varphi_R^C = (\bar{\varphi} - \varepsilon)$ reserves aside and invest $\bar{K} + \varepsilon D$ where $\bar{\varphi} = E[\varphi]$ and $\bar{\varphi}D + \bar{K} = D$. The interim shortfall is: εD.

Some countries will have to (fully) liquidate to pay $1 + r_S(\varepsilon) = \bar{\varphi} + \lambda \bar{K}/D - (1-\lambda)\varepsilon$ since their normal interim payments cannot be met. Let us denote $\ell(\varepsilon)$, the measure of countries that face a crisis. We have: $1 - \ell(\varepsilon) = F_\sigma(\widehat{\varphi}(\varepsilon))$ where:

$$\bar{\varphi} - \varepsilon = \int_0^{\widehat{\varphi}(\varepsilon)} \varphi dF_\sigma(\varphi) = G_\sigma(\widehat{\varphi}(\varepsilon)) \quad \Leftrightarrow \quad \widehat{\varphi}(\varepsilon) = G_\sigma^{-1}(\bar{\varphi} - \varepsilon)$$

So: $\ell(\varepsilon) = 1 - F_\sigma\left[G_\sigma^{-1}(\bar{\varphi} - \varepsilon)\right]$.

The reserves decision ε determines the probability $\ell(\varepsilon)$ that a country is in a sudden stop. The shortfall limits the interim insurance since the interim debt repayment of some countries, the ones with the largest shocks, cannot be met. We know:

- $\ell(0) = 0$ and $\ell(\bar{\varphi}) = 1$
- $\ell(\varepsilon)$ is strictly increasing in ε
- $G_\sigma(\varphi) = \frac{\sigma}{\sigma+1}\left[1 - \left(1 - \frac{1}{\sigma}\varphi\right)(1-\varphi)^{\frac{1}{\sigma}}\right] \quad \Rightarrow \quad \ell(\varepsilon) = \left[1 - G_\sigma^{-1}(\bar{\varphi} - \varepsilon)\right]^{\frac{1}{\sigma}}$

We now write the planner's problem as choice of reserves shortfall.

Planner's Problem. Noting that the interim decision has been solved above, the planner's problem is:

$$\max_\varepsilon \quad C$$
$$\text{subject to}$$

$$(\bar{\varphi}-\varepsilon)D+(\bar{K}+\varepsilon D)-D \leq 0 \tag{B.11}$$

$$C+\left[\int_0^{\widehat{\varphi}(\varepsilon)}(1-\varphi)\,dF_\sigma(\varphi)\right](1+r_N)D-A(1-\ell(\varepsilon))(\bar{K}+\varepsilon D) \leq 0 \tag{B.12}$$

$$\ell(\varepsilon)(1+r_S(\varepsilon))+\int_0^{\widehat{\varphi}(\varepsilon)}[\varphi+(1-\varphi)(1+r_N)]\,dF_\sigma(\varphi)-(1+r_W) \geq 0 \tag{B.13}$$

$$(1+r_S(\varepsilon))-[(\bar{\varphi}-\varepsilon)D+\lambda(\bar{K}+\varepsilon D)] \geq 0 \tag{B.14}$$

$$C,\ \varepsilon \geq 0 \tag{B.15}$$

Equations (B.11) - (B.15) represent initial resource constraint, final resource constraint, participation constraint, renegotiation proofness, and non-negativity constraint, which are analogous to
825 equations (2), (5), (8), (3) , and (6), respectively. This simplifies to:

$$\max_\varepsilon \quad C$$

subject to

$$C+[1-\ell(\varepsilon)-(\bar{\varphi}-\varepsilon)](1+r_N)D-A(1-\ell(\varepsilon))(\bar{K}+\varepsilon D) \leq 0 \tag{B.16}$$

$$\ell(\varepsilon)(1+r_S(\varepsilon))+\int_0^{\widehat{\varphi}(\varepsilon)}[\varphi+(1-\varphi)(1+r_N)]\,dF_\sigma(\varphi)-(1+r_W) \leq 0 \tag{B.17}$$

$$(1+r_S(\varepsilon))-[(\bar{\varphi}-\varepsilon)D+\lambda(\bar{K}+\varepsilon D)] \geq 0 \tag{B.18}$$

$$C,\ \varepsilon \geq 0 \tag{B.19}$$

Equation (B.17) can be written as:[40]

$$\ell(\varepsilon)(1+r_S(\varepsilon))+(\bar{\varphi}-\varepsilon)+(1+r_N)[1-\ell(\varepsilon)-(\bar{\varphi}-\varepsilon)] = (1+r_W) \tag{B.20}$$

Substituting (B.20) into (B.16) yields:

$$C+((1+r_W)-(\bar{\varphi}-\varepsilon)-\ell(\varepsilon)(1+r_S(\varepsilon)))D-A(1-\ell(\varepsilon))(\bar{K}+\varepsilon D) \leq 0$$

830 The planner's problem can then be written as:

$$\max_\varepsilon \quad A(1-\ell(\varepsilon))\left(\frac{\bar{K}}{D}+\varepsilon\right)-((1+r_W)-(\bar{\varphi}-\varepsilon)-\ell(\varepsilon)(1+r_S(\varepsilon)))$$

$$\Leftrightarrow$$

$$\max_\varepsilon \quad -\ell(\varepsilon)A\frac{\bar{K}}{D}+A(1-\ell(\varepsilon))\varepsilon-\varepsilon+\ell(\varepsilon)\left[\bar{\varphi}+\lambda\frac{\bar{K}}{D}-(1-\lambda)\varepsilon\right]$$

[40] This is assuming the shortfall is not too high. Otherwise, the consumption would be negative due to the high interest implied by the high reserves shortfall.

This is not a linear problem in ε since $\ell(\varepsilon)$ is not linear. However, we know that:

$$\begin{aligned}
\ell(\varepsilon) &= 1 - F_\sigma\left[G_\sigma^{-1}(\bar{\varphi}-\varepsilon)\right] \\
&\Downarrow \\
\ell'(\varepsilon) &= -F'_\sigma\left[G_\sigma^{-1}(\bar{\varphi}-\varepsilon)\right] \times \left(\frac{1}{G'_\sigma\left[G_\sigma^{-1}(\bar{\varphi}-\varepsilon)\right]}\right) \times (-1) \\
&= \frac{f\left[G_\sigma^{-1}(\bar{\varphi}-\varepsilon)\right]}{\left[G_\sigma^{-1}(\bar{\varphi}-\varepsilon)\right] f\left[G_\sigma^{-1}(\bar{\varphi}-\varepsilon)\right]} \quad \text{as } F' = f \text{ and } G'(\varphi) = \varphi f(\varphi) \\
&= \frac{1}{G_\sigma^{-1}(\bar{\varphi}-\varepsilon)} \\
&= \frac{1}{\widehat{\varphi}(\varepsilon)}
\end{aligned}$$

The F.O.C. w.r.t. ε gives:

$$-\ell'(\varepsilon) A \frac{\bar{K}}{D} - \ell'(\varepsilon) A\varepsilon + A(1-\ell(\varepsilon)) - 1 + \ell'(\varepsilon)\left[\bar{\varphi} + \lambda\frac{\bar{K}}{D} - (1-\lambda)\varepsilon\right] - (1-\lambda)\ell(\varepsilon) \geq 0$$

with equality if $\varepsilon > 0$. Rearranging yields:

$$\begin{aligned}
-\ell'(\varepsilon) A\left(\frac{\bar{K}}{D}+\varepsilon\right) + A(1-\ell(\varepsilon)) - (1-\lambda)\ell(\varepsilon) + \ell'(\varepsilon)(1+r_S(\varepsilon)) - 1 &\geq 0 \\
&\Leftrightarrow \\
(A-1) - (A+1-\lambda)\ell(\varepsilon) - \ell'(\varepsilon)\left[A\left(\frac{\bar{K}}{D}+\varepsilon\right) - (1+r_S(\varepsilon))\right] &\geq 0 \\
&\Leftrightarrow \\
(A+1-\lambda) F_\sigma(\widehat{\varphi}(\varepsilon)) - \frac{A\left(\frac{\bar{K}}{D}+\varepsilon\right) - \left(\bar{\varphi} + \lambda\frac{\bar{K}}{D} - (1-\lambda)\varepsilon\right)}{\widehat{\varphi}(\varepsilon)} - (2-\lambda) &\geq 0 \\
&\Leftrightarrow \\
(A+1-\lambda) F_\sigma(\widehat{\varphi}(\varepsilon)) - \frac{(A-\lambda) - (A+1-\lambda)(\bar{\varphi}-\varepsilon)}{\widehat{\varphi}(\varepsilon)} - (2-\lambda) &\geq 0
\end{aligned}$$

At $\varepsilon = 0$, the L.H.S. of the F.O.C. is:

$$\begin{aligned}
&(A+1-\lambda) - \left[(A-\lambda) - (A+1-\lambda)(\bar{\varphi})\right] - (2-\lambda) \\
&= (A+1-\lambda)\bar{\varphi} - (1-\lambda) \\
&> 0 \quad \text{iff } \sigma > \frac{1-\lambda}{A}
\end{aligned}$$

Therefore, $\varphi_R^C = \bar{\varphi}$ if and only if $\sigma \leq (1-\lambda)/A$. Otherwise, $\varphi_R^C < \bar{\varphi}$. Obviously, in any case: $\varphi_R^C \leq \bar{\varphi}$.

Finally, given that $\varphi_R^* = 1 - [(A-1)/(A-\lambda)\,\sigma/(\sigma+1)]^\sigma$ and $\bar{\varphi} = \frac{\sigma}{\sigma+1}$:

$$\varphi_R^* > \bar{\varphi} \Leftrightarrow \frac{1}{\sigma+1} > \left(\frac{A-1}{A-\lambda}\right)^\sigma \left(\frac{\sigma}{1+\sigma}\right)^\sigma$$

Since $\lambda < 1 < A$, it is sufficient to show that:

$$\frac{1}{\sigma+1} > \left(\frac{\sigma}{1+\sigma}\right)^\sigma,$$

which is true for $\sigma \in (0,1)$. □

B.4. Proof of Proposition 4

Given an arbitrary stage-0 allocation $\{\hat{R}_1, \hat{K}\}$ and normal time interest rate $\{\hat{r}_N\}$, we first characterize the thresholds for which the government is solvent. Then, after characterizing the allocations under normal times ($\psi < 1$) and under sudden stop ($\psi = 1$), we characterize the optimal sudden stop region. Finally, we characterize the implied interest rates. Without loss of generality, normalize $D = 1$.

Solvency Region. First, let us denote $d \in [0,1]$ the fraction of reserves $\hat{R}_1$ drawn in the interim. Similarly, let $\ell \in [0,1]$ denote the fraction of capital K liquidated in the interim. the non crisis resource constraints are:

$$\begin{array}{lccc} \text{[RC0]} & \hat{R}_1 & = & 1 + R_0 - \hat{K} \\ \text{[RC1]} & \varphi & = & d\hat{R}_1 + \lambda \ell \hat{K} \\ \text{[RC2]} & C + R' + (1-\varphi)(1+\hat{r}_N) & = & (1-d)R_1 + A(1-\ell)K \end{array}$$

with $d \in [0,1]$, $\ell \in [0,1]$, $K \in [0,1]$, $C \geq 0$, and $R' \geq 0$.

Substituting [RC1] into [RC2], any resource feasible liquidation $\ell \in [0,1]$ satisfies:

$$(A-\lambda)\ell\hat{K} \;\; \leq \;\; A\hat{K} + \hat{R}_1 - 1 - (1-\varphi)\hat{r}_N$$

Using [RC0], we obtain:

$$(A-\lambda)\ell\hat{K} \;\; \leq \;\; (A-1)\hat{K} + R_0 - (1-\varphi)\hat{r}_N$$

The level of liquidation which makes the government insolvent in period 2 is

$$\widehat{\ell}(\varphi) = \min\left\{\frac{(A-1)\hat{K} + R_0 - \hat{r}_N}{(A-\lambda)\hat{K}} + \frac{\hat{r}_N}{(A-\lambda)\hat{K}}\varphi, 1\right\}$$

Denote:

$$\varphi_{liq} \equiv \min_{[0,1]} \left\{ \varphi \text{ s.t. } \widehat{\ell}(\varphi) \geq 0 \right\} = \min \left\{ \max \left\{ -\frac{(A-1)K + R_0 - r_N}{r_N}, 0 \right\}, 1 \right\} \tag{B.21}$$

To further guarantee solvency in time 1, we need: $\varphi \leq \hat{R}_1 + \lambda \widehat{\ell}(\varphi)\hat{K}$. Denote:

$$[\varphi_{lo}, \varphi_{up}] \equiv \left\{ \varphi \in [0,1] \text{ s.t. } \varphi \leq \hat{R}_1 + \lambda \widehat{\ell}(\varphi)\hat{K} \right\} \tag{B.22}$$

The government is solvent over

$$[\varphi_{min}, \varphi_{max}] \equiv [\varphi_{liq}, 1] \cap [\varphi_{lo}, \varphi_{up}] \tag{B.23}$$

Denote $\varphi_R = \min\left\{1, \hat{R}_1\right\}$. Note that $\varphi_R \leq \varphi_{up}$ and $\varphi_R \leq \varphi_{max}$.

Normal Time Net Output. During normal times, when $\psi < 1$, net output is given by:

$$Y(\varphi) = A\hat{K} + \hat{R}_1 - 1 - \hat{r}_N(1-\varphi) - (A-\lambda)\ell\hat{K}$$

Since $Y(\varphi)$ is decreasing in ℓ, and thereby increasing in d, it is optimal to pay first with reserves. Hence, if $\varphi \leq \hat{R}_1$ then

$$(d, \ell) = \left(\frac{\varphi}{\hat{R}_1}, 0 \right) \tag{B.24}$$

$$Y(\varphi) = A\hat{K} + \hat{R}_1 - 1 - \hat{r}_N(1-\varphi) \tag{B.25}$$

Otherwise, if $\varphi > \hat{R}_1$ then

$$(d, \ell) = \left(1, \frac{\varphi - \hat{R}_1}{\lambda \hat{K}} \right) \tag{B.26}$$

$$Y(\varphi) = A\hat{K} + \hat{R}_1 - 1 - \hat{r}_N(1-\varphi) - (A-\lambda)\left(\frac{\varphi - \hat{R}_1}{\lambda} \right) \tag{B.27}$$

Overall, we have:

$$Y(\varphi) = A\hat{K} + \hat{R}_1 - 1 - \hat{r}_N(1-\varphi) - (A-\lambda)\left(\frac{\varphi - \hat{R}_1}{\lambda} \right)^+ \tag{B.28}$$

Sudden Stop Net Output. During a sudden stop, when $\psi = 1$, net output is

$$Y_{SS} = A\hat{K} + \hat{R}_1 - 1 - r_S - (A-\lambda)\ell\hat{K}$$

where

$$\text{[NP]} \quad 1 + r_S = \min\left\{1, \theta \frac{\hat{R}_1 + \lambda \hat{K}}{D}\right\}.$$

Since Y_{SS} is decreasing in ℓ, it is optimal to pay first with reserves. Hence, if $\hat{R}_1 \geq (1 + r_S)$ then

$$(d, \ell) = \left(\frac{1 + r_S}{\hat{R}_1}, 0\right) \tag{B.29}$$

$$Y_{SS} = A\hat{K} + \hat{R}_1 - 1 - r_S \tag{B.30}$$

Otherwise, if $\hat{R}_1 < (1 + r_S)$ then

$$(d, \ell) = \left(1, \frac{1 + r_S - \hat{R}_1}{\lambda \hat{K}}\right) \tag{B.31}$$

$$Y_{SS} = A\hat{K} + \hat{R}_1 - 1 - r_S - (A - \lambda)\left(\frac{1 + r_S - \hat{R}_1}{\lambda}\right) \tag{B.32}$$

Overall, we have:

$$Y_{SS} = A\hat{K} + \hat{R}_1 - 1 - r_S - (A - \lambda)\left(\frac{1 + r_S - \hat{R}_1}{\lambda}\right)^+ \tag{B.33}$$

Sudden Stop Region. A sudden stop is optimal when the net output in normal times is less than the sudden net output. That is:

$$\varphi \hat{r}_N - (A - \lambda)\left(\frac{\varphi - \hat{R}_1}{\lambda}\right)^+ \leq \hat{r}_N - r_S - (A - \lambda)\left(\frac{1 + r_S - \hat{R}_1}{\lambda}\right)^+$$

First, consider $\varphi \in [0, \hat{R}_1] \cap [\varphi_{min}, \varphi_{max}]$. Then $Y(\varphi) \leq Y_{SS}$ if and only if

$$\varphi \leq \frac{(\hat{r}_N - r_S) - \frac{A - \lambda}{\lambda}(1 + r_S - \hat{R}_1)^+}{\hat{r}_N}$$

Second, consider $\varphi \in [\hat{R}_1, 1] \cap [\varphi_{min}, \varphi_{max}]$. Then $Y(\varphi) \leq Y_{SS}$ if and only if

$$\varphi \geq \frac{\frac{A-\lambda}{\lambda}\hat{R}_1 + \frac{A-\lambda}{\lambda}(1 + r_S - \hat{R}_1)^+ - (\hat{r}_N - r_S)}{\frac{A-\lambda}{\lambda} - \hat{r}_N}$$

Denote:

$$\begin{aligned}
\underline{\varphi} &\equiv \min\left\{\hat{R}_1, \frac{(\hat{r}_N - r_S) - \frac{A-\lambda}{\lambda}(1+r_S-\hat{R}_1)^+}{\hat{r}_N}\right\} && \text{(B.34)}\\
\overline{\varphi} &\equiv \max\left\{\hat{R}_1, \frac{\frac{A-\lambda}{\lambda}\hat{R}_1 + \frac{A-\lambda}{\lambda}(1+r_S-\hat{R}_1)^+ - (\hat{r}_N - r_S)}{\frac{A-\lambda}{\lambda} - \hat{r}_N}\right\} && \text{(B.35)}\\
\varphi_N &\equiv \max\left\{\varphi_{min}, \underline{\varphi}\right\} && \text{(B.36)}\\
\varphi_S &\equiv \min\left\{\varphi_{max}, \overline{\varphi}\right\} && \text{(B.37)}
\end{aligned}$$

Note that $\overline{\varphi} \geq \varphi_R$ and $\varphi_{max} \geq \varphi_R$. Hence $\varphi_S \geq \varphi_R$. Therefore, by equation B.24, for any $\varphi \in (\varphi_N, \varphi_R)$, liquidation is zero.

Implied Interest Rate. Let $\mathcal{N} = [\varphi_N, \varphi_S]$. The normal time interest is then determined from

$$\text{[PC]} \quad (1+r_W) = G_\rho(\mathcal{N}) + (1+r_N)\left[F_\rho(\mathcal{N}) - G_\rho(\mathcal{N})\right] + (1+r_S)\left[1 - F_\rho(\mathcal{N})\right]$$

where $F_\rho(\varphi) = \rho f_{\sigma_L} + (1-\rho) f_{\sigma_H}$ and $G_\rho(\varphi) \equiv \int_0^\varphi x dF_\rho(x)$.

Since this characterization holds for any arbitrary stage-0 allocation, it also holds for the optimal stage-0 allocation. □

B.5. Proposition 5. Mutual Insurance with Correlated Shocks

Let $\gamma \in [0,1]$. Consider a measure of *ex ante* identical countries with randomly assigned identity $j \in [0,1]$ subject to liquidity shocks $\left\{\varphi^j\right\}_{j\in[0,1]}$. Correlation is modeled as follows:

$$\begin{cases} \varphi^j = \varphi(\gamma) & \forall j \in (0,\gamma) \quad \text{where } \varphi(\gamma) \sim F_\sigma \\ \varphi^j = \varphi(j) & \forall j \in (\gamma,1) \quad \text{where } \varphi(j) \sim F_\sigma \end{cases}$$

In other words, the liquidity shocks are perfectly correlated across the countries in $(0,\gamma)$ and i.i.d across the countries in $(\gamma,1)$. Hence, γ reflects the correlation of shocks across countries.
Under mutual insurance, the optimal reserves held by each country is

$$\gamma\varphi_R^* + (1-\gamma)\varphi_R^C \in \left[\varphi_R^*, \varphi_R^C\right]$$

Proof. Suppose that there are two liquidity insurance agreements setup in the interim, after the identity j is known. In particular, there is one insurance for $(0,\gamma)$ and another for $(\gamma,1)$.
From Proposition 1, we know that the optimal reserve for $(0,\gamma)$ is φ_R^* since the countries are identical *ex post* and cannot insure each other.
Similarly, from Proposition 3, the optimal reserve for $(\gamma,1)$ is φ_R^C.

Could the two pools cross-insure by distorting these reserve allocations? No. Moreover, *ex post*, no other insurance groups can be formed. Hence, *ex ante*, the optimal reserve allocation chosen is:

$$\gamma\varphi_R^* + (1-\gamma)\,\varphi_R^C \in \left[\varphi_R^*, \varphi_R^C\right].$$

This concludes the proof of proposition 5. □

B.6. Proposition 6. Conditional Distribution Function of Posterior Beliefs

$$\Pr\left[\rho' \leq x \mid \varphi^j,\, \rho\right] \;=\; \rho\left[1 - A_{N-1}\left(m(x,\varphi^j) \mid \sigma_L\right)\right] + (1-\rho)\left[1 - A_{N-1}\left(m(x,\varphi^j) \mid \sigma_H\right)\right]$$

where

$$m(x,\varphi^j) \;\equiv\; -\log(1-\varphi^j) + \left(\sigma_L^{-1} - \sigma_H^{-1}\right)\log\left[\left(x^{-1}-1\right) \times \left(\frac{\rho}{1-\rho}\right) \times \left(\frac{\sigma_L^{-1}}{\sigma_H^{-1}}\right)^N\right]$$

$$A_N(y \mid \sigma) \;\equiv\; \exp\left(\frac{y}{\sigma}\right)\sum_{n=1}^{N}(-1)^{n-1}\frac{1}{(n-1)!\,\sigma^{n-1}}y^{n-1}$$

Proof. See Online Appendix. □

Online Appendix

C. Computational Appendix

The government solves

$$W(R_0;\rho) = \max_{B\in\Gamma(R_0;\rho)} \mathbf{E}_{\vec{\varphi}|\rho}\left[C(R_0,\vec{\varphi};\rho) + \beta W\left(R_0'(R_0,\vec{\varphi};\rho);\rho'(\rho,\vec{\varphi})\right)\right]$$

This problem involves the N-dimensional vector $\vec{\varphi}$ as an interim contingency indexing each of the interim decisions $\{\psi, L, R_0'\}$. Therefore, the size of the vector of shocks $\vec{\varphi}$ substantially increases the computational burden of the problem. Below, we replace the interim state $\vec{\varphi}\in[0,1]^N$ with the sufficient representation $\{\varphi^j, \rho'(\rho,\vec{\varphi})\}\in[0,1]^2$.

$$\begin{aligned} W(R_0;\rho) &= \max_{K\in\widehat{\Gamma}(R_0;\rho)} \int \Bigg\{ \max_{R_0'\in[0,R_{max}(\varphi;\rho,R_0,K)]} \Bigg[u\Big(C\left(R_0',\varphi;\rho,R_0,K\right)\Big) \\ &\qquad + \beta \int W\left(R_0';\rho'\right) g(\rho';\varphi,\rho)\mathrm{d}\rho' \Bigg] \Bigg\} h^1_\rho(\varphi)\mathrm{d}\varphi \end{aligned}$$

In this representation, h^1_ρ is the perceived distribution of local shocks given a prior ρ. Hence, $h^1_\rho = \rho f_{\sigma_L} + (1-\rho) f_{\sigma_H}$. Most importantly, we derive an analytical expression for the conditional distribution g of posterior beliefs ρ' (see Proposition 6 in Appendix B.6).

C.1. Algorithm: Feasibility

We derive the feasible contract for each tuple of belief, incoming reserves and capital choice. The main step in deriving the feasible contracts is finding the interest rate satisfying the participation constraint, given the endogenous crisis regions. We derive the constraint set $\left\{(R_0,\rho,K) \mid K\in\widehat{\Gamma}(R_0;\rho)\right\}$ once and for all. A regular value function iteration is then computed.

For each ρ, R_0 and $K\in[0,1]$

1. Derive $R_1 = R_0 + 1 - K$ using [RC0]
2. Derive $\{r_N, r_S, \varphi_N, \varphi_S\}$ using Proposition 4, and r_S from [NPC]
3. Set feasibility indicator to 0 if no solution there is no solution to the participation constraint.
4. Store $R_1, r_N, r_S, \varphi_N, \varphi_S$

C.2. Algorithm Value Function Iteration

We can further rewrite the problem as:

$$W(R_0,\rho) = \max_{K\in\widehat{\Gamma}(R_0;\rho)} J(K,R_0,\rho)$$

where:

$$J\left(K, R_0, \rho\right) = \iint V\left(\varphi, \rho'; K, R_0, \rho\right) g(\rho'; \varphi, \rho) h^1_\rho(\varphi) \mathrm{d}\rho' \mathrm{d}\varphi$$

Given the constraint set, a simple value function iteration on a discrete state space is used to compute the optimal policies. Below is a sketch of the solution method. Guess a value function $W_{\text{old}}(R_0;\rho) \equiv W(R_0;\rho)$. Then, iterate on value function:

1. For each ρ, R_0, K
   (a) Skip if (R_o, ρ, K) is not in the constraint set.
   (b) Retrieve the policies $R_1, r_H, r_L, \underline{\varphi}, \overline{\varphi}$ derived from feasibility
      i. For each crisis state $\varphi \notin [\underline{\varphi}, \overline{\varphi}]$,
         - For each possible posterior $\rho' \in [0,1]$
           – Initialize crisis value function $V_{candidate}(R';\varphi,\rho') = 0$
           – Compute output Y_{SS} from (B.33)
           – For each candidate savings $R' \in [0, Y_{SS}]$, compute

$$V_{candidate}(R';\varphi,\rho') = u(Y_{SS} - R') + \beta W(R';\rho')$$

           – Store $V(\varphi,\rho') = \max_{R'} V_{candidate}(R';\varphi,\rho')$
      ii. For each non-crisis state $\varphi \in [\underline{\varphi}, \overline{\varphi}]$
         - For each possible posterior $\rho' \in [0,1]$
           – Initialize non crisis value function $V_{candidate}(R';\varphi,\rho') = 0$
           – Compute output $Y(\varphi)$ from (B.28)
           – For each candidate savings $R' \in [0, Y(\varphi)]$, compute

$$V_{candidate}(R';\varphi,\rho') = u(Y(\varphi) - R') + \beta W(R';\rho')$$

           – Store $V(\varphi,\rho') = \max_{R'} V_{candidate}(R';\varphi,\rho')$
      iii. Compute expected value

$$J(K) = \int_0^1 \int_0^1 V(\varphi,\rho') g(\rho';\varphi,\rho) h^1_\rho(\varphi) \mathrm{d}\rho' \mathrm{d}\varphi$$

   - Find optimal stage-0 allocation K by solving

$$W_{\text{new}}(R_0;\rho) = \max_K J(K)$$

$$K(R_0;\rho) = \arg\max_K J(K)$$

2. Update value function guess

$$W_{\text{old}} = \omega W_{\text{old}} + (1-\omega) W_{\text{new}}$$

3. Repeat (1) until convergence

$$||W_{\text{new}} - W_{\text{old}}|| < \varepsilon$$

We use 150 points for liquidity shocks, 5 points for incoming beliefs, 10 points for posterior beliefs, 40 points for incoming saved reserves, 60 points for capital, and 20 points for outgoing saved reserves on a state-contingent grid. We use linear interpolation when the optimal decision does not fall on a grid point. The results are robust to more grid points and alternative interpolation methods. The FORTRAN code used is available online.

C.3. Simulation and Calibration

For a given parameter configuration, we simulate the quarterly model a large number of times, each time for N_{EME} countries and $T = 1992Q1 \ldots 2006Q4$. Given the optimal policy functions and the updating rule for posterior beliefs, we simulate the model iterating on the rules to obtain the endogenous paths for reserves, beliefs, and sudden stops. In order to let countries fully learn about the initial regime, we first let each model simulation run many quarters leading up to $T = 1992Q1$. The calibration is accomplished by choosing the parameter configuration that minimizes the equally weighted mean-squared errors.

For each $\beta, \sigma_L, \sigma_H, \theta$ and for each $s = 1 : N_S$ simulations:

- Draw $N_{EME} * T_{END}$ random draws in $[0,1]$ such that $\sigma = \sigma_L$ for $t = 1 : 1996Q4$ and $\sigma = \sigma_H$ for $t = 1997Q1 : 2006Q4$
- Apply iteratively policy functions and belief updating rules
- Save average R_1 from $t = 1992Q1 : 1996Q4$ across time and countries
- Save average R_1 from $t = 2002Q1 : 2006Q4$ across time and countries
- Save average haircut $-r_S$ during sudden stops
- Save sudden stop frequency across $t = 1992Q1 : 1996Q4$ and $t = 2002Q1 : 2006Q4$

D. Additional Proofs

D.1. Lemma 1.1: Monotonicity of Interest Rate in Reserves

If $1 < (\sigma+1)(1-\lambda+r_W)$*, then* $r_N'(\varphi_R) < 0$.
Proof.

$$
\begin{aligned}
r_N'(\varphi_R) &= \frac{-f(\varphi_R)+f(\varphi_R)(\lambda+(1-\lambda)\varphi_R)-[1-F(\varphi_R)](1-\lambda)}{(F(\varphi_R)-G(\varphi_R))} \\
&\quad -\frac{1+r_W-F(\varphi_R)-[1-F(\varphi_R)](\lambda+(1-\lambda)\varphi_R)}{(F(\varphi_R)-G(\varphi_R))^2}(1-\varphi_R)f(\varphi_R)
\end{aligned}
$$

Using the Pareto Distribution properties, we have:

$$
\begin{aligned}
r_N'(\varphi_R) &= \frac{(1-\varphi_R)f(\varphi_R)}{(F(\varphi_R)-G(\varphi_R))^2}\Bigg\{[\lambda-(1-\lambda)\sigma](F(\varphi_R)-G(\varphi_R)) \\
&\qquad -[1-F(\varphi_R)](1-\lambda)(1-\varphi_R)-r_W\Bigg\}
\end{aligned}
$$

We also know that:

$$
\begin{aligned}
F(\varphi_R)-G(\varphi_R) &= \frac{1}{\sigma+1}-\frac{1}{\sigma+1}(1-\varphi_R)^{\frac{1}{\sigma}+1} \\
[1-F(\varphi_R)](1-\varphi_R) &= (1-\varphi_R)^{\frac{1}{\sigma}+1} \\
&= (\sigma+1)\left[-(F(\varphi_R)-G(\varphi_R))+\frac{1}{\sigma+1}\right]
\end{aligned}
$$

Therefore:

$$
r_N'(\varphi_R) = \frac{(1-\varphi_R)f(\varphi_R)}{(F(\varphi_R)-G(\varphi_R))^2}\left\{F(\varphi_R)-G(\varphi_R)-(1-\lambda)-r_W\right\}
$$

Finally,

$$
\begin{aligned}
r_N'(\varphi_R) &< 0 \\
&\Leftrightarrow \\
0 &< F(\varphi_R)-G(\varphi_R)-(1-\lambda)-r_W \\
&\Leftrightarrow \\
0 &< \frac{1}{\sigma+1}-\frac{1}{\sigma+1}(1-\varphi_R)^{\frac{1}{\sigma}+1}-(1-\lambda)-r_W,
\end{aligned}
$$

which holds if $1 < (\sigma+1)(1-\lambda+r_W)$. $\square$

D.2. Lemma 1.2: Convexity of Interest Rates

If $1 < (\sigma+1)(1-\lambda+r_W)$*, then* $r_N''(\varphi_R) > 0$.

Proof. From above lemma, we have:

$$r'_N(\varphi_R) = \frac{(1-\varphi_R)f(\varphi_R)}{F(\varphi_R)-G(\varphi_R)}\left[1-\frac{1-\lambda+r_W}{F(\varphi_R)-G(\varphi_R)}\right]$$

Then,

$$\begin{aligned}
r''_N(\varphi_R) &= \frac{-f(\varphi_R)+(1-\varphi_R)f'(\varphi_R)}{F(\varphi_R)-G(\varphi_R)}\left[1-\frac{1-\lambda+r_W}{F(\varphi_R)-G(\varphi_R)}\right] \\
&\quad -\frac{(1-\varphi_R)^2 f(\varphi_R)^2}{(F(\varphi_R)-G(\varphi_R))^2}\left[1-\frac{1-\lambda+r_W}{F(\varphi_R)-G(\varphi_R)}\right] \\
&\quad +\frac{(1-\varphi_R)f(\varphi_R)}{F(\varphi_R)-G(\varphi_R)}\frac{1-\lambda+r_W}{(F(\varphi_R)-G(\varphi_R))^2}(1-\varphi_R)f(\varphi_R) \\
&= \frac{-f(\varphi_R)+(1-\varphi_R)f'(\varphi_R)}{F(\varphi_R)-G(\varphi_R)}\left[1-\frac{1-\lambda+r_W}{F(\varphi_R)-G(\varphi_R)}\right] \\
&\quad -\frac{(1-\varphi_R)^2 f(\varphi_R)^2}{(F(\varphi_R)-G(\varphi_R))^2}\left[1-2\frac{1-\lambda+r_W}{F(\varphi_R)-G(\varphi_R)}\right]
\end{aligned}$$

Hence:

$$\begin{aligned}
r''_N(\varphi_R) &> 0 \\
&\Leftarrow \\
1 &< \frac{1-\lambda+r_W}{F(\varphi_R)-G(\varphi_R)}
\end{aligned}$$

which is the same condition needed for $r'_N(\varphi_R) < 0$. This lemma guarantees that the constraint set is convex. □

D.3. Lemma 1.3: Sufficiency of FOC

$$J''(\varphi_R) < 0$$

Proof. We know that

$$J'(\varphi_R) = [A-(1+r_N(\varphi_R))](1-\varphi_R)f(\varphi_R)-(A-1)F(\varphi_R)-r'_N(\varphi_R)(F(\varphi_R)-G(\varphi_R))$$

Then

$$
\begin{aligned}
J''(\varphi_R) &= -r_N'(\varphi_R)(1-\varphi_R)f(\varphi_R) \\
&\quad + [A-(1+r_N(\varphi_R))]\left[-f(\varphi_R)+(1-\varphi_R)f'(\varphi_R)\right] \\
&\quad -(A-1)f(\varphi_R) \\
&\quad -r_N''(\varphi_R)(\varphi_R)(F(\varphi_R)-G(\varphi_R)) \\
&\quad -r_N'(\varphi_R)(1-\varphi_R)f(\varphi_R) \\
\\
&= -\left[A-1+A-(1+r_N(\varphi_R))+2r_N'(\varphi_R)(1-\varphi_R)\right]f(\varphi_R) \\
&\quad + [A-(1+r_N(\varphi_R))](1-\varphi_R)f'(\varphi_R) \\
&\quad -r_N''(\varphi_R)(F(\varphi_R)-G(\varphi_R))
\end{aligned}
$$

It suffices to show

$$
2(A-1)-r_N(\varphi_R)+2r_N'(\varphi_R)(1-\varphi_R)>0
$$

$$
\Leftrightarrow
$$

$$
\begin{gathered}
2(A-1)(F(\varphi_R)-G(\varphi_R))^2 \\
-\{r_W+[1-F(\varphi_R)](1-\lambda)(1-\varphi_R)\}(F(\varphi_R)-G(\varphi_R)) \\
+2(1-\varphi_R)^2 f(\varphi_R)[F(\varphi_R)-G(\varphi_R)-(1-\lambda+r_W)]>0
\end{gathered}
$$

We know that

$$
\begin{aligned}
\lim_{A\to\infty}(1-\varphi_R^*) &= \lim_{A\to\infty}\left(\frac{A-1}{A-\lambda}\frac{\sigma}{\sigma+1}\right)^{\sigma} = \left(\frac{\sigma}{\sigma+1}\right)^{\sigma} \\
\lim_{A\to\infty} J''(\varphi_R^*)) &= -\infty
\end{aligned}
$$

Hence $\exists A^*(\lambda,\sigma,r_W)$ such that $\forall A \geq A^*$, $J''(\varphi_R)<0$. $\square$

D.4. Proof of Proposition 6

We know that by Bayes' rule:

$$\begin{aligned}\rho'(\rho,\vec{\varphi}) &= \left[1+\left(\frac{1-\rho}{\rho}\right)\times\frac{f^N_{\sigma_H}(\vec{\varphi})}{f^N_{\sigma_L}(\vec{\varphi})}\right]^{-1} \\ &= \left[1+\left(\frac{1-\rho}{\rho}\right)\times\left(\frac{\sigma_H^{-1}}{\sigma_L^{-1}}\right)^N\times\prod_{j=1}^N\left[1-\vec{\varphi}(j)\right]^{\sigma_H^{-1}-\sigma_L^{-1}}\right]^{-1}\end{aligned}$$

Let $\bar{x}(\rho)\equiv\left[1+\left(\frac{1-\rho}{\rho}\right)\times\left(\frac{\sigma_H^{-1}}{\sigma_L^{-1}}\right)^N\right]^{-1}$. Since $\prod_{j=1}^N\left[1-\vec{\varphi}(j)\right]\in[0,1]$, we know that $\rho'(\rho,\vec{\varphi})\in(0,\,\bar{x}(\rho))$.

Then, $\forall x\in(0,\bar{x}(\rho))$:

$$\begin{aligned}\Pr\left(\rho'\leq x\right) &= \Pr\left\{\left[1+\left(\frac{1-\rho}{\rho}\right)\times\left(\frac{\sigma_H^{-1}}{\sigma_L^{-1}}\right)^N\times\left(\prod_{j=1}^N\left[1-\vec{\varphi}(j)\right]\right)^{\sigma_H^{-1}-\sigma_L^{-1}}\right]^{-1}\leq x\right\} \\ &= \Pr\left\{\prod_{j=1}^N\left[1-\vec{\varphi}(j)\right]\leq\left[\left(x^{-1}-1\right)\times\left(\frac{\rho}{1-\rho}\right)\times\left(\frac{\sigma_L^{-1}}{\sigma_H^{-1}}\right)^N\right]^{\frac{1}{\sigma_H^{-1}-\sigma_L^{-1}}}\right\}\end{aligned}$$

Let us denote:

$$m(x,\rho)\equiv\frac{1}{\sigma_H^{-1}-\sigma_L^{-1}}\left[\log\left(x^{-1}-1\right)+\log\left(\frac{\rho}{1-\rho}\right)+N\log\left(\frac{\sigma_L^{-1}}{\sigma_H^{-1}}\right)\right]$$

Note that: $x\in(0,\bar{x})\Leftrightarrow m(x,\rho)\in(-\infty,0)\Leftrightarrow \exp m(x,\rho)\in(0,1)$.

We therefore derive:

$$\begin{aligned}\Pr\left(\rho'\leq x\right) &= \Pr\left[\prod_{j=1}^N\left[1-\vec{\varphi}(j)\right]\leq\exp m(x,\rho)\right] \\ &= \rho\Pr\left[\sum_{j=1}^N\log\left[1-\vec{\varphi}(j)\right]\leq m(x,\rho)\mid\sigma_L\right]+ \\ &\quad (1-\rho)\Pr\left[\sum_{j=1}^N\log\left[1-\vec{\varphi}(j)\right]\leq m(x,\rho)\mid\sigma_H\right]\end{aligned}$$

To characterize the distribution of the posterior, we establish the theorem below.

Theorem 6.1

$$\Pr\left[\sum_{j=1}^{N}\log\left[1-\vec{\varphi}(j)\right]\leq y\mid\sigma\right]=\frac{1}{\sigma}\times a_N(y)\times\exp\left(\frac{y}{\sigma}\right)\equiv A_N(y)$$

where:

$$\begin{aligned} a_1(y) &= \sigma \\ a_{n+1}(y) &= a_n(y)+\int_y^0\left[a_n'(z)+\frac{1}{\sigma}a_n(z)\right]\mathrm{d}z \end{aligned}$$

Proof by recursion. Let us denote: $Y_j\equiv\log\left(1-\varphi_j\right)$ where $\varphi_j\sim F_\sigma$. Define $Z_N\equiv\sum_{j=1}^{N}Y_j$. Let us derive by recursion, the c.d.f. A_N of Z_N. This will prove the first part of the theorem.

Case where $n=1$ First, we know that: $\Pr\left(1-\varphi\leq x\mid\sigma\right)=x^{\sigma^{-1}}$.
So: $\Pr\left(\log\left(1-\varphi\right)\leq y\mid\sigma\right)=\exp\left(\frac{y}{\sigma}\right)$.
Therefore, by observation, $a_1(y)=\sigma$ and $A_1(y)=\exp\left(\frac{y}{\sigma}\right)$.

Recursion Let us now prove that the property holds at $n+1$, assuming it holds for n. In other words, let us assume that:

$$\Pr\left(Z_n\leq y\mid\sigma\right)=\frac{1}{\sigma}\times a_n(y)\times\exp\left(\frac{y}{\sigma}\right)\equiv A_n(y)$$

We have $\forall\, y<0$:

$$\begin{aligned} \Pr\left(Z_{n+1}\leq y\mid\sigma\right) &= \Pr\left(\sum_{j=1}^{n+1}Y_j\leq y\mid\sigma\right) \\ &= \Pr\left(Z_n+Y_1\leq y\mid\sigma\right) \\ &= \Pr\left(Y_1\leq y-Z_n\mid\sigma\right) \\ &= \int_{-\infty}^{0}\Pr\left(Y_1\leq y-z_n\mid\sigma\right)\mathrm{d}A_n(z_n) \\ &= \int_{-\infty}^{y}\mathrm{d}A_n(z_n)+\int_y^0\Pr\left(Y_1\leq y-z_n\mid\sigma\right)\mathrm{d}A_n(z_n) \\ &= A_n(y)+\int_y^0\Pr\left(Y_1\leq y-z_n\mid\sigma\right)\mathrm{d}A_n(z_n) \end{aligned}$$

We know that:

$$
\begin{aligned}
A_n(z) &= \frac{1}{\sigma} \times a_n(z) \times \exp\left(\frac{z}{\sigma}\right) \\
&\Downarrow \\
\mathrm{d}A_n(z) &= \frac{1}{\sigma} \times \left[a_n'(z) + \frac{1}{\sigma} a_n(z)\right] \times \exp\left(\frac{z}{\sigma}\right)
\end{aligned}
$$

We can now plug this into the previous equation:

$$
\begin{aligned}
\Pr\left(Z_{n+1} \leq y \mid \sigma\right) &= A_n(y) + \int_y^0 \Pr\left(Y_1 \leq y - z_n \mid \sigma\right) \mathrm{d}A_n(z_n) \\
&= A_n(y) + \int_y^0 \exp\left(\frac{y - z_n}{\sigma}\right) \left[\frac{1}{\sigma}\left(a_n'(z_n) + \frac{1}{\sigma} a_n(z_n)\right) \exp\left(\frac{z_n}{\sigma}\right)\right] \mathrm{d}z_n \\
&= A_n(y) + \frac{1}{\sigma} \exp\left(\frac{y}{\sigma}\right) \int_y^0 \left(a_n'(z_n) + \frac{1}{\sigma} a_n(z_n)\right) \mathrm{d}z_n \\
&= \frac{1}{\sigma} a_n(y) \exp\left(\frac{y}{\sigma}\right) + \frac{1}{\sigma} \exp\left(\frac{y}{\sigma}\right) \int_y^0 \left(a_n'(z_n) + \frac{1}{\sigma} a_n(z_n)\right) \mathrm{d}z \\
&= \frac{1}{\sigma} \times \left[a_n(y) + \int_y^0 \left(a_n'(z) + \frac{1}{\sigma} a_n(z)\right) \mathrm{d}z_n\right] \times \exp\left(\frac{y}{\sigma}\right) \\
&= \frac{1}{\sigma} \times a_{n+1}(y) \times \exp\left(\frac{y}{\sigma}\right)
\end{aligned}
$$

Induction We can therefore conclude by induction that theorem 6.1 is true $\forall\, n$. □

The analytical expression of a_n. We now solve analytically for a_n using the recursive formulation proved above.
We know:

$$
\begin{aligned}
a_{n+1}(y) &= a_n(y) + \int_y^0 \left(a_n'(z) + \frac{1}{\sigma} a_n(z)\right) \mathrm{d}z_n \\
&= a_n(y) + a_n(0) - a_n(y) + \frac{1}{\sigma} \int_y^0 a_n(z) \mathrm{d}z \\
&= a_n(0) + \frac{1}{\sigma} \int_y^0 a_n(z) \mathrm{d}z
\end{aligned}
$$

Since $A_n(0) = 1\ \forall n$, we have: $a_n(0) = \sigma\ \forall n$. So: $a_{n+1}(y) = \sigma + \frac{1}{\sigma} \int_y^0 a_n(z) \mathrm{d}z$.

We have:

$$\begin{aligned}
a_1(y) &= \sigma \\
a_2(y) &= \sigma + \frac{1}{\sigma}\int_y^0 [\sigma]\,\mathrm{d}z = \sigma - y \\
a_3(y) &= \sigma + \frac{1}{\sigma}\int_y^0 [\sigma - y]\,\mathrm{d}z = a_2(y) + \frac{1}{2\sigma}y^2 \\
a_4(y) &= \sigma + \frac{1}{\sigma}\int_y^0 \left[\sigma - y + \frac{1}{2\sigma}y^2\right]\mathrm{d}z = a_3(y) - \frac{1}{3!\sigma^2}y^3
\end{aligned}$$

Lemma 6.2.

$$a_n(y) = a_{n-1}(y) + (-1)^{n-1}\frac{1}{(n-1)!\sigma^{n-2}}y^{n-1}$$

1025 *Proof by recursion.* Suppose:

$$a_n(y) = a_{n-1}(y) + (-1)^{n-1}\frac{1}{(n-1)!\sigma^{n-2}}y^{n-1}$$

Let us prove that this property holds at $n+1$. In other words, we want to show that:

$$a_{n+1}(y) = a_n(y) + (-1)^n\frac{1}{n!\sigma^{n-1}}y^n$$

We know that:

$$\begin{aligned}
a_{n+1}(y) &= \sigma + \frac{1}{\sigma}\int_y^0 a_n(z)\mathrm{d}z \\
&= \sigma + \frac{1}{\sigma}\int_y^0 \left[a_{n-1}(z) + (-1)^{n-1}\frac{1}{(n-1)!\sigma^{n-2}}z^{n-1}\right]\mathrm{d}z \\
&= \sigma + \frac{1}{\sigma}\int_y^0 [a_{n-1}(z)]\,\mathrm{d}z + \frac{1}{\sigma}\int_y^0 \left[(-1)^{n-1}\frac{1}{(n-1)!\sigma^{n-2}}z^{n-1}\right]\mathrm{d}z
\end{aligned}$$

So:

$$
\begin{aligned}
a_{n+1}(y) &= a_n(y) + \frac{1}{\sigma}\int_y^0 \left[(-1)^{n-1}\frac{1}{(n-1)!\sigma^{n-2}}z^{n-1}\right]\mathrm{d}z \\
&= a_n(y) + \frac{1}{\sigma}\left[(-1)^{n-1}\frac{1}{n}\frac{1}{(n-1)!\sigma^{n-2}}z^{n}\right]_y^0 \\
&= a_n(y) + (-1)^{n}\frac{1}{(n)!\sigma^{n-1}}y^{n}
\end{aligned}
$$

Induction We can therefore conclude by induction that the lemma is true $\forall\, n$. $\square$

1030 *Analytical Expression.* Using the recursive formula, we have:

$$
\begin{aligned}
a_{n+1}(y) &= a_n(y) + (-1)^{n}\frac{1}{n!\sigma^{n-1}}y^{n} \\
a_n(y) &= a_{n-1}(y) + (-1)^{n-1}\frac{1}{(n-1)!\sigma^{n-2}}y^{n-1} \\
\vdots &= \vdots \\
a_2(y) &= a_1(y) + (-1)^{1}\frac{1}{1!\sigma^{1-1}}y^{1} \\
a_1(y) &= 0 + (-1)^{0}\frac{1}{0!\sigma^{0-1}}y^{0}
\end{aligned}
$$

Therefore:

$$
a_{n+1}(y) = \sum_{k=0}^{n}(-1)^{k}\frac{1}{k!\sigma^{k-1}}y^{k}
$$

Unconditional Posterior Distribution

Finally, using the prior ρ, we obtain:

$$
\Pr\left(\rho' \leq x\right) = \rho A_N\left(m(x,\rho);\ \sigma_L\right) + (1-\rho)A_N\left(m(x,\rho)\ ;\ \sigma_H\right)
$$

Conditional Posterior Distribution

1035 We now characterize the conditional distribution of the posterior given a local shock. We know:

$$
\begin{aligned}
\rho'(\rho,\vec{\varphi}) &= \left[1+\left(\frac{1-\rho}{\rho}\right)\times\left(\frac{\sigma_H^{-1}}{\sigma_L^{-1}}\right)^{N}\times\left(\prod_{j=1}^{N}\left[1-\vec{\varphi}(j)\right]\right)^{\sigma_H^{-1}-\sigma_L^{-1}}\right]^{-1} \\
&= \left[1+\left(\frac{1-\rho}{\rho}\right)\times\left(\frac{\sigma_H^{-1}}{\sigma_L^{-1}}\right)^{N}\times\left(\prod_{k=1}^{N-1}\left[1-\vec{\varphi}(k)\right]\right)^{\sigma_H^{-1}-\sigma_L^{-1}}\left[1-\vec{\varphi}(j)\right]^{\sigma_H^{-1}-\sigma_L^{-1}}\right]^{-1}
\end{aligned}
$$

Let

$$\bar{x}(\rho,\varphi^j) \equiv \left[1+\left(1-\varphi^j\right)^{\sigma_H^{-1}-\sigma_L^{-1}}\left(\frac{1-\rho}{\rho}\right)\left(\frac{\sigma_H^{-1}}{\sigma_L^{-1}}\right)^N\right]^{-1}$$

Since $\prod_{k=1}^{N-1}\left[1-\vec{\varphi}(k)\right]\in[0,1]$, one can prove that $\rho'(\rho,\vec{\varphi})\in(0,\ \bar{x}(\rho,\vec{\varphi}(j)))$. To characterize the distribution over $(0,\bar{x}(\rho,\varphi^j))$, let us consider $x\in(0,\bar{x}(\rho,\varphi^j))$.

$$\begin{aligned}
\Pr\left(\rho'\leq x|\vec{\varphi}(j)=\varphi^j\right) &= \Pr\left\{\prod_{k=1}^{N}\left[1-\vec{\varphi}(k)\right]\leq\left[\left(x^{-1}-1\right)\left(\frac{\rho}{1-\rho}\right)\left(\frac{\sigma_L^{-1}}{\sigma_H^{-1}}\right)^N\right]^{\frac{1}{\sigma_H^{-1}-\sigma_L^{-1}}}\right\} \\
&= \Pr\left\{\prod_{k=1}^{N-1}\left[1-\vec{\varphi}(k)\right]\leq\frac{1}{1-\varphi^j}\left[\left(x^{-1}-1\right)\left(\frac{\rho}{1-\rho}\right)\left(\frac{\sigma_L^{-1}}{\sigma_H^{-1}}\right)^N\right]^{\frac{1}{\sigma_H^{-1}-\sigma_L^{-1}}}\right\}
\end{aligned}$$

Let:

$$\exp m(x,\varphi^j,\rho)\equiv\frac{1}{1-\varphi^j}\left[\left(x^{-1}-1\right)\times\left(\frac{\rho}{1-\rho}\right)\times\left(\frac{\sigma_L^{-1}}{\sigma_H^{-1}}\right)^N\right]^{\frac{1}{\sigma_H^{-1}-\sigma_L^{-1}}}\in[0,1]$$

Hence, we have:

$$\begin{aligned}
\Pr\left(\rho'\leq x|\vec{\varphi}(j)=\varphi^j\right) &= \Pr\left\{\prod_{k=1}^{N-1}\left[1-\vec{\varphi}(k)\right]\leq\exp m(x,\varphi^j,\rho)\right\} \\
&= \rho\Pr\left[\sum_{k=1}^{N-1}\log\left[1-\vec{\varphi}(k)\right]\leq m(x,\varphi^j,\rho)\mid\sigma_L\right]+ \\
&\quad (1-\rho)\Pr\left[\sum_{k=1}^{N-1}\log\left[1-\vec{\varphi}(k)\right]\leq m(x,\varphi^j,\rho)\mid\sigma_H\right] \\
&= \rho A_{N-1}\left(m(x,\varphi^j,\rho)\mid\sigma_L\right)+(1-\rho)A_{N-1}\left(m(x,\varphi^j,\rho)\mid\sigma_H\right)
\end{aligned}$$

The function $\mathscr{G}$ is the cumulative distribution function of the conditional posterior. This concludes the proof of Proposition 6. □

The probability density function is $g\equiv\mathscr{G}'$ and can be fully characterized.

Conditional Probability Density of Posterior Beliefs

By differentiating $\mathscr{G}$ with respect to x, we get:

$$g(x|\rho,\varphi^j) = \rho\left[A'_{N-1}\left(m(x,\varphi^j,\rho)|\ \sigma_L\right)m'(x)\right]+(1-\rho)\left[A'_{N-1}\left(m(x,\varphi^j,\rho)|\ \sigma_H\right)m'(x)\right]$$

To complete the analytical characterization, we also show that:

$$
\begin{aligned}
m(x,\varphi^j,\rho) &= -\log(1-\varphi^j) - \frac{1}{\sigma_L^{-1}-\sigma_H^{-1}} \log\left[\left(x^{-1}-1\right) \times \left(\frac{\rho}{1-\rho}\right) \times \left(\frac{\sigma_H}{\sigma_L}\right)^N \right] \\
m'(x,\varphi^j,\rho) &= -\frac{1}{\sigma_H^{-1}-\sigma_L^{-1}} \frac{1}{x(1-x)} \\
A_N(x;\sigma) &= \frac{1}{\sigma} a_N(x;\sigma) \exp\left(\frac{x}{\sigma}\right) \\
A'_N(x;\sigma) &= \frac{1}{\sigma} \left(a'_N(x;\sigma) + \frac{1}{\sigma} a_N(x;\sigma) \right) \exp\left(\frac{x}{\sigma}\right) \\
a_N(x;\sigma) &= \sum_{k=1}^{N} (-1)^{k-1} \frac{1}{(k-1)!\sigma^{k-2}} x^{k-1} \\
a'_N(x;\sigma) &= \sum_{k=1}^{N} (-1)^{k-1} \frac{1}{(k-2)!\sigma^{k-2}} x^{k-2} \ \text{ if } N \geq 2 \\
a'_1(x;\sigma) &= 0
\end{aligned}
$$