Sepsis, the tenth-leading cause of death in the United States, accounts for more than $16.7 billion in annual health care costs. A significant factor in these costs is hospital length of stay. The lack of standardized hospital discharge policies and an inadequate understanding of sepsis progression have resulted in unnecessarily long hospital lengths of stay. In this paper, a general model of when to discharge a patient with pneumonia-related sepsis from the hospital is presented. The model is parameterized using patient-based disease progression data from a large clinical study in order to characterize optimal discharge policies for various problem instances. In the presented experiments, patient health is represented by SOFA scores, which are commonly used to assess sepsis severity. Control-limit policies for specific patient cohorts defined by age and race are demonstrated.

Keywords: Medical decision making, patient management, Markov decision processes, sepsis, pneumonia

1. Introduction

Sepsis is a disease that results from an overwhelming inflammatory response to infection within the body. By over-producing or producing the wrong proportions of inflammatory and anti-inflammatory molecules (also known as cytokines), the body may negatively impact one or more of its organ systems, leading to organ dysfunction and possibly death (Riedemann et al., 2003). This serious condition, sometimes referred to as severe sepsis, septic shock or septicemia, is the tenth-leading cause of death in the United States (Hoyert et al., 2006). It develops in more than 750,000 people annually, of whom approximately 210,000 die (Angus et al., 2001).

Researchers continue to explore ways of reducing patient mortality through improving treatment efficacy at all stages of the disease, yet current therapy options are still mainly ad hoc (Hotchkiss and Karl, 2003) and highly dependent on the severity of the disease (Wheeler and Bernard, 1999).

Initially, antibiotics may be used to treat the underlying infection; however, factors such as polymicrobial infections and antimicrobial-resistant organisms make prompt diagnosis and treatment of infection difficult (Eggimann and Pittet, 2001). If the administered antibiotics are ineffective, or, even if effective, during the time required to treat the infection, organ support therapies such as fluid replacement, mechanical ventilation and blood transfusions, may be needed to prevent organ failure. Experimental medications aimed at controlling the body’s inflammatory response have had limited success (Bone, 1996; Zeni et al., 1997; Marshall, 2000; Burchardi and Schneider, 2004). In an attempt to learn more about the disease, research efforts such as the Genetic and Inflammatory Markers for Sepsis (GenIMS) trial led by the University of Pittsburgh School of Medicine (Anon, 2006) are trying to understand the body’s inflammatory response at a molecular level.

This paper utilizes patient-specific information obtained through the GenIMS trial to model the doctor’s decision-making process for the treatment of patients who are suspected of having sepsis. In particular, this work focuses on developing standard decision-making policies that can
inform clinical guidelines for patient treatment. For example, the last decision a doctor makes during sepsis treatment is when to discharge the patient from the hospital. In the GenIMS trial, a number of subjects discharged from the hospital died in subsequent weeks, with over 50% of the 90-day mortality occurring post-discharge (Anon, 2006). It is clear that robust discharge policies are essential, yet evidence has shown that the discharge decision is rarely made using patient-based standards (Halm et al., 1998).

With the annual costs associated with severe sepsis exceeding $16.7 billion in the United States (Angus et al., 2001), it is highly desirable to avoid unnecessary days in the hospital. Yet, despite attempts to decrease costs by reducing hospital length of stay (McCormick et al., 1999; Fine et al., 1997), concerns about the morbidity and mortality associated with premature hospital discharge have resulted in substantial differences in length of stay between and within institutions (Cleary et al., 1991; Fine et al., 1993). These differences suggest that decisions are made in an ad hoc fashion, often due to physician intuition and clinical uncertainty (Burns and Wholey, 1991; McCormick et al., 1999). Recent studies have focused on standardizing discharge procedures to reduce cost without increasing the risk of patient morbidity and mortality. In an attempt to develop patient-based discharge policies, these studies have explored modeling techniques that consider the cost-benefit tradeoff underlying the discharge decision. For example, Clermont et al. (2004) developed a dynamic microsimulation model to predict various outcomes for critically ill patients, including day of discharge from the Intensive Care Unit (ICU). While this model can be used as a predictive tool, it does not provide patient-specific optimal discharge policies. Similarly, Halm et al. (1998) used statistical modeling to measure the time to clinical stability inpatients with community-acquired pneumonia. The authors discuss how their results can be used to improve the efficiency of inpatient management by providing an evidence-based estimate for optimal length of stay. These estimates, however, cannot be easily translated into health-based discharge policies.

This paper further extends these and other previous modeling efforts by presenting a model and analysis of the hospital discharge decision using a Markov Decision Process (MDP) approach (Puterman, 1994; Bertsekas, 2001). Historically, MDPs have been applied in areas such as inventory control (D'Epenoux, 1963; Iglehart, 1963) and production planning (Bitran and Tirupati, 1993), but have recently seen increased application in medicine (Schaefer, 2004), including organ transplantation (Alagöz et al., 2004, 2007) and HIV therapy planning (Shechter et al., 2007). Within the limits of its assumptions, the MDP provides optimal decision policies. In addition, MDPs can more efficiently evaluate a larger number of policy alternatives than other modeling techniques used in sepsis research to date, such as statistics (Kasal et al., 2004), Markov modeling (Rangel-Frausto et al., 1995), and simulation (Clermont et al., 2003, 2007). Unfortunately, MDPs suffer from the “curse of dimensionality” in that the data requirements grow exponentially with the size of the state space.

A key contribution of this paper is to characterize discharge policies based on actual clinical data. To this end, we utilize a finite-horizon MDP framework to capture the time-varying nature of the progression of sepsis, while assuming that the state is completely observable at every stage. As will be discussed in the paper, this assumption is not unreasonable as the state description is actually a score that implicitly considers multiple aspects of the patient’s health as it varies stochastically over time. In addition, using the MDP framework with a single variable reduces the amount of data required to generate statistically valid transition probabilities based on the available patient data.

The characterization of the hospital discharge policies takes a two-pronged approach. The study begins by investigating the mathematical structure of the problem formulation and then considers a range of problem instances developed using patient-specific data. It will be shown that the optimal policies for several instances are similar to control-limit-type policies. A non-stationary control limit policy takes the following form: for every stage there exists a control limit health state at or below which it is optimal to discharge the patient from the hospital. Patients in all other health states should remain in the hospital. These types of policies are appealing since they are easy to understand and can be implemented as part of a general discharge strategy.

The remainder of this paper is organized as follows: Section 2 presents the model formulation; monotonicity results for the value function and conditions for the existence of a control limit policy are discussed in Section 3; data sources, problem instances and results for various computational experiments are presented and discussed in Section 4; finally, Section 5 concludes the paper with directions for future research.

2. Model formulation

This model considers the clinician’s decision problem of when to discharge an individual patient from the hospital in order to maximize that patient’s $T$-day expected survival as measured from hospital admission, where common values of $T$ include 30, 60 and 90 days (Quartin et al., 1997; Weycker et al., 2003; Davies et al., 2005). The problem is formulated as a finite-horizon MDP to capture time dependencies among state transitions and rewards. We assume a single measure of patient health characterizes the health state and standard methods of care guide patient treatment throughout the patient’s hospital stay. At each decision point, the clinician can choose either to continue treating the patient in the hospital with standard care or to discharge the patient from the hospital. It is assumed that decisions are made at the end of each time period.

We use the following notation:

$N = \{1, 2, \ldots, N\}$: discrete stages at which a decision is made by the clinician. If a patient has not died and has
not been discharged by stage \( N \), we assume the patient is discharged at stage \( N \). The finite-horizon model captures time dependencies among state transitions, where the value of \( N \) depends on the input data for computational experiments. This paper defines a stage as one day; however, as the data for solving such a model become available, the model is flexible enough to consider smaller time intervals (hours, for example).

\( T \): the observation horizon, as measured from admission to the hospital, in which a patient’s death is attributable to sepsis. 

\( h \): the patient’s health state vector. Let \( \mathcal{H} \) be the set of all possible realizations of \( h \) in order of decreasing health. The ordered elements of \( \mathcal{H} \) are represented as \( 1, 2, \ldots, H, H + 1 \), where \( H + 1 \) represents the patient being “Deceased” and is an absorbing state.

\( a_t(h) \): the action taken at time \( t \) and state \( h \). The possible actions are to continue treating the patient in the hospital (\( C \)) and to discharge the patient from the hospital (\( D \)).

\( p_t(j \mid h, a) \): the probability that the patient’s health state is \( j \) in stage \( t + 1 \) given that the patient’s health state is \( h \) in stage \( t \) and action \( a \) is chosen. Note that the process will terminate with reward \( r_t(H + 1, C) = 0 \) if a patient transitions to the “Deceased” state or with reward \( r_t(h, D) \) if action \( D \) is chosen. If a patient transitions to the “Deceased” state, it is assumed that patient death occurs at the beginning of stage \( t + 1 \).

\( r_t(h, D) \): the expected \((T - t)\)-day survival of a patient who is discharged from the hospital on day \( t \) in health state \( h \).

\( r_t(h, C) \): the expected reward received for deciding at stage \( t \) to keep a patient in health state \( h \) in the hospital for one more stage. This model uses an expected reward of one day for all stages and states. Note that this one-day reward is still received by a patient who transitions to the “Deceased” state in stage \( t + 1 \).

\( r_N(h) \): the expected \((T - N)\)-day survival of a patient who is discharged from the hospital at stage \( N \) in health state \( h \).

\( V_t(h) \): the value function representing the expected number of days survived from time \( t \) to the end of the observation horizon \( T \).

\( A_t^*(h) \): the set of optimal actions at stage \( t \) when the system is in state \( h \), where \( a_t^*(h) \in A_t^*(h) \) is an action that maximizes the value function \( V_t(h) \).

This problem can be formulated as an MDP with optimality equations given by

\[
V_t(h) = \max \left\{ r_t(h, D), \ r_t(h, C) + \sum_{j=1}^{H+1} p_t(j \mid h, C) V_{t+1}(j) \right\}
\]

for \( h = 1, \ldots, H \) and \( t = 1, \ldots, N - 1 \),

\[
V_N(h) = r_N(h) \quad \text{for} \quad h = 1, \ldots, H,
\]

\[
V_t(H + 1) = 0 \quad \text{for} \quad t = 1, \ldots, N.
\]

The next section discusses the structure of the optimal value function.

3. Structural properties

The mathematical framework of the MDP model allows us to study the structure of the optimal value function and policy, providing insight into results that can be expected in practice. For example, this section demonstrates the monotonicity of the value function obtained by solving Equations (1)–(3). It will be demonstrated that as a patient becomes sicker, the patient’s \( T \)-day expected survival does not increase. First, the following definition is provided.

**Definition 1.** (After Barlow and Proschan, 1965.) The \( N \times N \) transition probability matrix \( P(t) \), with entries \( [P(t)]_{ij} \), is said to be IFR (Increasing Failure Rate) if the rows of \( P(t) \) are in increasing stochastic order, that is, \( \sum_{j=1}^N [P(t)]_{ij} \) is monotonically increasing in \( h \) for \( k = 1, \ldots, N \).

The following assumptions are later verified in Section 4.4 for each of the problem instances presented in Section 4.3.

**Assumption 1.** The patient health transition probability matrix \( P(t) \), with entries \( [P(t)]_{ij} = p(t | h, C) \), is IFR for all \( t \in \mathcal{N} \).

Assumption 1 implies that for two patients in health states \( h \) and \( h + 1 \), respectively, the patient in health state \( h + 1 \) is more likely to transition to a health state worse than \( h \) in the next stage. In other words, sicker patients are more likely to progress to being even sicker than are healthier patients.

**Assumption 2.** The reward function \( r_t(h, D) \) is non-negative and monotone decreasing in \( h \) for all \( t \in \mathcal{N} \). It follows that the reward function \( r_N(h) \) is also non-negative and monotone decreasing in \( h \) since a patient who has not died or been discharged by stage \( N \) must be discharged at stage \( N \).

Assumption 2 says that sicker patients have worse expected survival after discharge than healthier patients.

**Assumption 3.** The reward function \( r_t(h, C) \) is monotone decreasing in \( h \) for all \( t \in \mathcal{N} \).

Assumptions 2 and 3 imply that as a patient’s health degrades, the value of remaining in the hospital for one additional day and the patient’s expected \((T - t)\)-day survival after discharge on day \( t \) do not increase.

Under these assumptions, it can be shown that the optimal value function, \( V_t^*(h) \), is non-increasing in \( h \).

**Theorem 1.** Under assumptions 1, 2 and 3 for \( h = 1, \ldots, H \),

\[
V_t^*(h) \geq V_t^*(h + 1) \quad \text{for all} \quad t \in \mathcal{N}.
\]

This result demonstrates the intuitive conclusion that as a patient’s health declines, the patient’s expected \( T \)-day survival does not improve. Kreke (2007) discusses several sufficient conditions for the existence of a control limit policy. Unfortunately, these sets of conditions appear to be highly
restrictive and are not fully satisfied by the GenIMS dataset. Kreke (2007) contains a discussion concerning standard control limit proof techniques (Puterman, 1994) and their inapplicability within the context of this problem.

Section 4 explores the existence of control limit policies for this model through various computational experiments.

4. Computational experiments

The main purpose of this paper is to explore the structure of hospital decision policies for patients with sepsis so that we can suggest general strategies for patient discharge. Due to the complexity of the disease and the availability of data at this time, we have chosen to define the model state space by a single parameter, the total Sepsis-related Organ Failure Assessment (SOFA) score. As will be discussed in Section 4.2, this score is calculated based on the complex interactions between multiple aspects of the patient’s health, all of which were captured as part of the GenIMS trial. The model was solved using the standard backward induction algorithm presented in Puterman (1994).

4.1. Data sources

The GenIMS trial data contains static and dynamic variables for 2320 patients who were identified as potentially having Community-Acquired Pneumonia (CAP). Of these patients, 2032 were admitted to the hospital and went on to develop varying degrees of sepsis. The computational experiments presented in this section utilize a sample of 2025 patients, with seven patients being excluded from the GenIMS inpatient cohort due to missing or irregular data. Static variables such as age and race are provided for each patient. Dynamic health variables are available on a daily basis, where missing data were estimated utilizing a clinically derived algorithm that combines last observation carried forward and other clinically based interpolation methods, as agreed upon by the GenIMS investigator team (Anon, 2006).

4.2. SOFA

The patient’s health state is represented by the total SOFA score, an integer value ranging between zero and 24, where 24 corresponds to the sickest health state. The score was developed by the Working Group on Sepsis-related Problems of the European Society of Intensive Care Medicine to describe quantitatively the degree of organ dysfunction/failure over time (Vincent et al., 1996). The correlation between organ dysfunction and mortality makes the SOFA score an ideal descriptor of patient health in a model of severe sepsis and its use is supported by previous models of severe sepsis in the literature that have used the SOFA score to describe patient health (Ferriera, 2001; Clermont et al., 2004).

The total SOFA is calculated based on six component scores that evaluate different organ systems (respiratory, coagulation, liver, central nervous system, renal and cardiac). Therefore, even though the total SOFA is a single value, the score actually captures a wide range of patient health variables. The daily component SOFA scores and the resulting total SOFA scores were calculated by the GenIMS investigators for all patients in the GenIMS cohort. By capturing the time-varying nature of each patient’s SOFA scores in the transition probabilities used as input to the model, we are able to model the evolution of patient health through all stages of the disease and the patient’s hospital stay.

Due to data sparseness, the 25 total SOFA score values are aggregated into four patient health states \{0,1\}, \{2,3\}, \{4,5,6,7\}, \{8, ..., 24\}, and defined as aggregated health states 1, 2, 3 and 4, respectively. This aggregation was chosen to capture changes in the SOFA score for those levels at which doctors would consider the discharge decision. Scores of eight or greater indicate a disease severity that would make the discharge decision improbable.

4.3. Problem instances

Based on conventions in the literature (Halm et al., 1998; Clermont et al., 2002, 2003, 2004; Kasal et al., 2004), we use the values $N = 30$ days and $T = 90$ days. Since age and race have been determined to be significant predictors of patient mortality (Kasal et al., 2004; Johnston, 2005), these static variables are used to define the 11 problem instances described in Table 1. The age breaks (45, 65) are used to describe the instances following the conventions in Clermont et al. (2004). Note that the instances are further stratified by race (Caucasian, non-Caucasian). Due to the small sample sizes associated with non-caucasian patients under 65 years of age, not all combinations of age and race groups could be tested with the available data.

The far right column of Table 1 describes a third component of the problem instances. Due to data sparsity,

<table>
<thead>
<tr>
<th>Instance</th>
<th>Sample size</th>
<th>Age</th>
<th>Race</th>
<th>Stationary during periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>468</td>
<td>&lt;65</td>
<td>Caucasian</td>
<td>1–3, 4–7, 8–29</td>
</tr>
<tr>
<td>2</td>
<td>1158</td>
<td>≥65</td>
<td>Caucasian</td>
<td>1–3, 4–7, 8–29</td>
</tr>
<tr>
<td>3</td>
<td>273</td>
<td>&lt;65</td>
<td>non-Caucasian</td>
<td>1–3, 4–7, 8–29</td>
</tr>
<tr>
<td>4</td>
<td>126</td>
<td>≥65</td>
<td>non-Caucasian</td>
<td>1–3, 4–7, 8–29</td>
</tr>
<tr>
<td>5</td>
<td>242</td>
<td>&lt;45</td>
<td>all</td>
<td>1, 2–9, 10–29</td>
</tr>
<tr>
<td>6</td>
<td>499</td>
<td>[45, 65]</td>
<td>all</td>
<td>1, 2–9, 10–29</td>
</tr>
<tr>
<td>7</td>
<td>1284</td>
<td>≥65</td>
<td>all</td>
<td>1, 2–9, 10–29</td>
</tr>
<tr>
<td>8</td>
<td>242</td>
<td>&lt;45</td>
<td>all</td>
<td>1–3, 4–7, 8–29</td>
</tr>
<tr>
<td>9</td>
<td>499</td>
<td>[45, 65]</td>
<td>all</td>
<td>1–3, 4–7, 8–29</td>
</tr>
<tr>
<td>10</td>
<td>741</td>
<td>&lt;65</td>
<td>all</td>
<td>1–3, 4–7, 8–29</td>
</tr>
<tr>
<td>11</td>
<td>1284</td>
<td>≥65</td>
<td>all</td>
<td>1–3, 4–7, 8–29</td>
</tr>
</tbody>
</table>
the transition probabilities and rewards are assumed to be piece-wise constant over specific stages as defined in the far right column of Table 1, but are allowed to be time-varying between the grouped stages. For example, problem instance 2 considers caucasian patients that are age 65 or older. By assumption, the transition probabilities and rewards for this problem instance are stationary during stages 1 through 3, stages 4 through 7, and stages 8 through 29, but can be time-varying between stages 3 and 4 and between stages 7 and 8. Note that there are two different groupings used to define different problem instances, (1–3, 4–7, 8–29) and (1, 2–9, 10–29). The former is based on the clinical expertise of the co-authors. The latter definition is similar to that used by Clermont et al. (2004).

These instances provide valuable insights into the effect of hospital length of stay on the hospital discharge decision for patients of varying age and race. These results are described in more detail in the next section.

4.4. Results

Table 2 presents the optimal policy for problem instance 2 including the optimal value function value and the optimal action for each stage and state. The optimal value function value, \( V^*_t(h) \), represents the \((90 - t)\)-day expected survival of a patient in state \( h \) at stage \( t \) given that the clinician chooses the optimal action in the current stage and in all stages moving forward. For example, for a patient in aggregated health state 2 on day 5, the optimal action is to Continue with an expected 85-day survival of 71.9 days given that the clinician chooses to keep the patient in the hospital and then act optimally in all future stages.

The optimal action to take at each stage and for each state is interpreted for problem instance 2 as follows. During days 1, 2 and 3, it is optimal to discharge patients in aggregated health states 1 and 2 (corresponding to a SOFA score of zero, one, two or three). For patients in all other aggregated health states (corresponding to a SOFA score of five or greater), it is optimal to keep the patient in the hospital for an additional day. During days 4, 5 and 6, it is optimal to discharge patients in the healthiest aggregated state only (corresponding to a SOFA score of zero or one) and to keep all other patients (corresponding to a SOFA score of two or greater) in the hospital for an additional day. Finally, during days 7 through 29, it is optimal to discharge all but the sickest patients and keep the remaining patients in the hospital for one more day (corresponding to SOFA scores of zero through seven and then eight through 24, respectively).

The optimal policy is control limit and not monotone over time. For example, a patient who has not been discharged by day 3 and who is in aggregated state 2 in day 4 would not be discharged under this policy. This means that the patient was in an aggregated health state of three or greater in all days prior to day 4 (or the patient would have been discharged previously). This policy is intuitive, because patients who are sicker may need to remain in the hospital for a longer period of time, even though they appear to improve over time. Comparing the same state across time periods, however, is difficult since the time component implicitly captures information regarding the positive (and potentially negative) probabilistic changes in health state based on length of stay as observed in the dataset.

For days 7 through 29, only patients in the sickest health state should be kept in the hospital, corresponding to similar results presented in the literature. For example, Halm et al. (1998) found that the median time to overall clinical stability in patients with CAP was between 3 days for the most lenient definition of clinical stability and 7 days for the most conservative definition. Studies looking at ICU length of stay for sepsis patients found the median length of stay to be between 7 and 14 days (Clermont et al., 2004; Roman-Marchant et al., 2004). Given that the current trend in research is to find ways to reduce excessively long ICU and hospital stays, the results found through this analysis are quite promising. Therefore, while this type of policy does not hold exactly for all stages in all problem instances, it does suggest an easy-to-implement decision-making strategy.

Table 3 provides a summary of the optimal action by stage and state for every problem instance. Control limit policies are indicated in bold text. For example, control limit policies exist for every stage and state for problem instance 6 and 9 in addition to problem instance 2. The optimal policies for most of the other problem instances are of control limit type for the majority of states and stages. For example, problem instance 3 has a control limit policy for all days except day 29 and problem instance 11 follows this type of
Kreke et al.

Table 3. States for which specified action (row) is optimal at indicated day (column) for problem instances 1 through 11

<table>
<thead>
<tr>
<th>Instance and action</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
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<tr>
<td>1</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
</tr>
<tr>
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Table 4. Verification of assumptions from Section 2

<table>
<thead>
<tr>
<th>Instance</th>
<th>Assumption 1 satisfied?</th>
<th>Assumption 2 satisfied?</th>
<th>Assumption 3 satisfied?</th>
<th>Control limit policy?</th>
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<td>11</td>
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<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

5. Conclusions and directions for future research

Through mathematical analysis and computational experiments, this study found that optimal hospital discharge strategies for patients with pneumonia-related sepsis tend to follow a non-stationary control-limit-type policy structure for specific problem instances. What this means is that the level of illness at which it is optimal to discharge a patient appears to change over the course of a hospital stay. These types of policies have an obvious advantage in that they are easy to understand and can be used to standardize an otherwise complicated and ad hoc procedure. Introducing the medical community to this type of policy structure is the first step in standardizing the hospital discharge decision.
There are limitations to describing patient health by a single dimension, such as total SOFA score. Future work will explore more complex state descriptions, such as those that include the component SOFA scores, which are necessary before such models can inform clinical practice. As more data become available, the model presented in this paper can be used to provide increasingly accurate values for the health-based non-stationary control limits. For example, the model is flexible enough to consider quality-adjusted survival in place of expected survival in the reward function.

Clearly, additional data would help to resolve any issues with data sparseness and would allow for the testing of additional cohort stratifications. For example, in addition to age and race, gender has also been cited as a key predictor of patient mortality among patients with severe sepsis. Additional data would allow for the testing of age, race and gender cohorts.

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Appendix

Proof of Theorem 1. The proof is by induction. From assumption 2 it follows that \( V^*_n(h) \geq V^*_n(h + 1) \) since \( V^*_n(h) = r_N(h) \) for all \( h \in \mathcal{H} \). Now suppose that \( V^*_n(h) \geq V^*_n(h + 1) \) for \( h = 1, \ldots, H \) and for \( n = 1 + 1, \ldots, \tilde{N} - 1 \). It remains to show that \( V^*_n(h) \geq V^*_n(h + 1) \) for \( h = 1, \ldots, H \). Note that

\[
V^*_n(h) = \max \left\{ r_i(h, D), r_i(h, C) + \sum_{j=1}^{H+1} p_i(j | h, C)V^*_{i+1}(j) \right\},
\]

(A1)

If \( V^*_n(1) = r_i(1, h, 1, D) \), then by definition of \( V^*_n(h) \) and assumption 2, \( V^*_n(h + 1) \geq r_i(h, 1, D) \) and the result follows. Otherwise,

\[
V^*_n(h) = V^*_n(h + 1) + \sum_{j=1}^{H+1} p_i(j | h, C)V^*_{i+1}(j) - \sum_{j=1}^{H+1} p_i(j | h + 1, C)V^*_{i+1}(j),
\]

(A2)

where Equation (A3) follows from the value functions (A1) and (A2) and the inequality (A3) follows from assumption 3. Following from the induction assumptions, Equation (A3) is non-negative and the desired result follows.

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