More Examples on Moments
For each case illustrated in Fig. 3–4, determine the moment of the force about point \( O \).

**SOLUTION (SCALAR ANALYSIS)**
The line of action of each force is extended as a dashed line in order to establish the moment arm \( d \). Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about \( O \) is shown as a colored curl. Thus,

- **Fig. 3–4a** \( M_O = (100 \text{ N})(2 \text{ m}) = 200 \text{ N} \cdot \text{m} \) \( \text{Ans.} \)
- **Fig. 3–4b** \( M_O = (50 \text{ N})(0.75 \text{ m}) = 37.5 \text{ N} \cdot \text{m} \) \( \text{Ans.} \)
- **Fig. 3–4c** \( M_O = (40 \text{ lb})(4 \text{ ft} + 2 \cos 30^\circ \text{ ft}) = 229 \text{ lb} \cdot \text{ft} \) \( \text{Ans.} \)
- **Fig. 3–4d** \( M_O = (60 \text{ lb})(1 \sin 45^\circ \text{ ft}) = 42.4 \text{ lb} \cdot \text{ft} \) \( \text{Ans.} \)
- **Fig. 3–4e** \( M_O = (7 \text{ kN})(4 \text{ m} - 1 \text{ m}) = 21.0 \text{ kN} \cdot \text{m} \) \( \text{Ans.} \)
Fig. 3-4
Determine the resultant moment of the four forces acting on the rod shown in Fig. 3–5 about point $O$.

**SOLUTION**
Assuming that positive moments act in the $+k$ direction, i.e., counterclockwise, we have

$$\sum M_{R_o} = \Sigma Fd;$$

$$M_{R_o} = -50 \text{ N}(2 \text{ m}) + 60 \text{ N}(0) + 20 \text{ N}(3 \sin 30^\circ \text{ m})$$

$$-40 \text{ N}(4 \text{ m} + 3 \cos 30^\circ \text{ m})$$

$$M_{R_o} = -334 \text{ N} \cdot \text{m} = 334 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

For this calculation, note how the moment-arm distances for the 20-N and 40-N forces are established from the extended (dashed) lines of action of each of these forces.
EXAMPLE 3

Determine the moment produced by the force $\mathbf{F}$ in Fig. 3–14a about point $O$. Express the result as a Cartesian vector.

**SOLUTION**

As shown in Fig. 3–14a, either $\mathbf{r}_A$ or $\mathbf{r}_B$ can be used to determine the moment about point $O$. These position vectors are

$$\mathbf{r}_A = \{12k\} \text{ m} \quad \text{and} \quad \mathbf{r}_B = \{4i + 12j\} \text{ m}$$

Force $\mathbf{F}$ expressed as a Cartesian vector is

$$\mathbf{F} = F\mathbf{u}_{AB} = 2 \text{ kN} \left[ \frac{\{4i + 12j - 12k\} \text{ m}}{\sqrt{(4 \text{ m})^2 + (12 \text{ m})^2 + (-12 \text{ m})^2}} \right]$$

$$= \{0.4588i + 1.376j - 1.376k\} \text{ kN}$$
Thus

\[ \mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix} \]

\[ = [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k} \]

\[ = \{-16.5\mathbf{i} + 5.51\mathbf{j}\} \text{ kN} \cdot \text{m} \]
or

$$M_O = r_B \times F = \begin{vmatrix} i & j & k \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [12(-1.376) - 0(1.376)]i - [4(-1.376) - 0(0.4588)]j + [4(1.376) - 12(0.4588)]k$$

$$= \{-16.5i + 5.51j\} \text{ kN}\cdot\text{m}$$

*Ans.*

**NOTE:** As shown in Fig. 3–14b, $M_O$ acts perpendicular to the plane that contains $F$, $r_A$, and $r_B$. Had this problem been worked using $M_O = Fd$, notice the difficulty that would arise in obtaining the moment arm $d$. 

*Fig. 3–14*
Two forces act on the rod shown in Fig. 3–15a. Determine the resultant moment they create about the flange at $O$. Express the result as a Cartesian vector.

\[ \mathbf{F}_1 = [-60i + 40j + 20k] \text{ lb} \]

\[ \mathbf{F}_2 = [80i + 40j - 30k] \text{ lb} \]
EXAMPLE CONTINUED

SOLUTION

Position vectors are directed from point \( O \) to each force as shown in Fig. 3–15b. These vectors are

\[
\mathbf{r}_A = \{5\mathbf{j}\} \text{ ft} \\
\mathbf{r}_B = \{4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\} \text{ ft}
\]

The resultant moment about \( O \) is therefore

\[
\mathbf{M}_{RO} = \Sigma (\mathbf{r} \times \mathbf{F})
\]

\[
= \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2
\]

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 5 & 0 \\
-60 & 40 & 20
\end{vmatrix} + \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
4 & 5 & -2 \\
80 & 40 & -30
\end{vmatrix}
\]

\[
= [5(20) - 0(40)]\mathbf{i} - [0] \mathbf{j} + [0(40) - (5)(-60)]\mathbf{k}
\]

\[
+ [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k}
\]

\[
= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\} \text{ lb} \cdot \text{ft}
\]

NOTE: This result is shown in Fig. 3–15c. The coordinate direction angles were determined from the unit vector for \( \mathbf{M}_{RO} \). Realize that the two forces tend to cause the rod to rotate about the moment axis in the manner shown by the curl indicated on the moment vector.
EXAMPLE 5

Determine the moment of the force in Fig. 3–18a about point O.

\[
\begin{align*}
F &= 5 \text{kN} \\
30^\circ & \quad 3 \text{ m} \\
75^\circ & \quad d
\end{align*}
\]

**SOLUTION I**

The moment arm \( d \) in Fig. 3–18a can be found from trigonometry.

\[
d = (3 \text{ m}) \sin 75^\circ = 2.898 \text{ m}
\]

Thus,

\[
M_O = Fd = (5 \text{kN})(2.898 \text{ m}) = 14.5 \text{kN} \cdot \text{m} \quad \text{Ans.}
\]

Since the force tends to rotate or orbit clockwise about point O, the moment is directed into the page.
EXAMPLE 5 CONTINUED

SOLUTION II
The \( x \) and \( y \) components of the force are indicated in Fig. 3–18b. Considering counterclockwise moments as positive, and applying the principle of moments, we have

\[
\sum M_O = -F_x d_y - F_y d_x
\]

\[
= -(5 \cos 45^\circ \text{ kN})(3 \sin 30^\circ \text{ m}) - (5 \sin 45^\circ \text{ kN})(3 \cos 30^\circ \text{ m})
\]

\[
= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \quad \text{Ans.}
\]

SOLUTION III
The \( x \) and \( y \) axes can be set parallel and perpendicular to the rod’s axis as shown in Fig. 3–18c. Here \( F_x \) produces no moment about point \( O \) since its line of action passes through this point. Therefore,

\[
\sum M_O = -F_y d_x
\]

\[
= -(5 \sin 75^\circ \text{ kN})(3 \text{ m})
\]

\[
= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \quad \text{Ans.}
\]
Force $F$ acts at the end of the angle bracket shown in Fig. 3–19a. Determine the moment of the force about point $O$.

**SOLUTION I (SCALAR ANALYSIS)**

The force is resolved into its $x$ and $y$ components as shown in Fig. 3–19b, then

\[ +M_O = 400 \sin 30^\circ \text{ N}(0.2 \text{ m}) - 400 \cos 30^\circ \text{ N}(0.4 \text{ m}) \]

\[ = -98.6 \text{ N} \cdot \text{m} = 98.6 \text{ N} \cdot \text{m} \downarrow \]

or

\[ M_O = \{-98.6k\} \text{ N} \cdot \text{m} \]

Ans.
SOLUTION II (VECTOR ANALYSIS)

Using a Cartesian vector approach, the force and position vectors shown in Fig. 3–19c are

\[
\mathbf{r} = \{0.4\mathbf{i} - 0.2\mathbf{j}\} \text{ m}
\]
\[
\mathbf{F} = \{400 \sin 30^\circ \mathbf{i} - 400 \cos 30^\circ \mathbf{j}\} \text{ N}
\]
\[
= \{200.0\mathbf{i} - 346.4\mathbf{j}\} \text{ N}
\]

The moment is therefore

\[
\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0.4 & -0.2 & 0 \\
200.0 & -346.4 & 0
\end{vmatrix}
\]
\[
= 0\mathbf{i} - 0\mathbf{j} + [0.4(-346.4) - (-0.2)(200.0)]\mathbf{k}
\]
\[
= \{-98.6\mathbf{k}\} \text{ N} \cdot \text{m}
\]

**NOTE:** It is seen that the scalar analysis (Solution I) provides a more convenient method for analysis than Solution II since the direction of the moment and the moment arm for each component force are easy to establish. Hence, this method is generally recommended for solving problems displayed in two dimensions, whereas a Cartesian vector analysis is generally recommended only for solving three-dimensional problems.
EXAMPLE

Determine the resultant moment of the three forces in Fig. 3–22 about the x axis, the y axis, and the z axis.

SOLUTION

A force that is parallel to a coordinate axis or has a line of action that passes through the axis does not produce any moment or tendency for turning about that axis. Therefore, defining the positive direction of the moment of a force according to the right-hand rule, as shown in the figure, we have

\[ M_x = (60 \text{ lb})(2 \text{ ft}) + (50 \text{ lb})(2 \text{ ft}) + 0 = 220 \text{ lb} \cdot \text{ft} \quad \text{Ans.} \]

\[ M_y = 0 - (50 \text{ lb})(3 \text{ ft}) - (40 \text{ lb})(2 \text{ ft}) = -230 \text{ lb} \cdot \text{ft} \quad \text{Ans.} \]

\[ M_z = 0 + 0 - (40 \text{ lb})(2 \text{ ft}) = -80 \text{ lb} \cdot \text{ft} \quad \text{Ans.} \]

The negative signs indicate that \( M_y \) and \( M_z \) act in the \(-y\) and \(-z\) directions, respectively.
EXAMPLE 8

Determine the moment $M_{AB}$ produced by the force $F$ in Fig. 3–23a, which tends to rotate the rod about the $AB$ axis.

**SOLUTION**

A vector analysis using $M_{AB} = \mathbf{u}_B \cdot (\mathbf{r} \times \mathbf{F})$ will be considered for the solution rather than trying to find the moment arm or perpendicular distance from the line of action of $F$ to the $AB$ axis. Each of the terms in the equation will now be identified.

Unit vector $\mathbf{u}_B$ defines the direction of the $AB$ axis of the rod, Fig. 3–23b, where

$$
\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{[0.4\mathbf{i} + 0.2\mathbf{j}] \text{ m}}{\sqrt{(0.4 \text{ m})^2 + (0.2 \text{ m})^2}} = 0.8944\mathbf{i} + 0.4472\mathbf{j}
$$

Vector $\mathbf{r}$ is directed from *any point* on the $AB$ axis to *any point* on the line of action of the force. For example, position vectors $\mathbf{r}_C$ and $\mathbf{r}_D$ are suitable, Fig. 3–23b. (Although not shown, $\mathbf{r}_{BC}$ or $\mathbf{r}_{BD}$ can also be used.) For simplicity, we choose $\mathbf{r}_D$, where

$$
\mathbf{r}_D = \{0.6\mathbf{i}\} \text{ m}
$$

The force is

$$
\mathbf{F} = \{-300\mathbf{k}\} \text{ N}
$$
Substituting these vectors into the determinant form and expanding, we have

\[
M_{AB} = \mathbf{u}_B \cdot (\mathbf{r}_D \times \mathbf{F}) = \begin{vmatrix}
0.8944 & 0.4472 & 0 \\
0.6 & 0 & 0 \\
0 & 0 & -300
\end{vmatrix}
\]

\[
= 0.8944[0(-300) - 0(0)] - 0.4472[0.6(-300) - 0(0)] \\
+ 0[0.6(0) - 0(0)]
\]

\[
= 80.50 \text{ N} \cdot \text{m}
\]

This positive result indicates that the sense of \( M_{AB} \) is in the same direction as \( \mathbf{u}_B \).

Expressing \( M_{AB} \) as a Cartesian vector yields

\[
M_{AB} = M_{AB} \mathbf{u}_B = (80.50 \text{ N} \cdot \text{m})(0.8944 \mathbf{i} + 0.4472 \mathbf{j})
\]

\[
= \{72.0 \mathbf{i} + 36.0 \mathbf{j}\} \text{ N} \cdot \text{m}
\]

\textbf{Ans.}

The result is shown in Fig. 3–23.

\textbf{NOTE:} If axis \( AB \) is defined using a unit vector directed from \( B \) toward \( A \), then in the above formulation \(-\mathbf{u}_B\) would have to be used. This would lead to \( M_{AB} = -80.50 \text{ N} \cdot \text{m} \). Consequently, \( M_{AB} = M_{AB}(-\mathbf{u}_B) \), and the same result would be obtained.
EXAMPLE 9

Determine the resultant couple moment of the three couples acting on the plate in Fig. 3–29.

SOLUTION

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4 \text{ ft}$, $d_2 = 3 \text{ ft}$, and $d_3 = 5 \text{ ft}$. Considering counterclockwise couple moments as positive, we have

\[ \tau + M_R = \sum M; \quad M_R = -F_1d_1 + F_2d_2 - F_3d_3 \]

\[ = (-200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft}) \]

\[ = -950 \text{ lb} \cdot \text{ft} = 950 \text{ lb} \cdot \text{ft} \quad \text{Ans.} \]

The negative sign indicates that $M_R$ has a clockwise rotational sense.
EXAMPLE 10

Determine the magnitude and direction of the couple moment acting on the gear in Fig. 3–30a.
**EXAMPLE CONTINUED**

**SOLUTION**

The easiest solution requires resolving each force into its components as shown in Fig. 3–30b. The couple moment can be determined by summing the moments of these force components about any point, for example, the center $O$ of the gear or point $A$. If we consider counterclockwise moments as positive, we have

$$\sum M = \sum M_O; \quad M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$

$$= 43.9 \text{ N} \cdot \text{m \uparrow} \quad \text{Ans.}$$

or

$$\sum M = \sum M_A; \quad M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$

$$= 43.9 \text{ N} \cdot \text{m \uparrow} \quad \text{Ans.}$$

This positive result indicates that $\mathbf{M}$ has a counterclockwise rotational sense, so it is directed outward, perpendicular to the page.

**NOTE:** The same result can also be obtained using $\mathbf{M} = Fd$, where $d$ is the perpendicular distance between the lines of action of the couple forces, Fig. 3–30c. However, the computation for $d$ is more involved. Realize that the couple moment is a free vector and can act at any point on the gear and produce the same turning effect about point $O$. 

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**Fig. 3–30**
EXAMPLE 11

Determine the couple moment acting on the pipe shown in Fig. 3–31a. Segment $AB$ is directed $30^\circ$ below the $x$–$y$ plane.

SOLUTION I (VECTOR ANALYSIS)
The moment of the two couple forces can be found about any point. If point $O$ is considered, Fig. 3–31b, we have

$$\mathbf{M} = \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k})$$

$$= (8\mathbf{j}) \times (-25\mathbf{k}) + (6\cos 30^\circ\mathbf{i} + 8\mathbf{j} - 6\sin 30^\circ\mathbf{k}) \times (25\mathbf{k})$$

$$= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i}$$

$$= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.} \quad \text{Ans.}$$
SOLUTION I (VECTOR ANALYSIS)
The moment of the two couple forces can be found about any point. If point \( O \) is considered, Fig. 3–31b, we have
\[
\mathbf{M} = \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k})
\]
\[
= (8\mathbf{j}) \times (-25\mathbf{k}) + (6 \cos 30^\circ \mathbf{i} + 8\mathbf{j} - 6 \sin 30^\circ \mathbf{k}) \times (25\mathbf{k})
\]
\[
= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i}
\]
\[
= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.}
\]
It is easier to take moments of the couple forces about a point lying on the line of action of one of the forces, e.g., point \( A \), Fig. 3–31c. In this case the moment of the force at \( A \) is zero, so that
\[
\mathbf{M} = \mathbf{r}_{AB} \times (25\mathbf{k})
\]
\[
= (6 \cos 30^\circ \mathbf{i} - 6 \sin 30^\circ \mathbf{k}) \times (25\mathbf{k})
\]
\[
= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.}
\]

SOLUTION II (SCALAR ANALYSIS)
Although this problem is shown in three dimensions, the geometry is simple enough to use the scalar equation \( M = Fd \). The perpendicular distance between the lines of action of the couple forces is \( d = 6 \cos 30^\circ = 5.196 \text{ in.} \), Fig. 3–31d. Hence, taking moments of the forces about either point \( A \) or point \( B \) yields
\[
M = Fd = 25 \text{ lb} \times (5.196 \text{ in.}) = 129.9 \text{ lb} \cdot \text{in.}
\]
Applying the right-hand rule, \( \mathbf{M} \) acts in the \(-\mathbf{j}\) direction. Thus,
\[
\mathbf{M} = \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.}
\]
Example 12

Replace the force and couple system shown in Fig. 3–35a by an equivalent resultant force and couple moment acting at point $O$.

(a) $4$ kN

(b) $\frac{3}{5}$ (5 kN)

Fig. 3–35
**Example Continued**

**Solution**

**Force Summation.** The 3 kN and 5 kN forces are resolved into their $x$ and $y$ components as shown in Fig. 3–35b. We have

$$ \uparrow (F_R)_x = \Sigma F_x; \quad (F_R)_x = (3 \text{ kN}) \cos 30^\circ + \left( \frac{4}{5} \right) (5 \text{ kN}) = 5.598 \text{ kN} \quad \uparrow $$

$$ + \downarrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = (3 \text{ kN}) \sin 30^\circ - \left( \frac{3}{5} \right) (5 \text{ kN}) - 4 \text{ kN} = -6.50 \text{ kN} = 6.50 \text{ kN} \quad \downarrow $$

Using the Pythagorean theorem, Fig. 3–35c, the magnitude of $F_R$ is

$$ F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598 \text{ kN})^2 + (6.50 \text{ kN})^2} = 8.58 \text{ kN} \quad \text{Ans.} $$

Its direction $\theta$ is

$$ \theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{6.50 \text{ kN}}{5.598 \text{ kN}} \right) = 49.3^\circ \quad \text{Ans.} $$

**Moment Summation.** The moments of 3 kN and 5 kN about point $O$ will be determined using their $x$ and $y$ components. Referring to Fig. 3–35b, we have

$$ \downarrow (M_R)_O = \Sigma M_O; $$

$$ (M_R)_O = (3 \text{ kN}) \sin 30^\circ (0.2 \text{ m}) - (3 \text{ kN}) \cos 30^\circ (0.1 \text{ m}) + \left( \frac{3}{5} \right) (5 \text{ kN}) (0.1 \text{ m}) $$

$$ - \left( \frac{4}{5} \right) (5 \text{ kN}) (0.5 \text{ m}) - (4 \text{ kN})(0.2 \text{ m}) $$

$$ = -2.46 \text{ kN} \cdot \text{m} = 2.46 \text{ kN} \cdot \text{m} \downarrow \quad \text{Ans.} $$

This clockwise moment is shown in Fig. 3–35c.

**Note:** Realize that the resultant force and couple moment in Fig. 3–35c will produce the same external effects or reactions at the supports as those produced by the force system, Fig. 3–35a.
EXAMPLE 13

Replace the force and couple system acting on the member in Fig. 3–36a by an equivalent resultant force and couple moment acting at point $O$.

Fig. 3–36
**SOLUTION**

**Force Summation.** Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its $x$ and $y$ components, thus,

$$\pm (F_R)_x = \sum F_x; \quad (F_R)_x = \left(\frac{3}{5}\right) (500 \text{ N}) = 300 \text{ N} \rightarrow$$

$$+ \uparrow (F_R)_y = \sum F_y; \quad (F_R)_y = (500 \text{ N}) \left(\frac{4}{5}\right) - 750 \text{ N} = -350 \text{ N} = 350 \text{ N} \downarrow$$

From Fig. 3–36b, the magnitude of $F_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300 \text{ N})^2 + (350 \text{ N})^2} = 461 \text{ N} \quad \text{Ans.}$$

And the angle $\theta$ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350 \text{ N}}{300 \text{ N}}\right) = 49.4^\circ \quad \text{Ans.}$$

**Moment Summation.** Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig. 3–36a, we have

$$\downarrow + (M_R)_O = \sum M_O + \sum M_c;$$

$$(M_R)_O = (500 \text{ N}) \left(\frac{4}{5}\right)(2.5 \text{ m}) - (500 \text{ N}) \left(\frac{3}{5}\right)(1 \text{ m})$$

$$- (750 \text{ N})(1.25 \text{ m}) + 200 \text{ N} \cdot \text{m}$$

$$= -37.5 \text{ N} \cdot \text{m} = 37.5 \text{ N} \cdot \text{m} \downarrow$$

This clockwise moment is shown in Fig. 3–36b.
EXAMPLE 14

The structural member is subjected to a couple moment \( \mathbf{M} \) and forces \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \) in Fig. 3–37a. Replace this system by an equivalent resultant force and couple moment acting at its base, point \( O \).

SOLUTION (VECTOR ANALYSIS)

The three-dimensional aspects of the problem can be simplified by using a Cartesian vector analysis. Expressing the forces and couple moment as Cartesian vectors, we have

\[
\mathbf{F}_1 = \{-800 \mathbf{k}\} \text{ N}
\]

\[
\mathbf{F}_2 = (300 \text{ N}) \mathbf{u}_{CB}
\]

\[
= (300 \text{ N}) \left( \frac{\mathbf{r}_{CB}}{r_{CB}} \right)
\]

\[
= 300 \text{ N} \left[ \frac{-0.15 \mathbf{i} + 0.1 \mathbf{j}}{\sqrt{(-0.15 \text{ m})^2 + (0.1 \text{ m})^2}} \right] = \{-249.6 \mathbf{i} + 166.4 \mathbf{j}\} \text{ N}
\]

\[
\mathbf{M} = -500 \left( \frac{4}{5} \right) \mathbf{j} + 500 \left( \frac{3}{5} \right) \mathbf{k} = \{-400 \mathbf{j} + 300 \mathbf{k}\} \text{ N} \cdot \text{m}
\]
Force Summation.

\[ \mathbf{F}_R = \sum \mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = -800 \mathbf{k} - 249.6 \mathbf{i} + 166.4 \mathbf{j} \]
\[ = \{-250 \mathbf{i} + 166 \mathbf{j} - 800 \mathbf{k}\} \text{ N} \quad \text{Ans.} \]

Moment Summation.

\[ \mathbf{M}_{R_0} = \sum \mathbf{M} + \sum \mathbf{M}_O \]

\[ \mathbf{M}_{R_0} = \mathbf{M} + \mathbf{r}_C \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2 \]

\[
\begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  -0.15 & 0.1 & 1 \\
  -249.6 & 166.4 & 0 \\
\end{vmatrix}
\]

\[ \mathbf{M}_{R_0} = (-400 \mathbf{j} + 300 \mathbf{k}) + (1 \mathbf{k}) \times (-800 \mathbf{k}) + (-166.4 \mathbf{i} - 249.6 \mathbf{j}) \]
\[ = (-400 \mathbf{j} + 300 \mathbf{k}) + (0) + (-166.4 \mathbf{i} - 249.6 \mathbf{j}) \]
\[ = \{-166 \mathbf{i} - 650 \mathbf{j} + 300 \mathbf{k}\} \text{ N} \cdot \text{m} \quad \text{Ans.} \]

The results are shown in Fig. 3–37b.
Replace the force and couple moment system acting on the beam in Fig. 3–41a by an equivalent resultant force, and find where its line of action intersects the beam, measured from point $O$.

**Fig. 3–41**
**EXAMPLE CONTINUED**

**SOLUTION**

**Force Summation.** Summing the force components,

\[ (F_R)_x = \Sigma F_x; \quad (F_R)_x = 8 \text{kN}(\frac{3}{5}) = 4.80 \text{kN} \rightarrow \]

\[ (F_R)_y = \Sigma F_y; \quad (F_R)_y = -4 \text{kN} + 8 \text{kN}(\frac{4}{5}) = 2.40 \text{kN} \uparrow \]

From Fig. 3–41b, the magnitude of \( F_R \) is

\[ F_R = \sqrt{(4.80 \text{kN})^2 + (2.40 \text{kN})^2} = 5.37 \text{kN} \quad \text{Ans.} \]

The angle \( \theta \) is

\[ \theta = \tan^{-1}\left(\frac{2.40 \text{kN}}{4.80 \text{kN}}\right) = 26.6^\circ \quad \text{Ans.} \]

**Moment Summation.** We must equate the moment of \( F_R \) about point \( O \) in Fig. 3–41b to the sum of the moments of the force and couple moment system about point \( O \) in Fig. 3–41a. Since the line of action of \( (F_R)_x \) acts through point \( O \), only \( (F_R)_y \) produces a moment about this point. Thus,

\[ \downarrow + (M_R)_O = \Sigma M_O; \quad 2.40 \text{kN}(d) = -(4 \text{kN})(1.5 \text{ m}) - 15 \text{kN} \cdot \text{m} \]

\[-[8 \text{kN}(\frac{3}{5})](0.5 \text{ m}) + [8 \text{kN}(\frac{4}{5})](4.5 \text{ m})\]

\[ d = 2.25 \text{ m} \quad \text{Ans.} \]
The jib crane shown in Fig. 3–42a is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant’s line of action intersects the column AB and boom BC.

**SOLUTION**

**Force Summation.** Resolving the 250-lb force into x and y components and summing the force components yields

\[ \downarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = -250 \text{ lb}(\frac{3}{5}) - 175 \text{ lb} = -325 \text{ lb} = 325 \text{ lb} \leftarrow \]

\[ + \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -250 \text{ lb}(\frac{4}{5}) - 60 \text{ lb} = -260 \text{ lb} = 260 \text{ lb} \downarrow \]

As shown by the vector addition in Fig. 3–42b,

\[ F_R = \sqrt{(325 \text{ lb})^2 + (260 \text{ lb})^2} = 416 \text{ lb} \quad \text{Ans.} \]

\[ \theta = \tan^{-1}\left(\frac{260 \text{ lb}}{325 \text{ lb}}\right) = 38.7^\circ \quad \text{Ans.} \]
**Moment Summation.** Moments will be summed about point A. Assuming the line of action of $F_R$ intersects $AB$ at a distance $y$ from $A$, Fig. 3-42b, we have

$$
\downarrow + M_{RA} = \Sigma M_A;
325 \text{ lb} (y) + 260 \text{ lb} (0)
= 175 \text{ lb} (5 \text{ ft}) - 60 \text{ lb} (3 \text{ ft}) + 250 \text{ lb} \left(\frac{3}{5}\right)(11 \text{ ft}) - 250 \text{ lb} \left(\frac{4}{5}\right)(8 \text{ ft})
$$

$$
y = 2.29 \text{ ft}
$$

By the principle of transmissibility, $F_R$ can be placed at a distance $x$ where it intersects $BC$, Fig. 3-42b. In this case we have

$$
\downarrow + M_{RA} = \Sigma M_A;
325 \text{ lb} (11 \text{ ft}) - 260 \text{ lb} (x)
= 175 \text{ lb} (5 \text{ ft}) - 60 \text{ lb} (3 \text{ ft}) + 250 \text{ lb} \left(\frac{3}{5}\right)(11 \text{ ft}) - 250 \text{ lb} \left(\frac{4}{5}\right)(8 \text{ ft})
$$

$$
x = 10.9 \text{ ft}
$$

*Ans.*
The slab in Fig. 3–43a is subjected to four parallel forces. Determine the magnitude and direction of a resultant force equivalent to the given force system and locate its point of application on the slab.

Fig. 3–43
SOLUTION (SCALAR ANALYSIS)

**Force Summation.** From Fig. 3–43a, the resultant force is

\[ +\uparrow F_R = \Sigma F; \quad -F_R = -600\text{ N} + 100\text{ N} - 400\text{ N} - 500\text{ N} \]

\[ = -1400\text{ N} = 1400\text{ N}\downarrow \quad \text{Ans.}\]

**Moment Summation.** We require the moment about the x axis of the resultant force, Fig. 3–43b, to be equal to the sum of the moments about the x axis of all the forces in the system, Fig. 3–43a. The moment arms are determined from the y coordinates since these coordinates represent the perpendicular distances from the x axis to the lines of action of the forces. Using the right-hand rule, we have

\[(M_R)_x = \Sigma M_x;\]

\[-(1400\text{ N})y = 600\text{ N}(0) + 100\text{ N}(5\:\text{m}) - 400\text{ N}(10\:\text{m}) + 500\text{ N}(0)\]

\[-1400y = -3500 \quad y = 2.50\:\text{m} \quad \text{Ans.}\]

In a similar manner, a moment equation can be written about the y axis using moment arms defined by the x coordinates of each force.

\[(M_R)_y = \Sigma M_y;\]

\[(1400\text{ N})x = 600\text{ N}(8\:\text{m}) - 100\text{ N}(6\:\text{m}) + 400\text{ N}(0) + 500\text{ N}(0)\]

\[1400x = 4200 \quad x = 3\:\text{m} \quad \text{Ans.}\]

**NOTE:** A force of \( F_R = 1400\text{ N} \) placed at point \( P(3.00\text{ m}, 2.50\text{ m}) \) on the slab, Fig. 3–43b, is therefore equivalent to the parallel force system acting on the slab in Fig. 3–43a.
EXAMPLE 18

Replace the force system in Fig. 3–44a by an equivalent resultant force and specify its point of application on the pedestal.

SOLUTION

Force Summation. Here we will demonstrate a vector analysis. Summing forces,

\[ \mathbf{F}_R = \sum \mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C \]

\[ = \{ -300 \text{k} \} \text{ lb} + \{ -500 \text{k} \} \text{ lb} + \{ 100 \text{k} \} \text{ lb} \]

\[ = \{ -700 \text{k} \} \text{ lb} \quad \text{Ans.} \]
**Location.** Moments will be summed about point \( O \). The resultant force \( \mathbf{F}_R \) is assumed to act through point \( P \) \((x, y, 0)\), Fig. 3–44b. Thus

\[
(M_R)_O = \Sigma M_O;
\]

\[
r_P \times \mathbf{F}_R = (r_A \times \mathbf{F}_A) + (r_B \times \mathbf{F}_B) + (r_C \times \mathbf{F}_C)
\]

\[
(x\mathbf{i} + y\mathbf{j}) \times (-700\mathbf{k}) = [(4\mathbf{i}) \times (-300\mathbf{k})]
\]  
+ [(-4\mathbf{i} + 2\mathbf{j}) \times (-500\mathbf{k})] + [(-4\mathbf{j}) \times (100\mathbf{k})]
\]

\[-700x(\mathbf{i} \times \mathbf{k}) - 700y(\mathbf{j} \times \mathbf{k}) = -1200(\mathbf{i} \times \mathbf{k}) + 2000(\mathbf{i} \times \mathbf{k})
\]

\[-1000(\mathbf{j} \times \mathbf{k}) - 400(\mathbf{j} \times \mathbf{k})
\]

\[700x\mathbf{j} - 700y\mathbf{i} = 1200\mathbf{j} - 2000\mathbf{j} - 1000\mathbf{i} - 400\mathbf{i}\]

Equating the \( \mathbf{i} \) and \( \mathbf{j} \) components,

\[-700y = -1400 \quad \text{(1)} \]

\[y = 2 \text{ in.} \quad \text{Ans.} \]

\[700x = -800 \quad \text{(2)} \]

\[x = -1.14 \text{ in.} \quad \text{Ans.} \]

The negative sign indicates that the \( x \) coordinate of point \( P \) is negative.

**NOTE:** It is also possible to establish Eq. 1 and 2 directly by summing moments about the \( x \) and \( y \) axes. Using the right-hand rule, we have

\[(M_R)_x = \Sigma M_x; \quad -700y = -100 \text{ lb}(4 \text{ in.}) - 500 \text{ lb}(2 \text{ in.})\]

\[(M_R)_y = \Sigma M_y; \quad 700x = 300 \text{ lb}(4 \text{ in.}) - 500 \text{ lb}(4 \text{ in.})\]