ENGR0135 - Design Project 1
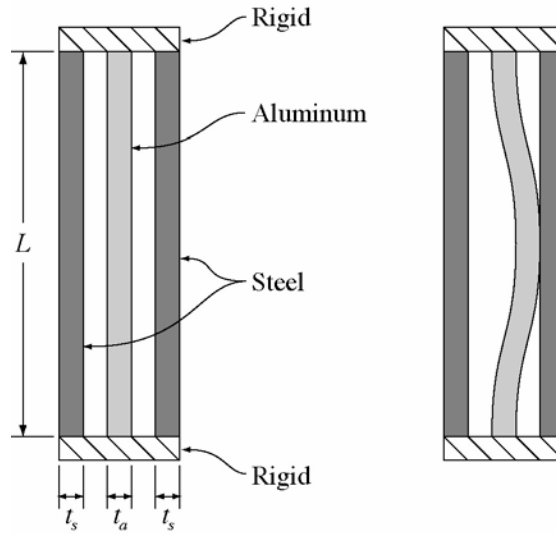## "Design of a Cheap Thermal Switch"
Due: 09/25/2020

Introduction

A thermal switch has been designed for incorporation into a low cost product. The switch consists of three metal strips clamped rigidly together in an assembly of insulating pieces, as shown below.

There is no external mechanical loading on the switch. Thus, the only loads are induced by temperature changes. In addition, there are no initial stresses.

The width of the aluminum strip normal to the plane of the figure shown above is w_a. The width of each steel strip, again normal to the plane of the figure, is w_s. You are given that

$$\alpha_a = 12.5 \times 10^{-6}/°F , \quad E_a = 10{,}000 \text{ ksi},$$
$$\alpha_s = 6.6 \times 10^{-6}/°F , \quad E_s = 30{,}000 \text{ ksi},$$

where α_a and α_s denote the coefficients of thermal expansion and E_a and E_s denote the Young's moduli for aluminum and steel, respectively. Both materials behave thermoelastically in the region of operation.

At the actuating temperature, the central strip is to snap aside—in the plane of the figure—thus making contact with one of the outer strips. These outer strips are electrically conducting and contact closes a circuit. The original design has (see figure)

$$t_a = \frac{1}{16} \text{ in}, \quad w_a = \frac{1}{4} \text{ in},$$
$$t_s = \frac{1}{16} \text{ in}, \quad w_s = \frac{1}{8} \text{ in},$$

and an overall length of $L = 4$ in. This switch has been designed to close when the temperature increases by about $180°$F.

Design Modification
By varying only the dimensions of the central strip, i.e., t_a and w_a, you are required to modify the design so that closure of the switch will occur when the temperature increases by 105ºF.

Additional Information
A formula that will be studied in ENGR0145 (Chapter 11 of Riley, Sturges, and Morris) is Euler's formula for the buckling of columns. When both ends of the column are built-in (as is the case for the aluminum strip), the formula provides the critical axial compressive load P_{cr} applied at the ends of the column for which buckling will first occur:

$$P_{cr} = \frac{4\pi^2 EI}{L^2} \ ,$$

where E is Young's modulus, $I = w_a t_a^3/12$ is the *minimum* second moment of inertia of the cross section, and L is the length of the column. For the designs considered here, always take $t_a < w_a$ so that the central strip buckles in the plane of the figure. Note that if $t_a > w_a$, the central strip will buckle out of the plane of the figure, neither contacting one of the steel outer strips nor closing the electrical circuit. You need not attempt to understand column theory in greater depth at this time.

Suggested Method

• As the temperature increases, the lengths of both aluminum and steel straps remain equal because of the rigid end pieces. Thus, the compressive force in the aluminum strip increases with temperature. Ultimately, when the critical temperature is attained, the aluminum strip buckles and makes contact with a straight steel strip. The spacing between the strips is not significant since this can easily be modified to guarantee contact upon buckling by using different insulating pieces at the ends.

• Generalize the problem outlined in the introduction to get an equation for the critical change in temperature ΔT_{cr} in terms of

$$E_a, \ E_s, \ \alpha_a, \ \alpha_s, \ t_a, \ t_s, \ w_a, \ w_s, \ L.$$

Check your formula for the original design.

• Use your formula to study different designs that will meet the requirements specified for the design modification. Thus, produce a design (or designs) which meet the necessary specifications.

Requirements
You must present your design calculations and recommended designs in a report. This is a group project and all members are expected to contribute equally.