MATLAB Workshop 2 - Using MATLAB to solve algebraic problems

Objectives: Learn about scalar variable properties in MATLAB, use MATLAB to solve algebraic engineering problems, create a diary, and archive results.

MATLAB Features:

scalar variable properties

Property	Comment
`var_name`	user chooses names • name should represent *meaning* of variable ex: use **`radius`** rather than **`x`** or **`s`** for radius • letters, digits (0-9), underscore (_) allowed in name • must start with a letter • case sensitive (**`Radius`** is different than **`radius`**) • can be any length, but first 31 characters must be unique
`value`	all numerical values are • double precision ~ 16 significant figures (not decimal places) • and imaginary has both real and imaginary parts **`a ± bi`** (where **`i = (`** $\sqrt{-1}$ **`)`**) both **`a`** and **`b`** are double precision if **`b = 0`**, MATLAB only displays real part, **`a`**
`memory location`	MATLAB assigns - you do not need to worry about this

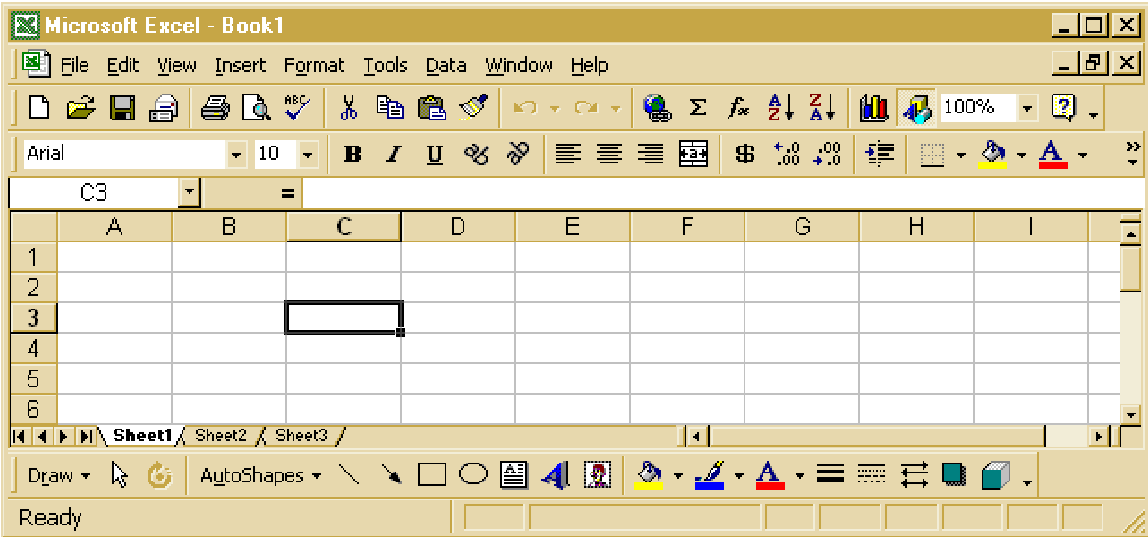
diary commands

MATLAB Command	Comment
`diary name`	creates and opens the specified diary • provide full directory path, filename, and extension ex: **`c:\temp\engr_hmwk`** • defaults current directory: **`engr_hmwk`** • if diary already exists opens for *appending* new session all lines following this command are placed in diary
`diary off`	closes the current diary
`diary on`	opens the most recently used diary

- **Variables in MATLAB**
  Variables are named as needed for use in MATLAB. For example, the MATLAB command

  » radius = 5

would create a variable named "radius" and assign the (real) value 5 to it. Behind the scenes, MATLAB would find an available location in computer memory to use for holding the value. Computer memory can be loosely regarded as similar to a spreadsheet as shown here.



MATLAB will look for an empty cell to use for storing the value; for example, cell **C3**. MATLAB will remember that the value for **radius** is stored in cell **C3** for us. We do not need to keep track of this. Whenever the name **radius** is used in the MATLAB assignment statement, MATLAB will either *retrieve* the stored value from cell **C3** (if **radius** appears on the right hand side of the assignment operator, **=**) or assign a new value to cell **C3** (if **radius** appears on the left hand side of the assignment operator.

MATLAB treats all numbers alike: they are all contain approximately 16 significant figures, termed *double precision* in computer jargon, and have both *real* and *imaginary* parts. If you are not familiar with imaginary numbers and imaginary number arithmetic and algebra - that is ok. The rules that govern real numbers and real number arithmetic are a subset of the rules that govern imaginary number arithmetic. This means that MATLAB will work with real numbers just fine. It also means that, when necessary, MATLAB will be able to work with imaginary numbers also.

If you violate the rules of real number arithmetic, you may be surprised by the result, but MATLAB will still work. Consider the quadratic equation

$$ax^2 + bx + c = 0$$

The roots of the quadratic equation are given by

$$roots = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If the value inside the square root is positve, the result is real. If the value inside the square root is negative, the result is imaginary. We can use MATLAB to find the roots for a quadratic expression as follows

(1) Using MATLAB to find the roots of a quadratic expression.

Find the roots of $x^2 - 4x + 3 = 0$

Enter the following at the Command Line prompt

```
» % roots of the quadratic equation
```

```
»   a = 1;
»   b = -4;
»   c = 3;
»   greater_root = (-b+sqrt(b^2-4*a*c))/(2*a)
greater_root =
     3
»   lesser_root = (-b-sqrt(b^2-4*a*c))/(2*a)
lesser_root =
     1
```

Find the roots of $x^2 - 16x + 5 = 0$

This is very similar to the preceding equation. Rather than reenter all of the previous lines, we can use the *arrow* keys to recall previously entered lines. Press the *up arrow* until

```
»   c = 3;
```

appears. Change the value 3 to the value 5 and press enter. Now press the *up arrow* until

```
»   greater_root = (-b+sqrt(b^2-4*a*c))/(2*a)
```

appears and press enter. MATLAB responds with

```
greater_root =
     2.0000 + 1.0000i
```

Now press the *up arrow* until

```
»   lesser_root = (-b-sqrt(b^2-4*a*c))/(2*a)
```

appears and press enter. MATLAB responds with

```
lesser_root =
     2.0000 - 1.0000i
```

The arrow keys, ↑ and ↓, can be used to scroll through commands on the command line.

- **Using MATLAB to solve engineering (algebraic) problems**

Activity 2: Pressure at the bottom of a tank

Problem Statement: The absolute pressure at the bottom of a liquid storage tank that is vented to the atmosphere is given by the relation, $P_{abs,b} = \rho gh + P_{out}$, where $P_{abs,b}$ is the absolute pressure, ρ is the liquid density, g is gravitational acceleration, h is the height of the liquid, and P_{out} is the outside atmospheric pressure. Find $P_{abs,b}$ in SI units if ρ = 1000 kg/m^3, g = 32.2 ft/s^2, h = 7 yd, and P_{out} = 1 atm.

Background: This is a problem in units conversion.

Solution strategy: Convert all units to SI before performing calculation

ρ is already in SI units.

need factors: ft_to_m = 0.3048
yd_to_m = 0.9144
atm_to_Pa = 1.013•10^5

Calculate $P_{abs,b}$

(2) Using MATLAB to find the pressure at the bottom of a tank.

Enter the following at the Command Line prompt

```
» diary  wkshp2_ac2  % or choose your own name/path
»
» % Solution to Workshop 2, Activity 2
» % Your Name(s)
» % Class (e.g. Engr 0012, TH 10:00, Instructor Name)
» % Today’s Date
» % Your e-mail address
»
»   format short e
»
» % conversion factors
»   ft_to_m = 0.3048; yd_to_m = 0.9144; atm_to_Pa = 1.013e5;
»
» % set problem parameters (note definitions)
»   rho = 1000;       % water density, kg/m^3
»   g = 32.2;         % gravitational acceleration, ft/s/s
»   height = 7;       % tank height, yd
»   pout = 1;         % outside pressure, atm
»
» % convert all parameters to SI
»   rho_si = rho;          % water density, kg/m^3
»   g_si = ft_to_m*g;% gravitational acceleration, m/s/s
»   height_si = yd_to_m*height;    % tank height, m
»   pout_si = atm_to_Pa*pout;      % outside pressure, Pa
»
» % calculate Pabs,b in Pa
»   pabs_b_si = rho_si*g_si*height_si+pout_si  % pabs,b, Pa
pabs_b_si =
     1.6412e+005
»
» diary off
```

Notes on programming style:

- All programming endeavors should have a header section that identifies
  - Who is doing the programming
  - What the program “solves”
  - Class for which the program is being prepared
  - Date the program was implemented or last revised
- Use lots of “whitespace” to make your program easier to read
  - An empty return at the command prompt adds space between lines
  - Indenting (using the spacebar or tab key) also adds whitespace
- Use comments judiciously to keep you and others informed of what is happening
  - **Define all variables (with units) first time used**
- More than one statement can be placed on a line!
  - If separated by a semicolon, `;`, the display is suppressed.
  - If separated by a comma, `,`, the display follows the first return.
- Solving problems requires a strategy!!
  - Identify what the desired result is.
  - Identify what is known.
  - Identify relations that move from what is known to what is desired.
  - Develop MATLAB solution working in logical progression with good annotation.

Be sure to use the **diary off** command to close the diary (otherwise, you may lose the diary file). Use your favorite text editor to open the file **wkshp2_ac2** in the current directory (or whatever you called it in whatever directory you placed it). Note that it contains a listing of all screen display (including white space and comments) that was entered from the time that the diary was initiated. This record can be edited (if necessary to remove errors and error statements that might occur), printed, and attached to a homework, report, or work file as an excellent record of your solution.

Activity 3: Spring mechanics

Problem Statement: A spring has a spring constant of 25 lb_f/in. What force is required to stretch the spring 3 in? How much work is done by the force in stretching the spring 3 in?

Background: From physics and conservation of momentum, the force required to stretch the spring is given by

$$F = kd$$

and the work performed by the spring is

$$W = kd^2 / 2$$

where F is the force, k is the spring constant, d is the distance, and W is the work.

Solution strategy: Convert all units to SI before performing calculation

in_to_m = 0.0254
lbf_to_N = 4.448

Find F and W by straightforward substitution into the equations.

(3) Using MATLAB to solve for spring mechanics.

Enter the following at the Command Line prompt

```
» diary  wkshp2_ac3 % or your choice
»
» % Solution to Workshop 2, Activity 3
» % Your Name(s)
» % Class (e.g. Engr 0012, TH 10:00, Instructor Name)
» % Today's Date
» % Your e-mail address
»
»   format short e
»
» % conversion factors
»   in_to_m = 0.0254; lbf_to_N = 4.448;
»
» % set problem parameters
»   spr_k = 25;       % spring constant, lbf/in
»   stretch = 3;      % spring stretch, in
»
» % convert to SI
»   spr_k_si = spr_k*lbf_to_N/in_to_m;% spring const, N/m
»   stretch_si = stretch *in_to_m;    % spring stretch, m
»
```

```
» % calculate force in N and work in J (SI units)
»   force_si = spr_k_si*stretch_si    % force, N
force_si =
     3.3360e+002
»
»   work_si = spr_k_si*stretch_si^2/2.0    % work, J
work_si =
     1.2710e+001
»
» diary off
```

Activity 4: Light bulb life expectancy

Problem Statement: The life of an incandescent light bulb has been experimentally determined to vary inversely as the 12th power of the applied voltage. A rated life of a bulb is 800 hours at 115 V. What is the life expectancy at 120 V? What is the life expectancy at 110 V?

Background: From the problem statement

$$L = AV^{-12}$$

where L is the expected life of the light bulb (hr), V is the applied voltage (V), and A is the proportionality constant (hr•V^{12}).

Solution strategy: Find A from L = 800 hours at 115 V. Calculate L at V = 120V. Calculate L at V = 110 V.

<u>(4) Using MATLAB to solve for light bulb life expectancy.</u>

Enter the following at the Command Line prompt

```
» diary  wkshp2_ac4 % or your choice
»
» % Solution to Workshop 2, Activity 4
» % Your Name(s)
» % Class (e.g. Engr 0012, TH 10:00, Instructor Name)
» % Today's Date
» % Your e-mail address
»
»   format short e
»
» % set problem parameters
»   life115 = 800;    % expected life at 115 V, hr
»   volts = 115;      % voltage, V
»
» % calculate proportionality constant (hr.V^12)
»   Aconst = life*volts^12    % proportionality const, hr.V^12
Aconst =
     4.2802e+027
»
» % expected lifetime at 120 V, hr
»   life120 = Aconst*120^(-12)    % expected life at 120 V, hr
life120 =
```

```
          4.8005e+002
»
» % expected lifetime at 110 V, hr
»   life110 = Aconst*110^(-12)    % expected life at 110 V, hr
life110 =
          1.3638e+003
»
» diary off
```

Exercises

1. Use MATLAB to solve the following problem.

   Problem Statement: A piece of cast iron, which has a density of 450 lb_m/ft^3, has a very irregular shape. You need to determine its volume in SI units. To do so, you submerge the specimen in a cylindrical water tank (d = 0.5 yd). The water rises 8.64 cm above its original level.

   Background: The volume of the specimen is equal to the volume of displaced water. The volume of displace water is equal to

   $$V = (\frac{\pi d^2}{4})h$$

   where h is the rise in water level.
   Need to convert everything to SI units for consistency.

   Solution strategy: Convert everything to SI units before calculating V
   Need conversion factors
   yd_to_m = 0.9144
   cm_to_m = 0.01
   (Note: density conversion not needed to find volume!!!!)
   Calculate volume, V

2. Use MATLAB to solve the following problem.

   Problem Statement: A pipeline in an oil refinery is carrying oil to a large storage tank. The pipe has a 20 in internal diameter. The oil is flowing at 5 ft/s. The density of the oil is 57 lb_m/ft^3. What is the mass flow rate of oil in SI units? What is the mass and volume of oil, in SI units, that flows in a 24-hour time period.

   Background: Need to be careful about units. By dimensional analysis, the mass flow rate of oil, $\dot{M}$ (kg/s) is

   $$\dot{M} = \rho \, vA$$

   where ρ is the density (kg/m^3), v is the flow speed (m/s), and A is the cross-sectional area of the pipe (m^2). The flows in any time period, T (s), are given by

   $$M = \dot{M}\,T \qquad \text{and} \qquad V = M/\rho$$

   where M is the mass (kg) and V is the volume (m^3).

   Solution strategy: Convert everything to SI units before proceeding.

Need conversion factors

in_to_m = 0.0254
ft_to_m = 0.3048
lbm_to_kg = 0.4535
hr_to_s = 3600

Make conversions for ρ, v, and d.
Calculate cross-sectional area, $A = \pi d^2/4$
Calculate mass flow rate, $\dot{M}$.
Calculate total mass in 24 hours, M.
Calculate equivalent volume, V.

3. Use MATLAB to solve the following problem.

Problem Statement: A researcher proposes to use a hollow wrought aluminum alloy sphere, 500 cm outside diameter with a wall thickness of 3 mm, as a buoy to mark the location of an underwater research site. Will the sphere float? If so, how high does the sphere rise above the water?

Background: This is buoyancy problem. The sphere will float if its average density is less than that of water (assume to be 1 kg/L = 1000 kg/m^3). Need to calculate mass of aluminum used

$$M = \rho_{Al}\left(\frac{\pi}{6}\right)\left(d_0^3 - d_i^3\right)$$

where M is the mass of aluminum (kg), ρ_{Al} is the density of aluminum (= 2800 kg/m^3), d_o is the outside wall diameter (m), and d_i is the inner wall diameter (m). The average density, ρ_{ave} (kg/m^3), is then

$$\rho_{ave} = \frac{M}{V_s} = \frac{6M}{\pi\, d_o^3}$$

where V_s is the volume of the sphere. If the sphere floats, the volume that it displaces is equivalent to a volume of water of equal mass, i.e.,

$$V_d = M / \rho_{H_2O}$$

where V_d is the submerged volume (m^3) and ρ_{H_20} is the density of water. This needs to be used in conjunction with the mensuration formula for the volume of a spherical sector,

$$V_h = \frac{\pi}{6} R^2 h$$

where V_h is the volume (m^3) of a sector of depth h (m). If $V_d > V_s/2$, need to think carefully about how to use the V_h relation (Why?).

Solution strategy: Make sure all parameters are in SI units.

Specify parameters: d_o, d_i, ρ_{Al}, ρ_{H_20}
Calculate sphere volume, V_s.
Calculate mass, M.
Calculate average density, ρ_{ave}.
If $\rho_{ave} > \rho_{H_20}$, sphere sinks; else
Calculate h.
Calculate height above water, $H = d_o - h$.

4. Use MATLAB to solve the following problem.

Problem Statement: A 10 m, medium carbon steel cable is needed to support a load of 100,000 N. The deflection (elongation) of the cable under the load must be less than 1 cm. Ignoring the mass of the cable, what cable mass is required to just support the load without permanent deformation? (E = 207,000 MPa, S_y = 552 MPa, S_t = 690 MPa, ρ = 7900 kg/m^3).

Background: This is a stress-strain problem (S = Ee)

Stress: $S = \dfrac{T}{A_0}$ where $A_0 = \dfrac{\pi d^2}{4}$

S: stress, T: tension, A_0: cable cross-sectional area, d: cable diameter

Strain: $e = \dfrac{\Delta l}{l_0}$ where $\Delta l = l - l_0$

e: strain, l_0: initial cable length, Δ*l*: change in cable length, *l*: cable length under load

Mass: $m = \rho V$ where $V = A_0 l = \pi d^2 l_0 / 4$

m: mass, V: cable volume

Solution strategy: Make sure all units are SI. Need to check two limits for stress

permanent deformation ==> $T/A_0 < S_y$ or $A_0 > T/S_y$

maximum deflection ==> $S = T/A_0 < Ee_{max}$ or $A_0 > T/(Ee_{max})$

strategy: calculate A_0 by both methods
use larger A_0 to compute mass

Recap: You should have learned

- How MATLAB treats scalar variables.
- How to use the arrow keys to scroll through commands at the command line.
- A strategic approach to solving problems.
- The use of white space and comments to make your sessions more readable.
- How to create a diary to record your MATLAB session.
- How to set up and solve problems with MATLAB