MATH 2301-ANALYSIS I, FALL 2013, HOMEWORK 1

DUE ON MONDAY SEPT. 16, 2013, AT THE BEGINNNING OF CLASS

If you use a theorem from the class, you should quote it and justify all of its assumptions.

1. Let $f_k : [a, b] \to \mathbb{R}$ be a sequence of functions converging point-wise to $f : [a, b] \to \mathbb{R}$ such that $V_{[a,b]}(f_k) \leq M < +\infty$ for all k. Show that $V_{[a,b]}(f) \leq M$ too.

2. Is any point-wise limit of functions of bounded variation also of bounded variation? What if the convergence is uniform, i.e. $\lim_{k\to 0} \sup_{x\in[a,b]} |f_k(x) - f(x)| \to 0$?

3. Let $f(x) = x \sin(1/x)$ for $x \in (0, 1]$ and $f(0) = 0$. Show that $f \notin BV([0, 1])$ but that $xf \in BV([0, 1])$.

4. Show that $\int_a^b f d\varphi$ exists iff for all $\varepsilon > 0$, there exists $\delta > 0$ such that $|R_\Gamma - R_{\Gamma'}| < \varepsilon$ if $|\Gamma|, |\Gamma'| < \delta$.

5. Suppose that $\varphi \in BV([a, b])$ and that $f \in C^0([a, b])$. Let $\psi(x) := \int_a^x f d\varphi$. Show that $\psi \in BV([a, b])$ and that for all $g \in C^0([a, b])$ we have

$$\int_a^b g d\psi = \int_a^b g f d\varphi.$$