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# The quality of information in electronic groups

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We examine some of the factors that might influence the quality of information produced in discussion groups on the internet, such as USENET and the WELL. In particular, we look at the impact of various different pricing structures, and compare regimes in which anonymity is enforced with regimes in which all contributors must identify themselves. Our main finding is that the flow of quality-weighted information within the group is maximized by a regime which front loads the cost of sending messages, and identification is required. If there is a positive spillover from the intra-group transmission of good quality information, however, benefiting society at large, then the social value of the flow of quality-weighted information may be maximized by a different regime, in which all replying is anonymous. Reputation effects play a key role in our analysis: posters who have sent high quality messages in the past are considered more likely to send high quality messages in the future, and are thus more likely to be taken notice of.

**Keywords:** discussion groups, Internet economics, reputation

## 1. Introduction

### 1.1. Newsgroups and democracy

This paper forms part of a larger study<sup>1</sup> whose central question is to what extent the Internet does, or could if appropriately constituted, enhance democracy. This question is many-faceted. In the present paper we consider one facet – the Internet’s role in disseminating information (in a broad sense of that word which includes opinion and theory as well as fact). We take it as indisputable that, for democracy to flourish, information of various sorts must be widely available to citizens. In particular, the beneficial effects of democracy require political choices to be based on a good rather than poor understanding of their consequences, and this in turn depends on the airing of a variety of views coming from a range of perspectives. However, information is a stuff of variable quality: assertions of purported fact may be accurate or inaccurate, opinions may be judicious or prejudiced, jokes may be witty or crass, arguments may be valid or invalid. For the flow

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<sup>1</sup> “The Information Superhighway: Market Structure, Access and Citizenship”, directed by Andrew Graham at the University of Oxford under E.S.R.C. contact L126251017.

of information to enhance democracy, therefore, the information must be not only easy to acquire but also good rather than bad.

Several branches of current Internet activity, notably the World Wide Web and several networks which support electronic information groups, have as a prime purpose the dissemination of information in our broad sense. For the sake of concreteness and precise argument, we shall focus in this paper on one kind of such activity, the operations of information groups such as news groups and discussion groups. It is often claimed that information groups such as those found in Usenet or the WELL cannot but be good for the quality of our democracy and – somewhat more controversially – that the freer their organizers and participants, the more democracy will benefit. Caricaturing a little, this view is: the more, and the less constrained, the better. It is held that they promote democracy by, for example, allowing and encouraging the unconstrained expression of views, by publishing important information which would otherwise remain in the private domain, and by encouraging the questioning of authority, as well as by offering a channel for campaigning across national boundaries in favour of democracy itself. All things considered, this libertarian line may be right. But a sophisticated version of it must consider carefully the quality dimension of the information which flows across interfaces into minds.

### *1.2. Electronic information groups*

The main structural features of existing electronic news and discussion groups are as follows. A member can do two things: she can send or ‘post’ messages, and she can read others’ messages. If she posts, her posting can be read by all members of the group. Since the size of electronic groups can run into the thousands, this group-wide broadcasting feature makes them a quite powerful means of information diffusion. Participation may or may not cost anything; if it does, the cost may or may not include a membership fee and can be related in different ways to what is sent or read. We shall find that the cost regime matters crucially for quality, and hence also for democracy. Electronic groups are usually dedicated, at their setting up, to specific subject matters (though these may be broad or narrow), ranging from the future of the planet, through drugs problems and academic freedom, to the Louise Woodward affair or vintage MGs. Each posting includes, in addition to the message body, a few items of ancillary information, displayed in a ‘header’, about the poster’s message, and possibly though not always about the poster herself. When a member accesses a group on a particular occasion, she sees a list of short legends recapping the headers of the most recent postings to the group. The ancillary information gives a reader some indication about which messages are worth her while reading: in particular, they allow people to engage – actively or passively – in a continuing discussion of a particular question, resembling those in the correspondence columns of newspapers.

There are typically few restrictions on the structural forms of interchanges; among the main forms in practice are multilateral debates, and questions followed by one or

more answers, perhaps acknowledged. The latter form is typical of Usenet groups (and is the one that the model below addresses).

Behavior in electronic information groups depends in a complex way on the procedural rules, the distribution over group members of initial information and of communication skills, and members' psychological motivations. For example, poorly informed or inarticulate posters will send low quality messages with the best will in the world. So too will 'spammers' who pose as disinterested informants in order to get business, and 'flamers' or 'trolls' who get satisfaction from disrupting others. Since attending to low quality messages consumes time and effort, and believing them may be damaging, readers may try to assess quality *ex ante* from the ancillary information, and may exit the group if average quality proves bad *ex post*. (If exiting readers tend to be high quality posters, groups can decline and collapse (or take off) through such feedback processes. A companion paper [2] is devoted to the dynamics of information groups.)

One way to make the *ex ante* assessments is by using the narrow identity information, such as the poster's 'username' or 'userid', which allows a reader to infer message-quality through the reputation the name-bearer has for providing good messages. This possibility is absent in anonymous groups, and limited in groups which allow posters to change id *ad libitum*. It is in principle present in 'deonymous' groups in which once-for-all self-naming is compulsory, but is eroded if there is a danger of impersonation.<sup>2</sup>

The motives people may have for participating in information groups are varied. That of a question-poster may be as simple as getting hold cheaply of factual information of practical use, but narrow economic motivation does not take one far in explaining information group behavior. It is notable, and has surprised some commentators, that many experts appear more than willing to give free advice, even advice which they sell professionally [6]. It appears that one motive of many regular posters is to become admired in the group, by acquiring a reputation as an expert. Conversely, members may be deterred from sending messages of whose value they are uncertain by the fear of exposure to ridicule, at least in deonymous groups. Others may give information, factual, theoretical or consisting of opinions, out of public- or at least group-spiritedness. It appears [9] that information groups are often groups in a strong sense, fostering a sense of group identity which motivates members to work for the group's implicit or declared objectives.

### 1.3. *Modelling electronic information groups*

It will be apparent from the foregoing account that a workable and intelligible model of electronic information groups must be highly selective, focusing on certain relationships and holding constant many features which can in fact vary. To use modelling methods to illuminate issues about the actual and potential role of Internet information groups in promoting democracy therefore calls for not one model but many.

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<sup>2</sup> Impersonation is a special form of identity mimicry, which also includes the use by posters of false 'broad identity' information, such as information about affiliation, profession or even gender [4].

The present model is thus offered as an initial contribution to a larger enterprise. It is directed at the relationship between three sets of factors: the quality of the information in the messages, and two aspects of the regime: the amount of ancillary identity information senders must give, and the costs of participating. The bottom line of our analysis reads: the flow of quality-weighted information transmitted within the group is maximized by a group regime which makes senders identify themselves, extracts all surplus from providers of information, and front-loads the costs of sending. If there is a positive spillover from the intra-group transmission of good information, benefiting society at large, then the social value of the flow of quality-weighted information may be maximized by a different regime, in which all replying is anonymous, but only if the spillover is considerable.

One reason why high quality is promoted by making it costly to participate is simply that charging is a means of inhibiting quality pollution. Low quality senders of information know that their participation will reduce average quality, and so the attention of their 'audience' (the group). If they have to pay to send, but only if they have to, will they therefore think twice about doing so. Another reason is to be found in the insight of signalling theory [10,11] that if a signal of a desirable property is cheap (enough) to send for those who have the property, and costly (enough) for those who do not, then the signal is a 'credible' sign of the property. In an electronic group, the very act of replying can, for appropriate cost structures, function in this way as a credible sign of high quality. What produces the differential cost or 'crossover' effect is a 'repeat-purchase' benefit which accrues only to high quality information providers (in this the mechanism is like those in [1,7]). A high quality sender may be able to afford an initial cost or membership fee where a low quality one cannot, because the *net* cost to him is reduced by this repeat-purchase benefit: if and when a receiver gets a message from him later she will remember her previous experiences of him, which are likely to have been favorable, and so be more likely to pay attention to it. Evidently this crossover mechanism can operate only in deonymous arenas.

In the next section we present a game-theoretic model of electronic information group interaction, and in section 3 we explore its equilibria in a range of styles of group. Section 4 uses a simple measure of the social benefit from the information that flows in the group to compare, very tentatively, the merits of these styles for the fostering of democracy. The analysis in section 3 is quite informal (details of proofs being relegated to the appendix); nevertheless, it is essentially mathematical, and might be skipped by those with no taste for such things.

## 2. The model

We aim to display conditions under which a high quality of information is produced in Internet discussion groups. The term 'information' is intended to be broadly construed, to cover opinions and conversation generally (including, for example, jokes) as well as purely factual information. We assume, however, that there is some degree of verifiability: it is possible to tell whether a particular piece of information is of high or

low quality, but only at some cost, for example, the risk of acting on it without knowing whether it is true. In this sense, information resembles an ‘experience good’ in the sense of Nelson [8]. We consider the implications of various different pricing structures for Internet discussion groups, and also the question of whether it is desirable to allow anonymous postings.

We model Internet interaction as a two-person Bayesian game (see, e.g., [5]). One of the players, the *questioner* (Q), posts a message, and the other, the *replier* (R) replies; the questioner can choose to absorb the reply, or else ignore it, and gets a positive payoff from absorbing a *good* reply, a negative payoff from absorbing a *bad* reply, and zero otherwise. The game is then repeated. The replier has private information about his type, HIGH or LOW, at the start of the game. HIGH quality repliers are more likely to produce good replies. We consider two possible arenas in which the game may take place. In the first, *deonymous* arena, the replier must mark himself in some indelible way, and the poster recognizes that she is dealing with the same person in round two of the game as in round one. (Note that ‘marking’ does not necessarily reveal off-line identity – cf. the *userid*s of the WELL.) In the second, *anonymous* arena, there is no marking, and she does not. (In Usenet, although posters choose whether they wish to mark (adopt a recognizable id) or not, in fact most groups have a convention for either anonymity or deonymity, and those abusing the convention tend not to be treated seriously.) We examine various different cost structures in each case.

The move order of the game is as follows:

1. Nature determines the type of the R: HIGH with probability  $q$ , LOW with probability  $1 - q$ , observed by R but not Q.
  2. R decides whether to join the group, incurring a cost  $c_j > 0$ , or not, incurring no cost.
  3. If he joined, R decides whether to reply, incurring a cost  $c_p > 0$ , or not, incurring no cost.
  4. Nature determines whether the reply is good or bad.
  5. Q decides whether to absorb or not, unobserved by R; if she absorbs, she observes whether the reply is good or bad, though R does not. R gets a benefit of  $i$ , and Q gets a benefit of  $g$  if the message is good, or suffers a loss of  $b$  if the message is bad.
- 6 – 8. Stages 3–5 are repeated.

The game trees for the anonymous and deonymous arenas are shown in figures 1 and 2 below. In both cases, the original move by Nature is omitted. Also omitted is R’s strategy of joining the group, and then failing to reply in either round. This is either dominated by (if  $c_j > 0$ ), or equivalent to (if  $c_j = 0$ ) not joining, and plays no role in the analysis. A few sample payoffs only are given, to avoid cluttering the diagrams. The labelling of the moves is explained in more detail later.

Although formally R is just one player, who is HIGH or LOW with probabilities  $q$  and  $1 - q$ , this may be considered a technical device for analyzing a population of

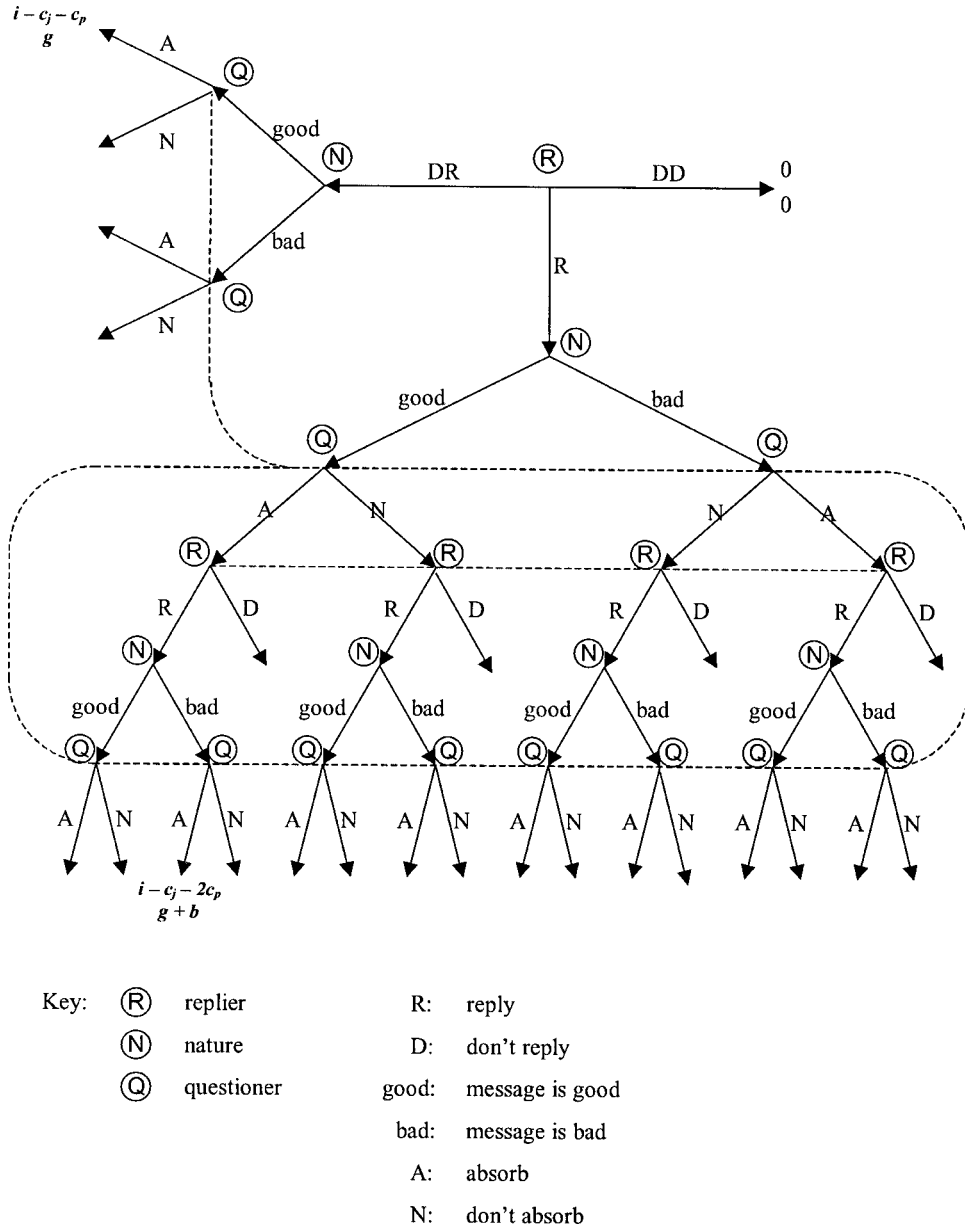


Figure 1. The anonymous arena.

repliers of whom a fraction  $q$  are HIGH and a fraction  $1 - q$  LOW. Similarly, one can interpret the mixed strategies which figure in the following analysis in terms of frequencies: for example, if in the model we find that a LOW type plays R and D in round 1 with particular probabilities, this means that some LOWs in the population play R and some play D, in the corresponding fractions.



We use the notation  $u_h$  and  $u_l$  to refer to Q's expected utility of absorbing a message from HIGH and LOW type R respectively.<sup>3</sup> That is,

$$u_h = hg + h'b, \quad u_l = lg + l'b.$$

In addition, we impose two restrictions on the parameter values, the point of which we shall explain shortly. First, we assume that the values of  $g$  and  $b$  are such that if, in equilibrium, *both* HIGH and LOW types always reply in both rounds, Q will not absorb either message. The condition required for this to be so is slightly different in the two arenas, since no updating of beliefs about R's type is possible in the anonymous arena. In the deonymous arena, on the other hand, Q's move in the second round can be contingent on whether she receives a good or bad reply in the first. We identify the condition for the deonymous arena first.

Consider the following strategy for Q: absorb in the first round, and absorb in the second round if and only if a good reply was received in the first. Suppose that this strategy yields strictly negative expected utility:

$$q(1+h)u_h + q'(1+l)u_l < 0. \quad (\text{A1})$$

Unless this strategy yields positive expected utility, no strategy involving absorption in either round will;<sup>4</sup> hence (A1) is the condition we seek.

The corresponding condition for the anonymous arena is simply

$$qu_h + q'u_l < 0. \quad (\text{A1}')$$

Note that (A1) implies (A1') (as shown in note 4).

Secondly, we assume that if *only* HIGH types send in each round, Q will absorb both messages:

$$2u_h > 0. \quad (\text{A2})$$

This condition is the same in the two arenas, since (*ex hypothesi*) there is no uncertainty about R's type.

### 2.1. Comments

In this simple model, the replier receives a fixed payoff  $i$ , whenever his message is absorbed, regardless of whether it turns out to be good or bad. This is supposed to represent the pleasure or satisfaction he gets from influencing another in some way, or from disseminating his opinions, or just from being 'visible'. The operation of such motives has been suggested elsewhere [4]. They should be distinguished from the altruistic

<sup>3</sup> In addition, we use  $x'$  as shorthand for  $1-x$ . Thus  $h' = 1-h$ ,  $l' = 1-l$ , etc.

<sup>4</sup>  $(qh/(qh+q'l))u_h + (q'l/(qh+q'l))u_l > qu_h + q'u_l$  since  $h > l$  and  $g > b$ . Thus it follows from (A1) that  $qu_h + q'u_l$ , the expected utility from absorbing in one round only, is less than zero. Similarly,  $(qh/(qh+q'l))u_h + (q'l/(qh+q'l))u_l > (qh'/(qh'+q'l'))u_h + (q'l'/(qh'+q'l'))u_l$ . Hence (unconditional) absorption in both rounds also yields negative expected utility.

motive of giving others the benefit of your knowledge (see, for example, [3]), since in this case the payoff to the replier depends on his own quality, which he knows, while  $i$  is independent of quality. We ignore, too, that people may sometimes reply to a posting for the sake of reciprocation by the poster in the future.

The two rounds of the model give two ‘opportunities to reply’ to a replier. We think of a typical replier as deciding first of all whether or not to join the group, and if he does join, having various opportunities to reply to questions posted to the group. We limit these further opportunities to two per questioner only for the sake of simplicity. The costs  $c_j$  and  $c_p$  are intended to represent the costs of joining the group in order to be able to participate by sending a reply, and the cost of each participation. They may be zero. We consider a variety of patterns for  $c_j$  and  $c_p$ , which broadly correspond to the charging policies of actual systems. For example,  $c_j = 0$ ,  $c_p > 0$  gives a WELL-like or AOL-like structure and  $c_j = c_p = 0$  a USENET-like structure. That there are no analogous costs for the questioner is without loss of generality – the payoffs  $g$  and  $b$  are taken to be net of any such costs.

The precise interpretation of the act of absorption will depend on the type of information being considered, and hence on the subject matter of the discussion group. Certain messages, such as jokes or mathematical proofs, are easy to verify, and in these cases  $b$  will represent merely the cost of reading the message. But other messages, such as advice about which laptop to buy, where to go on holiday, or which political causes to support, may be verifiable only with the passage of time – for example, by acting on them. In these cases information is a full ‘experience good’, and  $b$  may be very high.

The assumptions (A1) and (A2) do severely limit the number of discussion groups to which the model is applicable, but this is intentional. The focus of the analysis is the quality of information produced in discussion groups, and it is when (A1) and (A2) are jointly satisfied that this becomes a particularly important issue. If (A1) is not satisfied, all information (some of it good, some of it bad) will be absorbed whatever the cost structure of the group. Thus (A1) rules out the case of *idle chat*, where the information content of the messages is of no particular consequence to the parties involved; rather, it is from the act of discussion itself that the participants derive utility. Hence  $b$  is small or perhaps even positive. Examples include discussions about sport and television serials. (A1) also rules out groups whose subject matter is such that the information exchanged is less of an experience good: its quality is easily verifiable. Algebra problems and jokes fall into this category. Again  $b$  will be small, since poor quality information is recognizable as such before it can do any harm.<sup>5</sup> If (A2) is not satisfied, on the other hand, information will never be absorbed, even if the questioner is certain that the replier is a HIGH type. Here, the information sought is of such a sensitive nature and the risks

<sup>5</sup> It is fully consistent with the results of our model that the majority of USENET groups are of one or other of these two types. Indeed, our model predicts that, if posting is free (as it is in USENET), no information will be exchanged if (A1) is satisfied, and hence the group will cease to be active. Thus successful USENET groups should be ones for which (A1) fails (cf. [rec.sport.tennis](#); [alt.tv.star-trek.voyager](#); [alt.algebra.help](#); [rec.humor](#)).

if is it of poor quality are so high that discussion groups are not a suitable medium for its exchange: even the best informed are not well informed enough. Medical and financial advice are good examples of this type of information. It is the ‘in-between’ cases, when both (A1) and (A2) are satisfied, where the structure of the group has a role to play in affecting quality of information.

Clearly, our two-round two-person model is a drastic simplification of actual discussion group interaction. We view our model as a *snapshot* of a more general model in which there are many questioners and many repliers, and each replier gets precisely two opportunities to reply to each questioner. Within these parameters, in each round chance decides who gets the opportunity to reply to a given question. Thus for a questioner who receives a reply from an unrecognised replier, the probability that this is someone taking his first opportunity to reply to her, rather than his second opportunity after passing at his first, depends not only on the strategies of the repliers but also on the current composition of the group. In particular, for a given questioner, call the repliers who have not yet had an opportunity to reply to that questioner *new*, and those who have *old*. Then as long as the group size is fairly constant and large, in any round the expected number of new repliers and old replier will be approximately the same.<sup>6</sup>

Of course, even this general model is a simplification of real-life discussion group interaction. Many pairs may interact only once, or more than twice. Nevertheless, as long as there is a possibility of re-meets, some reputation effect will come into play, and the second period is intended to pick up this effect. The other simplification is that each interaction is assumed to involve only two people. In practice, many people give responses to any given question, with varying degrees of interdependence. In such a setting a questioner has ways of judging the quality of a reply before having to decide whether to act on it, for example, by comparing it with other replies; and the influence

<sup>6</sup> To make this statement more precise, consider a group containing precisely  $N$  potential repliers to a given questioner. In any round, the group can be in one of  $N + 1$  states  $s = 0, 1, \dots, N$ , where  $s$  is the number of new repliers to the questioner. Whenever a new replier is picked, he becomes old, and whenever an old replier is picked, he drops out of the group and is replaced by a new one. Thus the group dynamics can be modelled as a Markov process with matrix of transition probabilities given by

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots \\ \frac{1}{N} & 0 & \frac{N-1}{N} & 0 & \\ 0 & \frac{2}{N} & 0 & \frac{N-2}{N} & \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{pmatrix},$$

where the rows represent the state in period  $t$ , and the columns the state in period  $t + 1$ . The system is irreducible, and so converges to a limiting probability distribution  $p^*$  from any given state, where  $p^*$  is given by

$$p^*A = p^*.$$

It is easy to check that  $p^*$  is simply the symmetric binomial distribution. Thus  $s$  has mean of  $N/2$ , as required.

payoff is diluted if a replier thinks he is one of many. Analysis of the interaction between multiple responders is beyond the scope of this model.

### 3. Equilibrium analysis

Clearly, if  $c_j = c_p = 0$ , there can be no equilibrium in which any messages are absorbed. For if there is any chance that a message will be absorbed, both types of  $R$  will reply, yielding a contradiction with (A1). We now compare two possible nonzero pricing structures, first for the anonymous and then for the deonymous arena, with the aim of finding out which generates the higher quality of information.

#### 3.1. The anonymous arena

In the anonymous arena, the poster is unable to identify repliers from their previous postings, if any. Formally, the game is one of imperfect recall (see figure 1), with the poster unsure whether she is at the first round or the second round (that is, whether she has met the replier before or not). Thus there is a single information set at which  $Q$  is on move, and so there are only two pure strategies available to her: *absorb* ( $A$ ) or *not absorb* ( $N$ ). In other words, she is unable to condition her second round behavior on whether she received a good or bad response to the first question. We assume that, *a priori*, she assigns equal probabilities to each round.<sup>7</sup> Both types of  $R$ <sup>8</sup> have four pure strategies,  $RR$ ,  $RD$ ,  $DR$ , and  $DD$ , where  $R$  denotes ‘reply’, and  $D$  denotes ‘don’t reply’; ‘not join’ is labelled  $DD$  for notational convenience.<sup>9</sup>

In the following analysis, we restrict our attention to equilibria in which HIGH  $R$  plays  $RR$  (with probability one).<sup>10</sup> This immediately puts an upper limit on the total costs of joining and participation in the group:  $c_j + 2c_p \leq 2i$ . If this inequality is not satisfied, it will never be worthwhile for  $R$  to reply, since the costs of doing so exceed the potential influence payoff even if all messages are absorbed. We consider the two pricing structures in turn. The first of these, case 1, assumes that there is a cost for joining the group, but thereafter participation is free. The second, case 2, assumes that it is free to join, but there is a fixed charge for each message sent.

<sup>7</sup> This corresponds to an assumption of constant group size (see note 6).

<sup>8</sup> Since formally  $R$  is just one player, his choice of strategy should be conditioned on whether Nature has chosen HIGH or LOW. The abuse of notation here is convenient and harmless.

<sup>9</sup> We are assuming that  $R$  does not find out whether his first message was absorbed or not, nor what its quality was, by the time he has to make his decision about whether or not to reply for a second time. This assumption simplifies the model (since  $R$  is unable to condition his second round action on  $Q$ 's first round behavior), but is also realistic: acknowledgement of replies seems to be rare in electronic discussion groups. In any case, the more complex model, with additional conditional strategies, would enhance reputation effects and increase the difference between the two arenas and cost structures, strengthening our conclusions.

<sup>10</sup> See the appendix for possible justifications of this restriction.

### 3.1.1. Case 1: $c_j > 0, c_p = 0$

We have assumed that HIGH R is playing *RR*. For this to be a best response, Q must be playing *A* with strictly positive probability, in which case it is easy to see that *RR* for LOW R does strictly better than *RD* and *DR*. But LOW R cannot be playing only *RR*, since if this were the case (A1) implies that there would be no absorption of messages by Q; nor can he be playing only *DD*, unless  $c_j = 2i$ , since (A2) then implies that all messages would be absorbed.<sup>11</sup> So we suppose he plays a strict mix between *RR* and *DD*, playing *RR* with probability  $\tau$ . And for LOW R to be willing to mix<sup>12</sup> between *RR* and *DD*, it must be the case that  $c_j = 2\sigma i$ , where  $\sigma$  is the probability with which Q plays *A*. Similarly, for Q to be willing to mix between *A* and *N*, we require,

$$qu_h + q'\tau u_l = 0,$$

or

$$\tau = Z,$$

where

$$Z = -\frac{q}{q'} \frac{u_h}{u_l}.$$

(A1) and (A2) imply that  $0 < \tau < 1$ , and  $\sigma$  will be strictly between zero and one as long as  $0 < c_j/i < 2$ . We call this type of equilibrium *E1*.

### 3.1.2. Case 2: $c_j = 0, c_p > 0$

As in case 1, LOW R cannot be playing only *RR*, or only *DD* (unless  $c_p = i$ ). A necessary and sufficient condition for him to be willing to play *any* strictly mixed strategy is that  $c_p = \sigma i$ , where  $\sigma$  is as denned above, since Q's behavior in both rounds is exactly the same. As long as  $0 < c_p/i < 1$ ,  $\sigma$  lies strictly between zero and one, and Q is playing a strict mix between *A* and *N*. We characterize LOW R's strategy with two parameters  $\lambda$  and  $\mu$ , where  $\lambda$  is the probability that he sends (at least) one message, and  $\mu$  is the probability that he sends two. So he plays *RR* with probability  $\mu$ , *RD* or *DR* with probability  $\lambda - \mu$ , and *DD* with probability  $\lambda'$ . Note that  $0 \leq \mu \leq \lambda \leq 1$ ,  $\lambda > 0$  and  $\mu < 1$ . We do not need to assign separate weights to *RD* and *DR* since they are indistinguishable as far as Q is concerned, and payoff-equivalent for R. Then for Q to be willing to mix, we require:

$$2qu_h + q'(\lambda + \mu)u_l = 0,$$

or

$$\lambda + \mu = 2Z.$$

<sup>11</sup> Here and henceforth we give results only for 'generic' cases of the game, i.e., we exclude cases that require 'fluke' parameter values which lie in a vanishingly small subset of possible values.

<sup>12</sup> A basic property of equilibria in which a player mixes between the pure strategies  $s_1$  and  $s_2$  is that her expected payoffs from  $s_1$  and  $s_2$  are equal.

Given (A1) and (A2) we can always find values of  $\lambda$  and  $\mu$  such that this condition is satisfied. We call this type of equilibrium  $E2$ .

Before continuing to the analysis of the deonymous arena, we shall summarize the results of this section. Notice that the two types of equilibria look very similar: in both cases, LOW R is mixing between replying and not replying in just the correct proportions to make Q indifferent between absorption and ignoring. Whether LOW R always replies to precisely one message ( $RD$  or  $DR$ ), or replies to both ( $RR$ ) half the time and to neither ( $DD$ ) the rest, does not matter as far as Q is concerned, since she is unable to distinguish between first and second messages; all that is important to her is the *average quality of information*. Since in all the equilibria we consider, HIGH R replies all the time, a suitable index of average quality would be the degree of *non-participation* by LOW R, or

$$I = 1 - \text{prob}(\text{LOW R replies to a given question}),$$

where equal weight is given to both rounds of the game. Note that  $I$  ranges between 0 (when LOW R replies all the time), and 1 (when LOW R never replies). Computing average quality for the equilibria  $E1$ <sup>13</sup> and  $E2$ ,<sup>14</sup> we find that in both cases  $I = 1 - Z$ . This figure does not differ from case 1 to case 2, nor does it vary with the *level* of costs (i.e., the particular values of  $c_j$  and  $c_p$ ).

It is instructive to consider what prevents average quality from falling to zero. In a scenario in which individuals are not recognized in the event of any re-meets, we might think that LOW R's would masquerade as HIGH R's, and mimic their behavior, replying all the time. But although no reputation effect at the individual level is possible, there is a group-level reputation effect at work in the anonymous arena: if too many LOW R's start replying, and the average quality of information in the group falls below the critical equilibrium level, Q's will stop absorbing any messages and it will cease to be worthwhile for *any* R to reply. This is very different from the individual-level reputation effect we shall see emerging in the anonymous arena.<sup>15</sup>

The other important feature of the two equilibria derives from the mixing conditions for LOW R. For LOW R to be indifferent between joining the group and not, and thus willing to randomize between his various strategies, it must be the case that as costs rise, so do does the equilibrium level of absorption by Q. In the anonymous arena this is illustrated starkly, with the percentage of messages absorbed by Q rising linearly from

<sup>13</sup> Here,  $\text{prob}(\text{LOW R replies to a given question})$  is simply  $\tau$ , so  $I = 1 - \tau$ .

<sup>14</sup> If LOW R plays  $RD$  or  $DR$  then he replies to a given question with probability  $1/2$ . Thus  $I = 1 - (1/2)(\lambda - \mu) - \mu$ .

<sup>15</sup> Note that the absence of any reputation effect at the individual level means that there is no 'single-crossing property' to distinguish HIGH and LOW R's. This creates a certain indeterminacy in the equilibria: in general, the expected payoffs of the two types will be the same, so just as LOW R is indifferent between  $RR$  and  $DD$ , so too is HIGH R. But this is nothing other than the indeterminacy problem that is present whenever we consider equilibria in mixed strategies (which we do in our analysis of both arenas). Similar indeterminacies are ubiquitous in economic theory.

zero to one hundred as total costs<sup>16</sup> rise from 0 to  $2i$ , the highest possible influence pay-off if all of one's messages are absorbed. But this intuition is robust, and the same effect is also present in the deonymous arena.

### 3.2. *The deonymous arena*

In the deonymous arena, posters of messages have to mark themselves in a unique and indelible manner, and are thus recognized in the event of repeat transactions. As far as the game is concerned, this means that Q, if she received and absorbed a reply in the first round, might gain by conditioning her second-round action on whether she received good or bad information. There are, then, six possible pure strategies available to her which we must consider. We label these  $AA$ ,  $AG$ ,  $AB$ ,  $AN$ ,  $NA$ , and  $NN$ , where  $A$  denotes 'absorb',  $N$  denotes 'ignore', and  $G$  and  $B$  denote absorption conditional on respectively good and bad replies in the first round. Note that the first letter of each label specifies what she does at her first *information set*, i.e., in response to R's first reply. This might be in the first round of the game, or in the second round, if he declined his first opportunity to reply. Q is unable to distinguish between these two situations, never having met R before in either case. The same four pure strategies are available to both types of R as before, and again we restrict our attention to equilibria in which HIGH R replies in both rounds ( $RR$ ). This enables us to rule out  $AN$  and  $AB$  as possible equilibrium strategies for Q. The proof is immediate from fact 1:

**Fact 1.** In any equilibrium in which HIGH R plays  $RR$  with probability one, whenever Q absorbs a good message in the first round she will also absorb in the second round.

This is proved in the appendix. We now consider the two pricing structures in more detail.

#### 3.2.1. *Case 1: $c_j > 0$ , $c_p = 0$*

We have assumed that HIGH R plays  $RR$ . The following fact tells us that we have only to consider a very narrow set of strategies for LOW R.

**Fact 2.** In any equilibrium in which HIGH R plays  $RR$ , LOW R plays a strict mix between  $RR$  and  $DD$ .

The intuition behind the proof (given in the appendix) is straightforward: any strategy involving  $RD$  will be dominated by  $RR$  if there is any chance of absorption in the second round, since  $c_p$ , the cost of participating once one has joined the group, is zero. And the pure strategies  $RR$  and  $DD$  are ruled out in equilibrium since (A1) and (A2) imply that there would be no absorption and complete absorption respectively, in which case the other strategy would do better.

<sup>16</sup> I.e., the costs of joining and full participation:  $c_j + 2c_p$ .

This in turn implies that we only have to consider a small subset of Q's strategy space:

**Fact 3.** In any equilibrium in which HIGH R plays *RR*, Q plays either a strict mix between *AG* and *NN* or a strict mix between *AG* and *AA*.

Again, the proof is given in the appendix. We consider these two mixed strategies in turn. In both cases, we assume that LOW R plays *RR* with probability  $\tau$ , and *DD* with probability  $\tau'$ , where  $0 < \tau < 1$  (from fact 2).

First, suppose that Q is mixing between *AG* (with probability  $\sigma$ ) and *NN* (with probability  $\sigma'$ ). The mixing conditions for LOW R and Q respectively are:

$$c_j = i\sigma(1+l), \quad q(1+h)u_h + q'\tau(1+l)u_l = 0.$$

(A1) and (A2) imply that the Q-condition is always satisfied for some  $\tau \in (0, 1)$ , and the LOW R-mixing condition can be satisfied for some  $\sigma \in (0, 1)$  whenever  $0 < c_j/i < 1+l$ . Solving for  $\tau$ , we obtain

$$\tau = \frac{(1+h)}{(1+l)}Z.$$

It is easily verified that, for this value of  $\tau$ , *AA* does strictly worse. We call this type of equilibrium *E3*.

Suppose now that Q is mixing instead between *AG* (with probability  $\sigma$ ) and *AA* (with probability  $\sigma'$ ). In this case, the mixing conditions for LOW R and Q are:

$$c_j = i(2 - \sigma l'), \quad q(1+h)u_h + q'\tau(1+l)u_l = 2qu_h + 2q'\tau u_l.$$

Again, (A1) and (A2) imply that the Q-condition is always satisfied for some  $\tau \in (0, 1)$ . The LOW R-mixing condition can be satisfied as long as  $1+l < c_j/i < 2$ . Solving for  $\tau$ , we obtain

$$\tau = \frac{h'}{l'}Z.$$

At this value of  $\tau$ , *AG* and *AA* do better than *NN*. Such an equilibrium we label *E4*.

Since  $0 \leq l < h \leq 1$ , the value of  $\tau$  is larger in *E3* than in *E4*. The intuition behind this result is fairly straightforward. In *E3*, Q is mixing between *AG* and *NN*, and so her expected utility must be zero; in *E4*, Q is mixing between *AA* and *AG*, and so her expected utility from absorbing in round 2 conditional on having received a bad message in round 1 must be zero; hence her expected utility from absorbing after a good message has been received must be strictly positive; but then so is that from unconditional absorbing in round 2, and so too from doing so in round 1. Thus the expected utility from *AA*, and therefore also from *AG*, is strictly positive. But if Q is receiving a lower expected utility in *E3* than in *E4*, this can only be because LOW R is participating with higher probability, since HIGH R plays the same strategy in both.

Table 1

Range of $c_j/i$	Type of equilibrium	$\tau$	$I$
$0 < \frac{c_j}{i} < 1+l$	$E3$	$\frac{1+h}{1+l}Z$	$1 - \frac{1+h}{1+l}Z$
$1+l < \frac{c_j}{i} < 2$	$E4$	$\frac{1-h}{1-l}Z$	$1 - \frac{1-h}{1-l}Z$

We summarize the results of this section in table 1. It is interesting to note that, unlike in the anonymous arena, the value of  $c_j$  does affect the average quality of information, but only in as much as it affects the existence of the two types of equilibrium. Although average quality does rise with  $c_j$ , it does so discontinuously. Also worthy of note is that we now have an individual-level reputation effect, as well as the group-level effect present in the anonymous arena. The play of the strategy  $AG$  by  $Q$  creates a ‘single-crossing property’, whereby the benefit to HIGH R from playing  $RR$  is higher than that to LOW R, since he is more likely to have sent a good message in the first round, and hence more likely to have his second message absorbed.

### 3.2.2. Case 2: $c_j = 0, c_p > 0$

As before, we search for equilibria in which HIGH types reply in both rounds. We can again restrict our attention to a particular subset of LOW R’s strategy space:

**Fact 4.** In any equilibrium in which HIGH R plays  $RR$ , LOW R plays a mixed strategy which gives strictly positive weight to  $RR$  and to at least one of  $RD$  and  $DR$ .

The proof is given in the appendix.

Adopting the same notation as for case 2 in the anonymous arena, we assume that LOW R plays  $RR$  with probability  $\mu$  and  $DD$  with probability  $\lambda'$  (so  $\lambda$  is the probability that he sends (at least) one message, and  $\mu$  is the probability that he sends two).<sup>17</sup> Note that  $0 < \mu < \lambda \leq 1$ , since both  $RR$  and at least one of  $RD$  and  $DR$  are played with strictly positive probability.

**Fact 5.** In any equilibrium in which HIGH R plays  $RR$ ,  $Q$  plays a strict mix between (i)  $AG$  and  $NN$ ; (ii)  $AG$  and  $NA$ ; or (iii)  $AG$ ,  $AA$  and  $NA$ .

The proof is given in the appendix. We now consider each of these three cases in turn. First, we note that against HIGH and LOW R’s strategies, the expected utilities from  $Q$ ’s pure strategies are:

$$u_{AA} = 2qu_h + q'(\lambda + \mu)u_l,$$

$$u_{AG} = q(1+h)u_h + q'(\lambda + l\mu)u_l,$$

<sup>17</sup> As before,  $DR$  and  $RD$  are interchangeable since they are indistinguishable to  $Q$  at the point at which she has to move and they yield the same payoff to R.

$$\begin{aligned} u_{NA} &= qu_h + q'\mu u_l, \\ u_{NN} &= 0. \end{aligned}$$

1. *AG, NN.* Suppose that Q plays *AG* with probability  $\sigma$ , and *NN* with probability  $\sigma'$ , where  $0 < \sigma < 1$ . Given that strictly positive weight is placed on *AG*, the expected utility for R from sending a second reply must be strictly lower than from sending a first. Hence, if LOW R is indifferent between *RR*, *RD*, and *DR*, he must strictly prefer all three to *DD*. This implies that  $\lambda = 1$ . For Q to be willing to mix, her expected utility from playing *AG* must be equal to zero:

$$u_{AG} = q(1+h)u_h + q'(\lambda + l\mu)u_l = 0,$$

whence

$$\mu = \frac{1+h}{l}Z - \frac{1}{l}.$$

We know that  $\mu < 1$ , since if  $\mu = 1$ ,  $u_{AG} < 0$  from (A1). But we need a condition to ensure that  $\mu > 0$ . This is equivalent to  $Z(1+h) > 1$ . In addition, we require that *AG* and *NN* do at least as well as *AA* and *NA*. It is easily verified that a necessary and sufficient condition for this is that  $Z(1+h-1) \leq 1$ . For LOW R to be willing to mix, we require  $c_p = i\sigma l$ . This condition also implies that LOW R will send a first message, since the cost of participation in both rounds is the same. It can be met, for appropriate choice of  $\sigma$ , as long as  $0 < c_p/i < l$ . Call this type of equilibrium *E5*.

2. *AG, NA.* Suppose that Q plays *AG* with probability  $\sigma$  and *NA* with probability  $\sigma'$ , where  $0 < \sigma < 1$ . For Q to be indifferent between *AG* and *NA*, we require:

$$u_{AG} = q(1+h)u_h + q'(\lambda + l\mu)u_l = qu_h + q'\mu u_l = u_{NA},$$

whence

$$\frac{\lambda - l'\mu}{h} = Z.$$

For LOW R to be indifferent between *RR*, *RD*, and *DR* we must have

$$c_p = i(l + \sigma'l'),$$

whence

$$\sigma = \frac{1 - c_p/i}{l'}.$$

For LOW R to be playing *DD* with strictly positive probability as well, it must be the case that  $c_p = \sigma i$ . This is only satisfied for non-generic parameter values (precisely, if  $c_p/i = 1/(2-l)$ ). Hence we shall assume that no weight is placed on *DD*, and so  $\lambda = 1$ . Solving for  $\mu$  then gives us:

$$\mu = \frac{1}{l'} - \frac{h}{l'}Z.$$

It is easy to show that  $0 < \mu < 1$ , as required. But we must also check that *AG* and *NA* do better than *NN* and *AA*. Given that  $\lambda = 1$ , we know from (A1) that *NA* does better than *AA*; and a necessary and sufficient condition for *NA* to do better than *NN* is that  $Z(1 + h - l) < 1$ . Finally, we require that  $\sigma > 0$ , for the mixing to be strict, and  $\sigma \geq c_p/i$  for *RR*, *RD*, and *DR* to do as well as *DD*. These conditions are met as long as  $l < c_p/i < 1/(2 - l)$ . We call this type of equilibrium *E6*.

3. *AG*, *AA*, *NA*. Mixing by *Q* requires  $u_{AG} = u_{AA} = u_{NA}$ . The first equality gives us  $\mu = (h'/l')Z$ , and the second  $\lambda = Z$ . These values satisfy  $0 < \mu < \lambda \leq 1$ , and imply that each strategy beats *NN*. Suppose then that *Q* plays *AG* with probability  $\phi$  and *NA* with probability  $\psi'$ . Then for *LOW R* to be indifferent between *RR*, *RD*, *DR*, and *DD*, it must be the case that  $c_p = (l + l'\phi')i$  and  $2c_p = (\psi + l + l'\phi')i$ . Hence,  $\psi = c_p/i$  and  $\phi = \psi^0/l^0$ . Since  $\psi > \phi$  if strictly positive weight is to be placed on *AA*, the second equation implies that  $\psi > 1/(2 - l)$ . So an equilibrium of this type, which we call *E7*, exists as long as  $1/(2 - l) < c_p/i < 1$ .

Table 2 summarizes these results. As in case 1, average quality rises as rising costs shift us from one equilibrium to the next. Note that the individual-level reputation effect is again present, since *Q* plays *AG* with positive probability. But *LOW R* now has a response to this strategy: he can play *RD* or *DR*, replying only once and then leaving the group before his reputation is sullied. This *hit-and-run* entry is ruled out in case 1 where messages are free to send once one has joined the group, and *LOW R* would rather not join at all unless there is a sufficiently high probability that both of his messages will be absorbed. In fact, the same is true in the anonymous arena, where we also observe play of *RD* and *DR* by *LOW R* in case 2 (pay-per-message) only. But there cost structure does not make a substantive difference. For in the anonymous arena, where she cannot recognize repliers from their previous postings, and so cannot distinguish first and second messages, these strategies are equivalent as far as *Q* is concerned to a fifty-fifty mixture of *RR* and *DD*; whereas in the deonymous arena they are very different. In particular, *Q* now has a strategy to combat hit-and-run entry: *NA*. Playing *NA*, she absorbs second messages only, thereby avoiding all those who play *RD* and *DR*. This makes hit-and-run entry less attractive, but as we can see from the analysis above, does not rule it out altogether.

Table 2

Range of $c_p/i$	Type of equilibrium	$\lambda$	$\mu$	$I$
$0 < \frac{c_p}{i} < l$	<i>E5</i>	1	$\frac{(1+h)Z-1}{l}$	$\frac{1+l-(1+h)Z}{2l}$
$l < \frac{c_p}{i} < \frac{1}{2-l}$	<i>E6</i>	1	$\frac{1-hZ}{l'}$	$\frac{hZ-l}{2l'}$
$\frac{1}{2-l} < \frac{c_p}{i} < 1$	<i>E7</i>	$Z$	$\frac{h'Z}{l'}$	$1 - \frac{(l'+h')Z}{2l'}$

In the final section, we compare the results from the two arenas and cost structures, and give some concluding remarks.

#### 4. Conclusions

We argued in the introduction that it is crucial for the proper functioning of democracy for there to be high quality information widely available to citizens. Accordingly, the main aim of the model was to determine how the cost structure of an electronic group may influence the quality of information produced. Of equal importance is how much of this information is actually taken in and acted on, or ‘absorbed’, to use the terminology above. An immediate measure of the welfare effect of this information transmission is the expected utility of  $Q$ . But since our concern is the enhancement of democracy, and not just the welfare of members of the group, it may be appropriate to weight the benefits and costs of good and bad information to take into account potential externalities if messages are passed on. If, as seems likely, good messages are more likely to be passed on than bad ones, the weight on  $g$  should be larger than that on  $b$ , and so without loss of generality we normalize and set the former equal to  $k > 1$  and the latter equal to 1. Our measure of democratic gain is then simply the expected utility of  $Q$  with  $g$  replaced by  $kg$ . We denote this by  $U_h$  or  $U_l$ , depending on whether the message was sent by a HIGH or LOW R (so  $U_h = hkg + h'b$  and  $U_l = lkg + l'b$ ). Depending on the size of  $k$ , it may be the case that only messages from HIGH Rs yield positive expected democratic gain (i.e.,  $U_l < 0 < U_h$ ), or that messages from both types are democratically beneficial (i.e.,  $0 < U_l < U_h$ ).

In the diagrams, we show by means of a numerical example how the different groups compare, first with respect to average quality of information produced (figure 3), and then with respect to democratic gain, when  $U_l < 0$  (figure 4) and when  $0 < U_l$  (figure 5). The parameter values we use are:  $q = q' = \frac{1}{2}$ ;  $h = \frac{3}{4}$ ,  $l = \frac{1}{4}$ ,  $g = 5$ ,  $b = -6$ .

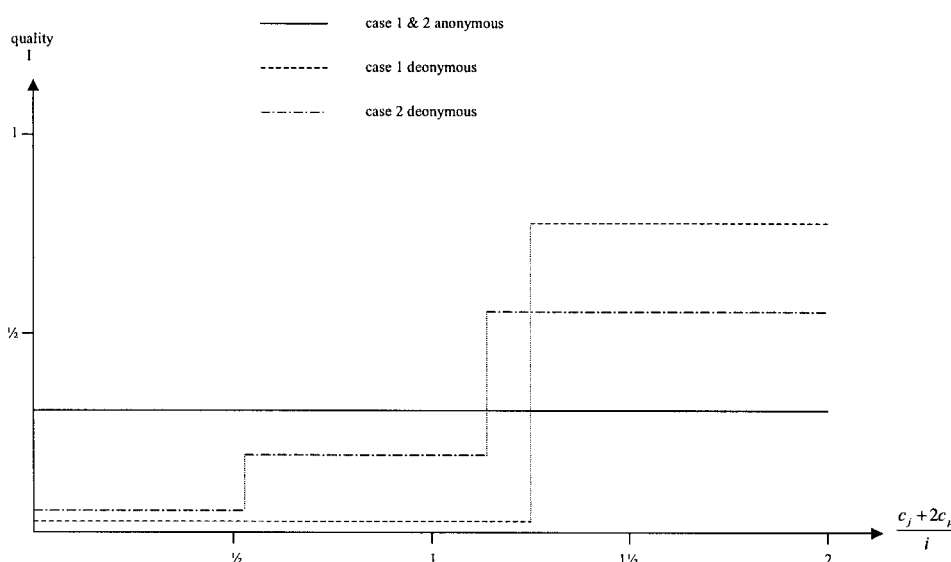


Figure 3. Average quality of information.

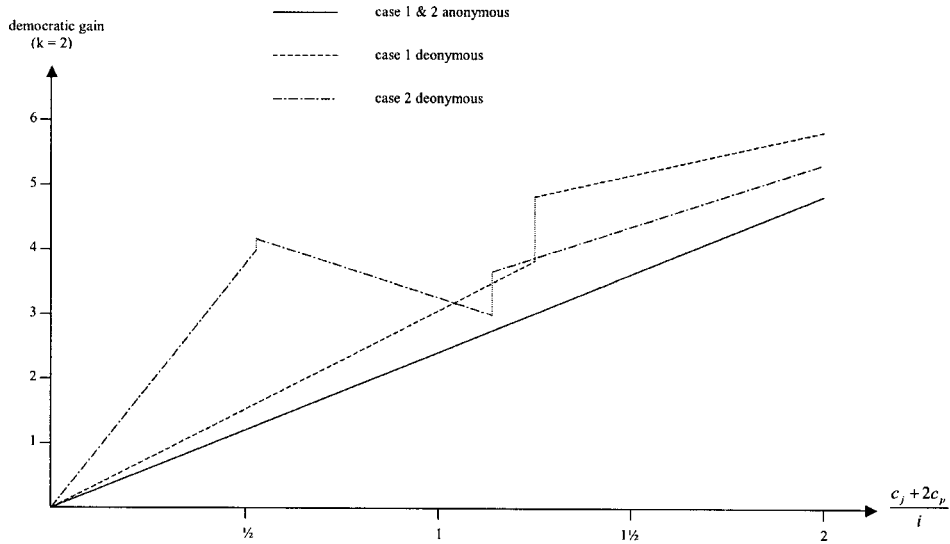


Figure 4. Democratic gain ( $k = 2$ ).

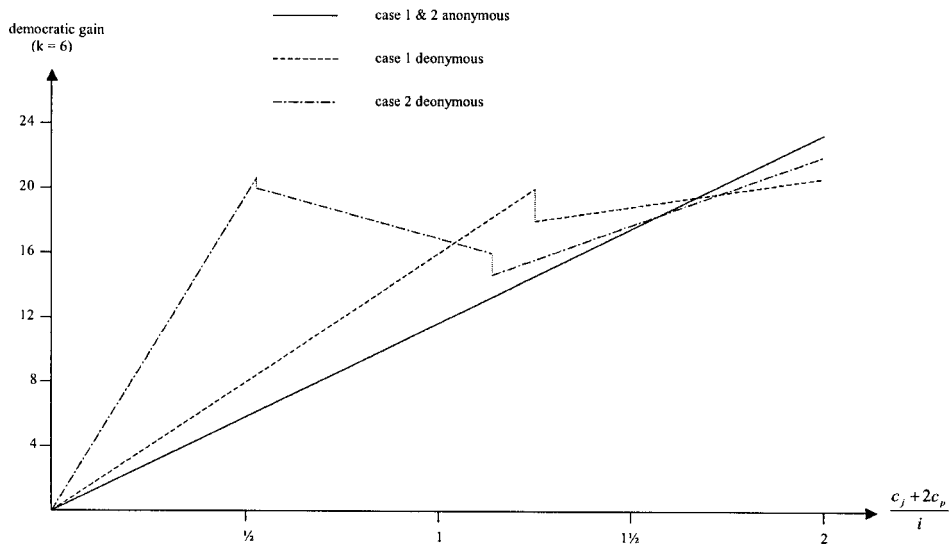


Figure 5. Democratic gain ( $k = 6$ ).

In figure 4 we take  $k = 2$ , and in figure 5,  $k = 6$ . As before, we use  $I$  as our index of the quality of information (recall that  $I = 1 - \text{prob}(\text{LOW R replies to a given question})$ ).

If we are free to choose the level of costs, the diagrams seem to suggest that, as far as the quality of information is concerned, and also from the point of view of democratic gain, case 1 in the deonymous arena comes out on top, as long as  $k$  is not too large. For large  $k$ , amount of information transmitted becomes more important relatively to quality,

and an anonymous regime is favoured. We should stress, however, that there exist other equilibria involving lower degrees of participation by HIGH R (see the appendix on ‘alternative equilibria’). If the choice of cost level influences whether or not one of these evolves (e.g., if high costs scare people away from the group), these policy conclusions may not hold.

## Appendix A.

### A.1. *Alternative equilibria*

In our equilibrium analysis above, we restricted our attention to equilibria in which HIGH R replied in both rounds, and showed that, for generic parameter values, there was a unique equilibrium for values of total costs ( $c_j + 2c_p$ ) between zero and two.<sup>18</sup> There are, however, alternative equilibria in which HIGH R plays other strategies. In particular, there is always an equilibrium in which both types of R play *DD*, and Q plays *NN*, with appropriate out-of-equilibrium beliefs (i.e., Q must believe that if she does receive a message, it is from LOW R with sufficiently high probability). We consider other possible equilibria for the deonymous and anonymous arenas in turn.

In the deonymous arena, all of these alternative equilibria involve inactivity in round 2 of the game (i.e., no messages are absorbed in that round). We present here a heuristic proof of this proposition. Start by noting that if there is to be any absorption in the second round, HIGH R must play *RR* with strictly positive probability. The only strategies for Q which involve absorption in the second round are *AA*, *NA*, *AG*, and *AB*. We can rule out *AB* at once, since it is strictly dominated by *AG*; we can also rule out *AA* or *NA* alone, except for non-generic parameter values, and any mix between just them: if Q is mixing between *AA* and *NA*, then both types of R must be playing either *RR* or *DD*. This implies that Q’s expected utility from absorbing is the same in round 1 as in round 2 (i.e., zero given that she is mixing), and hence *AG* must do better. So Q must play *AG* with some positive probability. If Q plays *AG* with positive probability, but not *AB*, then HIGH R must be getting higher expected utility in round 2 than LOW R. And we know this expected utility is zero, since HIGH R mixes between *RR* and either *RD* or *DD*. Thus, LOW R cannot be playing *RR*. But then *AA* does better than *AG*, which contradicts the argument.

In the anonymous arena, there is a continuum of alternative equilibria. Any reduced level of activity of HIGH R has to be matched by a correspondingly reduced level of LOW R, so that the average quality of information produced stays constant, and Q remains willing to mix between *A* and *N*. At the one end of the extreme HIGH R plays *RR* with probability one, and at the other both types of R play *DD* with probability one, with Q mixing between *A* and *N* in the correct proportion so that R is indifferent between replying and not.

Without any dynamics in the model, there is no strong argument to exclude any of these alternative equilibria. Work in progress [2] is analyzing various dynamic models

<sup>18</sup> We needed an additional condition to guarantee existence of equilibria of types *E5* and *E6*.

of group membership, and we thus hope to provide some answers to the equilibrium selection problem. There is a general consideration which makes high activity equilibria more plausible than low activity ones. In electronic groups of the kind we have been describing, if there is believed to be no activity by HIGH's, LOW R's achieve nothing by participating, since  $u_l$  is negative. This means that LOW R's could never get a group started, and that groups can only be founded by a sufficient proportion of HIGH's. Once this happens, we may expect to see invasion by parasitic LOW's (a decrease in  $q$ ), and an equilibrating decrease in absorption. Plausibly, this dynamic adjustment of absorption will only go as far as it needs to, that is, to that equilibrium among the new set of equilibria which has the highest level of absorption.

A proper theory to select among the multiple equilibria must wait on the spelling out of such dynamic stories as these. Meanwhile, as the main aim of the paper is to compare different electronic group structures, we feel justified in considering only the equilibria in which HIGH R plays  $RR$  as this provides some kind of level playing field. Nevertheless, in the light of this multiple equilibria problem, we should be wary of taking the results too seriously as policy ascriptions.

#### A.2. *Proofs*

Note that all of these proofs relate to games in the deonymous arena.

**Fact 1.** In any equilibrium in which HIGH R plays  $RR$  with probability one, whenever Q absorbs a good message in the first round she will absorb also in the second round.

*Proof.* Given HIGH R's strategy, the probability of R being HIGH in the second round conditional on a good message being sent in the first must be strictly greater than the probability of R being HIGH in the first round (unless that probability is one, in which case the conclusion follows immediately from (A2)). Hence, the expected utility of absorbing in the first round is strictly less than the (conditional) expected utility of absorbing in the second, and again the conclusion follows.  $\square$

**Fact 2.** In any equilibrium in which HIGH R plays  $RR$ , LOW R plays a strict mix between  $RR$  and  $DD$ .

*Proof.* HIGH R is playing  $RR$  by assumption. There can be no equilibrium in which LOW R chooses  $RR$  only, since by (A1) there would then be no absorption, and he would be better off not participating ( $DD$ ). Nor can there be an equilibrium in which he chooses  $RD$  or  $DR$  only: by (A2), Q would always absorb second messages from R (since they are certain to be from HIGH types), and hence it would be strictly better for LOW to reply in both rounds. Similarly, there cannot be an equilibrium in which he chooses just  $DD$ , unless  $c_j = 2i$ : by (A2), Q would absorb in both rounds, whatever message is received in the first, and so if it is worthwhile for HIGH R to reply, it must be worthwhile for LOW R also; so they must both be indifferent between  $RR$  and  $DD$ .

This leaves us with the eleven (strictly) mixed strategies. Given that we can rule out equilibria in which there is no absorption, fact 1 implies that second messages are absorbed with some strictly positive probability. This in turn enables us to rule out any mixes involving  $RD$  or  $DR$  – since the (conditional) cost of replying for a second time is zero, and the expected utility from doing so is strictly positive,  $RR$  does strictly better. Thus we have to consider only mixes between  $RR$  and  $DD$ .  $\square$

**Fact 3.** In any equilibrium in which HIGH R plays  $RR$ , Q plays either a strict mix between  $AG$  and  $NN$  or a strict mix between  $AG$  and  $AA$ .

*Proof.* Recall from fact 2 that LOW R must be playing a strict mix between  $RR$  and  $DD$ . It is easy to verify  $NA$  is not optimal given R's strategy: if the expected utility of  $NA$  is negative,  $NN$  does better, and if not,  $AG$  does better. This leaves us with just  $AA$ ,  $AG$ , and  $NN$ . Equilibria in which Q plays  $AA$  only exist only non-generically (for LOW R to be mixing, it must be the case that  $c_j = 2i$ ), as do equilibria in which she plays  $AG$  only (mixing by LOW R implies that  $c_j = i(1 + l)$ ); and equilibria in which she plays  $NN$  only cannot exist at all, since both types of R would be strictly better off not replying at all. It remains to consider her possible (strictly) mixed strategies. Any mix involving both  $AA$  and  $NN$  cannot be an equilibrium: for Q to be willing to mix, the two strategies would have to yield the same expected utility, i.e., zero, and then we can show that  $AG$  does strictly better. This leaves only mixes between  $AG$  and  $NN$ , and between  $AG$  and  $AA$ .  $\square$

**Fact 4.** In any equilibrium in which HIGH R plays  $RR$ , LOW R plays a mixed strategy which gives strictly positive weight to  $RR$  and to at least one of  $RD$  and  $DR$ .

*Proof.* We start by ruling out the pure strategies. If LOW R plays  $RR$ , there will be no absorption, and if he plays  $RD$  or  $DR$ , Q will absorb second messages only. In both cases  $DD$  would be better. If he plays  $DD$ , Q will absorb in both rounds, and this can be an equilibrium only if  $c_p = i$ . So we are left with the eleven strictly mixed strategies.

Consider first mixes between  $RR$  and  $DD$  only. As we noted in the proof of fact 3, this enables us to rule out  $NA$  for Q, leaving us with  $AA$ ,  $AG$  and  $NN$ . Given that these are the only possible responses, we can rule out equilibria in which any weight is placed on  $AG$ : if it were, LOW R's expected utility from replying for the second time would be strictly less than from replying the first, and hence less than zero, since total expected utility must equal zero; so  $RD$  or  $DR$  would be better than  $DD$ . Equilibria in which Q plays  $AA$  with probability one exist only non-generically (when  $c_p = i$ ), and equilibria in which she plays  $NN$  only cannot exist, since both types of R would be strictly better off not replying at all. This leaves us with mixes between  $AA$  and  $NN$ , and these cannot exist because if  $AA$  yields zero expected utility,  $AG$  must do strictly better.

Any mix involving only  $RD$ ,  $DR$  and  $DD$  is also ruled out unless  $c_p = i$ : (A2) implies that Q would absorb all second messages; hence if it is worthwhile for HIGH R to

send in both rounds, it would be worthwhile for LOW R also. This leaves us with mixes between  $RR$  and  $RD$ , between  $RR$  and  $DR$ , and three- and four-way mixes involving  $RR$ , all of which place strictly positive weight on  $RR$  and at least one of  $RD$  and  $DR$ .  $\square$

**Fact 5.** In any equilibrium in which HIGH R plays  $RR$ , Q plays a strict mix between (i)  $AG$  and  $NN$ ; (ii)  $AG$  and  $NA$ ; or (iii)  $AG$ ,  $AA$  and  $NA$ .

*Proof.* Recall from fact 4 that LOW R must be playing a mixed strategy that gives strictly positive weight to  $RR$  and at least one of  $RD$  and  $DR$ . We start by ruling out any equilibrium in which Q plays a pure strategy: if Q is playing the pure strategy  $AG$ , for LOW R to be indifferent between replying and not replying for a second time we require  $c_p = li$ ; similarly, if Q is playing  $AA$ , we require  $c_p = i$ ; against  $NA$ , R's expected utility from sending a second message must be strictly positive to make up for the loss of  $c_p$  on the first. Hence  $RR$  does strictly better than  $RD$  or  $DR$ ; and against  $NN$ , R is strictly better off not replying at all.

Next, we note that any mixed strategy involving  $NN$  and  $AA$  cannot be played in equilibrium either. The expected utility of absorbing first messages must be lower than the expected utility of absorbing second messages, and hence strictly negative, since total expected utility must be zero. But then  $NA$  does strictly better. This leaves us with seven possible mixed strategies for Q. Four of these are ruled out below, leaving us with just (i), (ii) and (iii). In each case, HIGH R is playing  $RR$  with probability one, and LOW R is playing  $RR$  with probability  $\mu$  and  $DD$  with probability  $\lambda'$ , where  $0 < \mu < \lambda \leq 1$ . This gives us the following equations for the expected utility Q gets from playing each of her pure strategies:

$$u_{AA} = 2qu_h + q'(\lambda + \mu)u_l,$$

$$u_{AG} = q(1 + h)u_h + q'(\lambda + l\mu)u_l,$$

$$u_{NA} = qu_h + q'\mu u_l,$$

$$u_{NN} = 0.$$

1.  $AG, AA$ : suppose Q plays  $AG$  with probability  $\sigma$  and  $AA$  with probability  $\sigma'$ . Then, for LOW R to be willing to be willing to mix between  $RR$  and  $RD$  or  $DR$  it must be the case that  $c_p = i(l + \sigma'l')$ . This implies that  $c_p < i$ , and hence that  $RR, RD$  and  $DR$  yield strictly positive expected utility. So the weight on  $DD$  must be zero, i.e.,  $\lambda = 1$ . For Q to be willing to mix, we require the following equivalent equalities.

$$u_{AA} = u_{AG},$$

$$2qu_h + q'(\lambda + \mu)u_l = q(1 + h)u_h + q'(\lambda + l\mu)u_l,$$

$$qh'u_h + q'l'\mu u_l = 0,$$

$$\mu = -\frac{q}{q'} \frac{h'}{l'} \frac{u_h}{u_l}.$$

In addition, both strategies must do at least as well as *NA*. This requires:

$$\begin{aligned} u_{AA} &\geq u_{NA}, \\ 2qu_h + q'(\lambda + \mu)u_l &\geq qu_h + q'\mu u_l, \\ qu_h + q'u_l &\geq 0, \end{aligned}$$

which contradicts (A1).

2. *AG, NN, NA*: Mixing by Q implies that  $u_{AG} = u_{NN} = u_{NA}$ . The first equality implies that  $\lambda + \mu = -2(q/q')(u_h/u_l)$  and the second that  $\mu = -(q/q')(u_h/u_l)$ . Hence  $\lambda = \mu$ , But we need  $\lambda > \mu$ .
3. *AA, NA*: Against this strategy by Q, mixing by LOW R between *RR* and *RD* or *DR* implies that  $c_p = i$ . But then with any positive weight on *NA*, LOW R would do better to play *DD* and not participate at all.
4. *NN, NA*: cannot be an equilibrium, since *RD* and *DR* would always yield negative payoff for LOW R.

□

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