ECON 2200: Problem Set 5

Due 11/19/08

1. Consider a finite strategic form game $G = \langle N, \{S_i, u_i\}_{i \in N} \rangle$, and information model $\langle \Omega, p, \{\mathcal{P}_i\}_{i \in N} \rangle$, where Ω is a set of states, p is a (common) prior on Ω with the property that $p(\omega) > 0$ for all $\omega \in \Omega$, and $\mathcal{P}_i$ is a partition of Ω representing player is information; let $f : \Omega \to S$ be a *strategy function* telling us which strategy profile is chosen at each state, and suppose that $f_i(\omega') = f_i(\omega)$ for all $\omega' \in \mathcal{P}_i(\omega)$ (i.e. each player knows her own strategy choice). Show that if every player is rational at every state, then the distribution over S induced by p is a correlated equilibrium.

2. Consider a variant of Rubinstein's infinite-horizon bargaining game (see FT §4.4 or OR ch. 7) where a dollar is to be divided between two players but divisions must be whole numbers of cents (i.e. if the split is $(x, 1-x)$, x can be $0, 0.01, 0.02, \ldots, 0.99$, or 1). Assume that utilities for player 1 and player 2 from a division of $(x, 1-x)$ at time $t = 0, 1, 2, \ldots$ are $(\delta^t x, \delta^t (1-x))$. Characterize the set of subgame perfect equilibria for $\delta = \frac{1}{2}$ and for δ very close to (but strictly less than) 1.

3. Consider a bargaining problem (U, d) with 2 agents, where $U \subset \mathbb{R}^2$ is a set of feasible utility outcomes (U is convex, closed, bounded above, and satisfies free disposal: $U - \mathbb{R}^2_+ \subset U$), and $d \in U$ is the disagreement outcome. If $\mathcal{B}$ is the set of all such bargaining problems, then a *bargaining solution* is a function $f : \mathcal{B} \to \mathbb{R}^2$ such that $f(U, d) \in U$. For example, the *Kalai-Smorodinsky solution* assigns to a given bargaining problem the agreement Pareto efficient agreement x such that:

$$\frac{x_1 - d_1}{x_2 - d_1} = \frac{b_1 - d_1}{b_2 - d_2},$$

where b_i is the highest payoff player i can receive in any agreement in which player j get at least d_j.

   (a) Give an example of a bargaining problem in which the Kalai-Smorodinsky solution differs from the Nash solution.

   (b) Which of the following axioms does the Kalai-Smorodinsky solution satisfy: invariance to equivalent utility representations, Pareto efficiency, symmetry, and independence of irrelevant alternatives? In each case, offer a proof or a counterexample.

4. A buyer and a seller are bargaining. The seller owns an object for which the buyer has value $v > 0$ (the seller's value is zero). This value is known to the buyer, but not to the seller. Its prior distribution is common knowledge. There are two periods of bargaining. The seller makes a take-it-or-leave-it offer at the start of each period (a price) which the buyer may accept or reject. The game ends when an offer is accepted, or after two periods, whichever comes first (in the event of no agreement, the seller keeps the object). Both players discount period-two payoffs with a discount factor of $\delta < 1$. Assume throughout that the buyer always accepts the seller's offer whenever the buyer is indifferent.

   (a) Characterize the (pure strategy) perfect Bayesian equilibria for the case where v can take two values, $\{v_L, v_H\}$, with $0 < v_L < v_H$. Use λ to denote the probability that $v = v_H$.

   (b) Do the same for the case where v is uniformly distributed on the interval $[v_L, v_H]$.