ECON 2200: Problem Set 3

Due October 6th, 2008

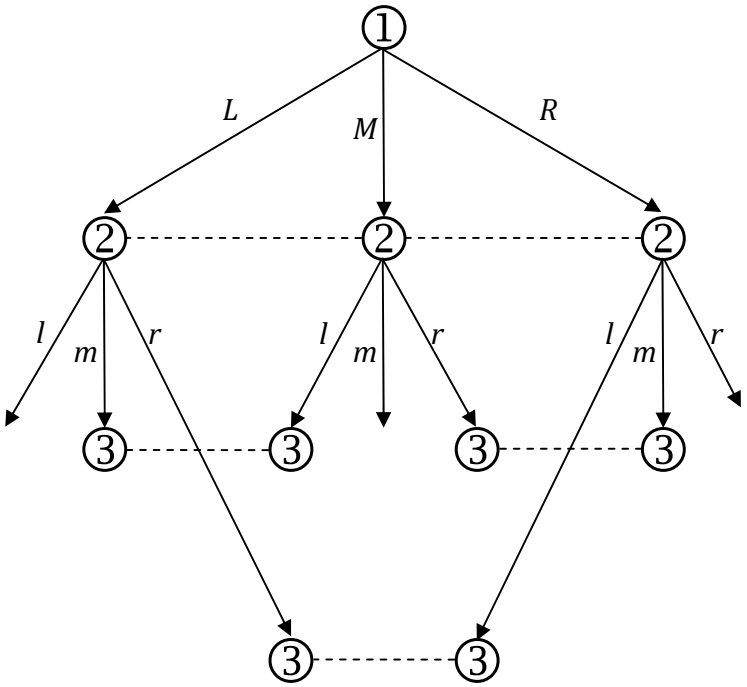
1. Alice and Bob are trying to split \$100. In the first round of bargaining, Alice makes an offer at cost c (to herself), proposing to keep x_A and give $x_B = 100 - x_A$ to Bob. Bob either accepts (ending the game) or rejects. In round 2, Bob makes an offer of (y_A, y_B), at cost of 10 to himself, and Alice either accepts or rejects. In round 3, Alice makes an offer of (z_A, z_B) at cost of c to herself, and Bob accepts or rejects. If he rejects, the money is lost. Assume that the players are risk-neutral (utility is equal to money obtained minus any costs), and there is no discounting.

   (a) If $c = 0$, what is the subgame-perfect equilibrium outcome?

   (b) If $c = 80$, what is the subgame-perfect equilibrium outcome?

   (c) If the value of c is private information to Alice, and Bob's prior is that it is equally likely to be 0 or 80, what is the perfect Bayesian equilibrium outcome?

2. Consider the following game tree (player 3's choices and the payoffs have been omitted, since they will not affect the answer).



   Suppose player 3's beliefs are given by: $\alpha_1 = \beta_1 = \gamma_1 = \frac{2}{3}$, $\alpha_2 = \beta_2 = \gamma_2 = \frac{1}{3}$.

   (a) Do these beliefs form part of any consistent assessment?

   (b) Are these beliefs structurally consistent?

3. There are two firms, an incumbent and a (potential) entrant. In the first period, the incumbent can give away some of its product to consumers, as a signal to the entrant about its costs. The amount given away does not influence consumers' demand in the second period. After observing the amount given away, denoted by y, the entrant decides whether or not to enter. If it decides to stay out, the incumbent monopolizes the market, and the entrant receives zero profit. If it enters, the firms engage in Cournot competition.
   Let $x_i \geqslant 0$ denote the second-period output of firm i $(i = 1, 2)$. The inverse demand function in period 2 is $p = 10 - x_1 - x_2$. The firms have constant marginal costs: C for the incumbent (in both periods) and 4 for the entrant. The entrant must pay a fixed cost of 2 to enter.
   At the time of the entry decision, the entrant is ignorant of the incumbent's marginal cost C, believing it is $H = 4$ with probability $\frac{1}{2}$ and $L = 1$ with probability $\frac{1}{2}$. The entrant discovers the cost as soon as it enters.

   (a) For each of the cases $C = H$ and $C = L$, compute the two firms' profits if entry occurs and the incumbent's profit if entry does not take place.

   (b) The is a perfect Bayesian equilibrium (PBE) in which there is always entry. Construct this equilibrium and show that it satisfies the conditions for a PBE.

   (c) Does the PBE constructed pass the test of dominated messages? Explain why or why not.

   (d) There is a PBE in which the high-cost incumbent chooses $y = 0$ and the low-cost incumbent chooses $y = 2$. Does this PBE satisfy the test of dominated messages? Explain why or why not.

4. (*optional*) The following entry model is drawn from Kreps and Wilson (JET 1982). There is a monopolist and an entrant. The entrant first decides whether to enter the market, and, if he does, the monopolist chooses whether to fight or acquiesce. If the entrant stays out, the monopolist gets $a > 1$, and the entrant gets 0. If the entrant enters, the monopolist gets 0 from fighting and -1 from acquiescing if he is a "tough" monopolist, and -1 from fighting and 0 from acquiescing if he is a "normal" monopolist. The entrant gets b if the monopolist acquiesces and $b-1$ if he fights, where $0 < b < 1$. Suppose the entrant believes the monopolist to be tough with probability p (the monopolist knows his own type).

   (a) Solve for the sequential equilibrium of this game.

   (b) Suppose the monopolist plays 2 entrants in sequence, and the second entrant observes the outcome of the first game (there is no discounting). Solve for the sequential equilibrium. (*Hint*: ignore the case where $p = b$, and consider the cases $p < b$ and $p > b$ seperately.)

   (c) Would the monopolist be better or worse off if the second entrant could not observe the first game?