ECON 2200: Problem Set 2

Due September 24th, 2008

1. Consider the following simultaneous-move game:

	L	C	R
T	$3,1$	$0,0$	$5,0$
M	$2,1$	$1,2$	$3,1$
B	$1,2$	$0,1$	$4,4$

   (a) Find all pure-strategy Nash equilibria of this game.

   (b) Suppose that the game is played twice, with the outcome from the first stage observed before the second stage begins. Can the outcome $(4,4)$ be achieved in the first stage in a pure strategy subgame-perfect equilibrium? If so, describe a strategy profile that does so and prove that it is a subgame perfect Nash equilibrium. If not, prove why not.

2. Consider an infinite repetition of the game below, and consider the following repeated game strategy (which can be used by either player): "Play A in even periods and B in odd periods until someone deviates from this. Once anyone has deviated, play C forever." The discount factor for both players is δ. For what values of δ is it a subgame perfect equilibrium for both players to follow this strategy?

	A	B	C
A	$3,3$	$3,5$	$0,0$
B	$5,3$	$2,2$	$0,0$
C	$0,0$	$0,0$	$1,1$

3. Arthur and Beatrix compete in a race. At the start of the race, both players are 6 steps away from the finish line. Who gets the first turn is determined by a toss of a fair coin; the players then alternate turns, with the results of all previous turns being observed before the current turn occurs.
   During a turn, a player chooses from these four options:

   - Do nothing at cost 0;
   - Advance 1 step at cost 2;
   - Advance 2 steps at cost 7;
   - Advance 3 steps of at cost 15.

   The race ends when the first player crosses the finish line. The winner of the race receives a payoff of 20, while the loser gets nothing. Finally, there is discounting: after each turn, payoffs are discounted by a factor of δ, where δ is less than but very close to 1. Find all subgame perfect equilibria of this game.

4. Show that more information may hurt a player by constructing a two-player game with the following features: player 1 is fully informed while player 2 is not; the game has a unique Bayesian Nash equilibrium, in which player 2's payoff is higher than his payoff in the unique equilibrium of any of the related games in which he knows player 1's type.

5. Consider a first-price, sealed-bid auction in which bidders simultaneously submit bids with the object going to the highest bidder at a price equal to their bid. Suppose that there are two bidders and that their values for the object are chosen independently from a uniform distribution over $[0, 1]$. Think of a player's type as being the value that the player places on the object. A player's payoff is $v - b$ when she wins the object by bidding b and her value is v; her payoff is 0 if she does not win the object.

   (a) Formulate this as a Bayesian game.

   (b) Let $b_i(v)$ denote the bid made by player i of type v. Show that there is a Bayesian Nash equilibrium in which $b_i(v) = \alpha + \beta v$ for all i and v. Determine the values of α and β.

6. Modify the first-price, sealed-bid auction in question 5 so that the loser also pays his bid (but does not win the object). This modified auction is called an *all-pay* auction.

   (a) Show that there is a Bayesian Nash equilibrium in which $b_i(v) = \gamma + \delta v + \phi v^2$ for all i and v.

   (b) How do the player's bids compare to those in the first-price auction? What is the intuition behind this difference in bids?

   (c) Show that, *ex ante*, the first-price auction and the all-pay auction generate the same expected revenue for the seller.