ECON 2200: Problem Set 1: solutions

1. ***game 1*:** $\{U; L\}$ and $\{D; R\}$. Note that in the second of these, both players play weakly dominated strategies.

   ***game 2*:** there is a continuum of Nash equilibria:
   $\{U; p \circ L, (1-p) \circ R\}$ and $\{p \circ U, (1-p) \circ D; L\}$, for all $p \in [0,1]$

   ***game 3*:** $\{U; l\}$, $\{M, r\}$, $\{D, L\}$, $\left\{\frac{11}{20} \circ M, \frac{9}{20} \circ D; p \circ L, (1-p) \circ r\right\}_{p \in [0,1]}$

2. (a) Each player i solves

$$\begin{aligned} & \max \pi_i = p q_i \\ \Rightarrow \quad & \frac{d\pi_i}{dq_i} = 1 - 2q_i - q_{-i} = 0 \\ \Rightarrow \quad & q_i = \frac{1 - q_{-i}}{2} \quad \text{or} \quad q_i = 1 - Q \end{aligned}$$

   Since this condition is the same for all i, the equilibrium must be symmetric. Solving, we obtain

$$q_i = \frac{1}{1+n}; \quad Q = \frac{n}{1+n}; \quad p = \frac{1}{1+n}; \quad \pi_i = \frac{1}{(1+n)^2}$$

   (b) Since $q_2 \geqslant 0$, the first-order condition above gives us $q_1 \leqslant \frac{1}{2}$ (all other strategies are strictly dominated by $q_1 = \frac{1}{2}$). And since $q_1 \leqslant \frac{1}{2}$, we have $q_2 \geqslant \frac{1}{4}$; this gives us $q_1 \leqslant \frac{3}{8}$, so $q_2 \geqslant \frac{5}{16}$. Repeated iterations and symmetry give us $q_1 = q_2 = \frac{1}{3}$. This is not a surprise: supermodular games with a unique Nash equilibrium have a unique strategy profile surviving iterated deletion of strictly dominated strategies.

   (c) Since $q_2, q_2 \geqslant 0$, the first-order condition above gives us $q_1 \leqslant \frac{1}{2}$ (all other strategies are strictly dominated by $q_1 = \frac{1}{2}$). Since $q_1 \leqslant \frac{1}{2}$ and $q_3 \leqslant \frac{1}{2}$, we have $q_2 \geqslant 0$. Iterated deletion of strictly dominated strategies implies only that $0 \leqslant q_1, q_2, q_3 \leqslant \frac{1}{2}$. This game is not supermodular, so even though it has a unique Nash equilibrium there may be many strategy profiles surviving IDSDS.

3. (a) Suppose each citizen contributes with probability p. For this to be an equilibrium, each must be indifferent between contributing and not, given that the two others contribute with probability p, i.e.

$$\begin{aligned} & EU(\text{contribute}) = 1 - (1-p)^2 - \frac{3}{8} = p^2 = EU(\text{don't}) \\ \Rightarrow \ & p = \frac{1}{4} \text{ or } \frac{3}{4} \end{aligned}$$

   The good will be provided with probability

$$3p^2(1-p) + p^3,$$

   i.e. $\frac{5}{32}$ for $p = \frac{1}{4}$ and $\frac{27}{32}$ for $p = \frac{3}{4}$.

(b) For equilibrium, we now need

$$EU(\text{contribute}) = 1 - (1-p)^3 - \frac{3}{8} = 3p^2(1-p) + p^3 = EU(\text{don't})$$
$$\Rightarrow \; p = \frac{1}{2} \text{ or } \frac{3-\sqrt{5}}{4} \;\; (\text{or } \frac{3+\sqrt{5}}{4})$$

The good will be provided with probability

$$6p^2(1-p)^2 + 4p^3(1-p) + p^4,$$

i.e. $\frac{11}{16}$ for $p = \frac{1}{2}$ and $\frac{7(3-\sqrt{5})}{32}$ for $p = \frac{3-\sqrt{5}}{4}$.

4. In the first round of elimination every action $a_i \leqslant 50$ of each player is weakly dominated by $a_i = 51$. No other action is weakly dominated, because every action a_i with $51 \leqslant a_i \leqslant 99$ is a best response to $a_i + 1$, and 100 is a best response to 0. Thus, we can eliminate in the first round all the integers in $[0, 50]$. In the second round, 100 is weakly dominated by 51 for each player, and every other remaining action a_i of player i is a strict best response to $a_i + 1$, so no other action is weakly dominated. Similarly in the third round 99 is weakly dominated by 51, and no other action is weakly dominated. Continuing with this type of argument we can eliminate all integers in $[52, 100]$, so we are left with the single action pair $(51, 51)$, and with the payoffs $(50, 50)$.