ECON 2200: Midterm — solutions

10/08/08, 10:00 – 1:00 p.m.

1. (a) Yes: for player 1, E is strictly dominated by A; for player 2, for player 2, a is strictly dominated by any strict mix between d and e.
   (b) B, E, a, and d.
   (c) (C, c).
2. (a) Consider each pure strategy profile:
   (U, L): player 2 will deviate to R
   (U, R): player 1 will deviate to D
   (D, L): player 1 will deviate to U
   (D, R): player 2 will deviate to L
   (b) In the mixed strategy Nash equilibrium, each player must be indifferent between their two strategies. Suppose 1 plays U with probability p and 2 plays L with probability q. Then:

$$EU_1(U) = qu + (1-q)m = qw + (1-q)y = EU_1(D)$$
$$EU_2(L) = pv + (1-p)x = pn + (1-p)z = EU_2(R)$$
$$\Rightarrow p = \frac{x-z}{(x-z)+(n-v)}$$
$$q = \frac{y-m}{(y-m)+(u-w)}$$

3. (a) (Pub, Pub), $(Cafe, Cafe)$, and a mixed-strategy equilibrium:

$$\text{Alice:} \quad Pub \text{ with prob.} \frac{1}{4+\epsilon_B}, \; Cafe \text{ with prob.} \frac{3+\epsilon_B}{4+\epsilon_B}$$
$$\text{Bob:} \quad Pub \text{ with prob. } \frac{3+\epsilon_A}{4+\epsilon_A}, \; Cafe \text{ with prob. } \frac{1}{4+\epsilon_A}.$$

   (b) Suppose Alice and Bob play the following strategies:
   Alice: Pub if $\epsilon_A \leq \epsilon_A^*$; Cafe if $\epsilon_A > \epsilon_A^*$.
   Bob: Cafe if $\epsilon_B \leq \epsilon_B^*$; Pub if $\epsilon_B > \varepsilon_B^*$.

   For this to be an equilibrium, we need the agents to be indifferent between the two strategies at the cutoff value of ϵ. This is sufficient for equilibrium, since for lower values of ϵ, Cafe (Pub) becomes relatively more attractive for Alice (Bob) and *vice versa*. So we need:

$$\begin{aligned}
1 \cdot \Pr(\epsilon_B > \epsilon_B^*) &= (3+\epsilon_A^*) \cdot \Pr(\epsilon_B \le \epsilon_B^*) \text{ and} \\
(3+\epsilon_B^*) \cdot \Pr(\epsilon_A \le \epsilon_A^*) &= 1 \cdot \Pr(\epsilon_A > \epsilon_A^*) \\
\Rightarrow 1 \cdot (1 - \frac{\epsilon_B^*}{\bar{\epsilon}}) &= (3+\epsilon_A^*)\frac{\epsilon_B^*}{\bar{\epsilon}} \text{ and} \\
(3+\epsilon_B^*) \cdot \frac{\epsilon_A^*}{\bar{\epsilon}} &= 1 \cdot (1 - \frac{\epsilon_A^*}{\bar{\epsilon}}) \\
\Rightarrow \epsilon_A^* = \epsilon_B^* &= \sqrt{4+\bar{\epsilon}} - 2.
\end{aligned}$$

(c) As $\bar{\epsilon} \to 0$, $\frac{\epsilon_A^*}{\bar{\epsilon}} \to \frac{1}{4}$ and $\frac{\epsilon_B^*}{\bar{\epsilon}} \to \frac{1}{4}$. This equilibrium of the game with private information approximates the mixed-strategy equilibrium of the game where $\epsilon_A = \epsilon_B = 0$.

4. (a) Game 1: $(A, p \circ l \oplus (1-p) \circ r)$ for $p \in [0, \frac{1}{3}]$ and (L, l)
   Game 2: (DL, l)

   (b) Game 1: $(A, p \circ l \oplus (1-p) \circ r; \mu(L) = \frac{1}{4}, \mu(R) = \frac{1}{4})$ for $p \in [0, \frac{1}{3}]$
   and $(A, r; \mu(L) = q, \mu(R) = (1-q))$ for $q \in [0, \frac{1}{4})$
   and $(L, l; \mu(L) = 1, \mu(R) = 0)$
   Game 2: $(DL, l;\ \mu(L) = 1, \mu(R) = 0)$

5. (a) There is a continuum of pooling equilibria: $e_L = e_H = 0$; $w_0 = 2\frac{1}{2}$, $w_1 = 4 - 3p$; $\Pr(L \mid e = 0) = \frac{1}{2}$, $\Pr(L \mid e = 1) = p$, where $p \in [0, 1]$. $\pi_w^L = \pi_w^H = 2\frac{1}{2}$.
   There is one separating equilibrium: $e_L = 0$, $e_H = 1$; $w_0 = 1$, $w_1 = 4$; $\Pr(L \mid e = 0) = 1$, $\Pr(L \mid e = 1) = 0$. $\pi_w^L = 1$, $\pi_w^H = 2$.

   (b) All of them: in each pooling equilibria, it is equilibrium dominated for *both* types to choose $e = 1$.

   (c) There is a continuum of pooling equilibria: $e_L = e_H = 0$; $w_0 = 3$, $w_1 = 5 - 4p$; $\Pr(L \mid e = 0) = \frac{1}{2}$, $\Pr(L \mid e = 1) = p$, where $p \in \left[\frac{1}{10}, 1\right]$. $\pi_w^L = \pi_w^H = 3$.
   There is one separating equilibrium: $e_L = 0$, $e_H = 1$; $w_0 = 1$, $w_1 = 5$; $\Pr(L \mid e = 0) = 1$, $\Pr(L \mid e = 1) = 0$. $\pi_w^L = 1$, $\pi_w^H = 3\frac{2}{5}$.
   None of the pooling equilibria survives the intuitive criterion, because it is equilibrium dominated for the low type but not for the high type to choose $e = 1$.

   (d) There is a continuum of pooling equilibria: $e_L = e_H = 0$; $w_0 = 2\frac{1}{2}$, $w_1 = 4 - 3p$; $\Pr(L \mid e = 0) = \frac{1}{2}$, $\Pr(L \mid e = 1) = p$, where $p \in [0, 1]$. $\pi_w^L = \pi_w^H = 2\frac{1}{2}$.
   There is a continuum of separating equilibria; here are some of them: $e_L = 0$, $e_H = e^*$; $w_0 = 1$ if $e < e^*$, $w_1 = 4$ if $e \ge e^*$; $\Pr(L \mid e < e^*) = 1$, $\Pr(L \mid e \ge e^*) = 0$, where $e^* \in \left[\frac{3}{8}, 1\frac{1}{2}\right]$. $\pi_w^L = 1$, $\pi_w^H = 4 - 2e^*$. All the rest are outcome equivalent to one of these, but may have different beliefs off-the-equilibrium path.
   Of the above equilibria, only the separating equilibrium with $e^* = \frac{3}{8}$ survives the intuitive criterion.