ECON 2100: Problem Set 6

Due 12/5/08

1. In an economy with two types of consumer, each type has the respective utility function and endowment:

$$\begin{aligned} u_1(x_{11}, x_{21}) &= x_{11}x_{21} \quad \text{and} \quad \omega_1 = (8, 2), \\ u_2(x_{12}, y_{22}) &= x_{12}x_{22} \quad \text{and} \quad \omega_2 = (2, 8). \end{aligned}$$

   (a) Draw an Edgeworth box for this economy when there is one consumer of each type.

   (b) Characterize as precisely as possible the set of allocations that are in the core of this two-consumer economy.

   (c) Show that the allocation $x_1 = (4, 4)$ and $x_2 = (6, 6)$ is in the core.

   (d) Now consider the two-fold replica of this economy. Show that the allocation which give both types of consumer 1 $(4, 4)$ and both types of consumer 2 $(6, 6)$ is not in the core.

   (e) Characterize the core of the two-fold replica economy.

2. Consider a one-consumer, one-producer (Robinson Crusoe) economy. There are two goods, leisure/labor (l) and grain (g). The price of leisure is w, and the price of food is p, which you can normalize to 1. Compute the equilibrium prices, profits, and allocation when Robinson's utility function is $u(l, g) = \log l + \log g$, Crusoe produces grain according to the production function is $g = \sqrt{l}$, and the total endowment of leisure is $\bar{L} = 1$ (and the endowment of grain is 0).

3. Ancient Rome was (for a while) ruled by two *consuls*, each with veto power over the other's actions. We can represent this situation by the social choice function such that xRy if and only if xR_1y or xR_2y (x and y are social states, and R_1 and R_2 are the weak preference orderings of the two consuls; the preferences of the other citizens, $3, \ldots, N$ do not matter). There are at least three social states.

   (a) Show that this social choice function satisfies the conditions WP, IIA, and ND of Arrow's theorem.

   (b) Show that the social choice ordering R generated by this rule need not be transitive.

   (c) Show that the social choice ordering R is *quasi-transitive*, i.e. strict preference P is transitive but indifference I might not be.

4. Suppose that the set of social states X is the interval $[0, 1]$ and that each agent's preferences are *single-peaked*, i.e. for each agent i there is some social state $a_i^* \in X$ such that if $a_i^* \leq b < c$ or if $c < b \leq a_i^*$, then bP_ic. Show that if there is an odd number of agents, and we restrict the domain of the social choice function to inculde only single-peaked preferences, then the majority rule induces complete and transitive social preferences.