ECON 2100: Problem Set 5

Due 11/19/08 in class

1. Consider an Edgeworth-box economy in which the two consumers have locally non-satiated preferences. Let $x_{li}(p)$ be consumer i's demand for good l at prices (p_1, p_2).

   (a) Show that $p_1 \left(\sum_i x_{1i}(p) - \bar{\omega}_1\right) + p_2 \left(\sum_i x_{2i}(p) - \bar{\omega}_2\right) = 0$ for all prices p.

   (b) Argue that if the market for good 1 clears at prices $p^* \gg 0$, then so does the market for good 2; hence p^* is a Walrasian equilibrium price vector.

2. Consider an Edgeworth-box economy in which the consumers have the Cobb-Douglas utility functions $u_1(x_{11}, x_{21}) = x_{11}^{\alpha} x_{21}^{1-\alpha}$ and $u_1(x_{12}, x_{22}) = x_{12}^{\beta} x_{22}^{1-\beta}$. Consumer i's endowment is $(\omega_{1i}, \omega_{2i}) \gg 0$, for $i = 1, 2$.

   (a) Solve for the equilibrium price ratio and allocation.

   (b) How does your answer to (a) change with a differential change in ω_{11}?

3. Compute all equilibria of the following Edgeworth box economy:

$$u_1(x_{11}, x_{21}) = \left(x_{11}^{-2} + \left(\frac{12}{37}\right)^3 x_{21}^{-2}\right)^{-\frac{1}{2}}, \qquad \omega_1 = (1, 0)$$

$$u_2(x_{12}, x_{22}) = \left(\left(\frac{12}{37}\right)^3 x_{12}^{-2} + x_{22}^{-2}\right)^{-\frac{1}{2}}, \qquad \omega_2 = (0, 1)$$

4. Consider a pure exchange economy with two consumers (1 and 2) and two commodities (x and y). The utility functions of the two consumers are given by

$$u_1(x_1, y_1) = x_1 y_1 \qquad \text{and} \qquad u_2(x_2, y_2) = x_2 + y_2.$$

   The endowments are $\omega_1 = (7, 2)$ and $\omega_2 = (3, 2)$.

   (a) Find the Pareto set of this economy.

   (b) Find the core of this economy.

   (c) Find the Walrasian equilibrium.

   (d) Is consumer 1 better off at this equilibrium than at his endowment? What about consumer 2?

   (e) Do the marginal rates of substitution of the two consumers coincide at this equilibrium? Explain why or why not.

   (f) Find an allocation that is in the core of this economy, but not in the core of its 2-fold replica.

5. Consider a pure exchange economy with two consumers (1 and 2) and two commodities (x and y). Suppose WEA and WEA' are two Walrasian equilibrium allocations corresponding to two different initial endowments for the consumers, (ω_1, ω_2) and (ω_1', ω_2'). Is it possible that $\omega_1 \gg \omega_1'$, and yet consumer 1 strictly prefers her allocation under WEA' to her allocation under WEA? An diagrammatic explanation (proof or example) will suffice.

6. Consider an exchange economy with two identical consumers. Their common utility function is $u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha} x_{2i}^{1-\alpha}$, where $0 < \alpha < 1$. Society has 10 units of x_1 and 10 units of x_2. Find endowments ω_1 and ω_2, and corresponding Walrasian equilibrium prices, that will "support" as a Walrasian equilibrium allocation the equal-division allocation giving both consumers the bundle $(5, 5)$.