ECON 2100: Problem Set 3

Due 10/14/08 in class

1. Calculate the supply function $y(p)$ and the profit function $\pi(p)$ for each of the following production functions:

   (a) $f(a) = a_1^{\alpha}$, where $0 < \alpha \leq 1$.

   (b) $g(a) = \alpha a_1 + \beta a_2$, where $\alpha, \beta > 0$.

   (c) $h(a) = \min\{a_1, a_2\}$.

   (d) $i(a) = (a_1^{\alpha} + a_2^{\alpha})^{\frac{1}{\alpha}}$, where $0 < \alpha \leq 1$

2. Consider a firm with conditional factor demands of the form

$$x_1 = 1 + 3w_1^{-\frac{1}{2}} w_2^{a}$$
$$x_2 = 1 + b w_1^{\frac{1}{2}} w_2^{c}$$

   Output has been set to one for convenience. What are the values of the parameters a, b and c? Explain your reasoning carefully.

3. Suppose a consumer's preferences over lotteries satisfy completeness, transitivity, continuity and the *betweenness* axiom:

$$\text{for all lotteries } l, l' \text{ and } \alpha \in (0,1), \text{ if } l \sim l' \text{ then } \alpha \circ l \oplus (1-\alpha) \circ l' \sim l'$$

   (a) Show that betweenness is implied by the independence axiom.

   (b) Consider the set of lotteries with three possible outcomes, $a < b < c$. We can represent this set by a right-angled triangle, where the x-axis measures the probability of winning prize a, and the y-axis the probability of winning prize c. Show that betweenness implies that the consumer's indifference curves plotted in this space must be straight lines. Conversely, show that if the indifference curves are straight lines, then the betweenness axiom is satisfied.

   (c) Do these straight lines need to be parallel? Explain.

   (d) Are the (typical) choices of the Allais paradox compatible with the betweenness axiom?

4. Let $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ be a strictly increasing expected utility function. Show that:

   (a) $u(\cdot)$ exhibits constant relative risk aversion equal to $\rho \neq 1$ if and only if $u(x) = \beta x^{1-\rho} + \gamma$, where $\beta > 0$ if $\rho < 1$ and $\beta < 0$ if $\rho > 1$.

   (b) $u(\cdot)$ exhibits constant relative risk aversion equal to 1 if and only if $u(x) = \beta \log x + \gamma$, where $\beta > 0$.

   (c) $\lim_{\rho \rightarrow 1} \left(\frac{x^{1-\rho}}{1-\rho} \right) = \log x$ for all $x > 0$.

5. One way to construct preferences over lotteries with monetary prizes is by evaluating each lottery l on the basis of two numbers, $E[l]$, the expected value of l, and $var[l]$, the variance of l. Such a procedure may or may not be consistent with the vNM assumptions.

   (a) Show that $u(l) = E[l] - \frac{1}{4}var[l]$ induces a preference relation that is not consistent with the vNM assumptions.

   (b) Show that $u(l) = E[x] - (E[l])^2 - var[l]$ is consistent with the vNM assumptions.

6. A coin has probability $\frac{1}{2}$ of landing heads. You are offered a bet in which you will be paid $\$2^j$ if the first head occurs on the jth flip.

   (a) What is the expected value of this bet?

   (b) Suppose that your expected utility function is $u(x) = \ln x$. Express the utility of this game to you as a sum.

   (c) Evaluate the sum.

   (d) Let w_0 be the amount of money that would give you the same utility you would have if you played this game. Solve for w_0.