ECON 2100: Problem Set 2

Due 9/24/08 in class

1. A consumer of two goods faces positive prices and has positive income. His utility function is

$$u(x_1, x_2) = \max\{ax_1, ax_2\} + \min\{x_1, x_2\}, \qquad \text{where } 0 < \alpha < 1$$

Compute the Walrasian demand functions.

2. Consider a three-good setting in which the consumer has utility function $u(x) = (x_1 - b_1)^{\alpha} (x_2 - b_2)^{\beta} (x_3 - b_3)^{\gamma}$ $(\alpha, \beta, \gamma > 0)$.

   (a) Explain why you can assume that $\alpha + \beta + \gamma = 1$ without loss of generality. Do so for the rest of the problem.

   (b) Write down the first-order conditions for the utility maximization problem, and derive the consumer's Walrasian demand and indirect utility functions.

3. Determine which of the following behavior patterns are consistent with rational consumer choice (assuming there are only two goods). In each case, either provide a preference relation (or utility function) which rationalizes the behavior, or prove why it is not rational.

   (a) The consumer's Walrasian demand function is $x(p, w) = \left(\frac{2w}{2p_1+p_2}, \frac{w}{2p_1+p_2}\right)$.

   (b) The consumer consumes up to a fixed amount, k, of commodity 1 and spends the rest of his income on commodity 2.

   (c) The consumer chooses the bundle (x_1, x_2) which satisfies $\frac{x_1}{x_2} = \frac{p_1}{p_2}$ and costs w.

4. Consider the utility function

$$u = 2\sqrt{x_1} + 4\sqrt{x_2}.$$

   (a) Find the demand functions for each good.

   (b) Find the compensated demand functions $h(\cdot)$.

   (c) Find the expenditure function, and verify Shepherd's lemma.

   (d) Find the indirect utility function, and verify Roy's identity.

5. The substitution matrix of a utility-maximizing consumer's demand system at prices $(8, p)$ is

$$\begin{pmatrix} a & b \\ 2 & -\frac{1}{2} \end{pmatrix}$$

Find a, b, and p, explaining your reasoning carefully.