**ECON 2100: Problem Set 1**

Due September 8th in class

1. Suppose there is some finite set of alternatives $X$. An individual's weak preference relation on $X$ is denoted by $R$ and her strict preference relation (derived from $R$) by $P$. $R$ is reflexive and complete.

   $R$ is said to be *transitive* if for all $x, y, z \in X$, $xPy$ and $yPz$ implies $xPz$. $R$ is said to be *acyclic* if for all $x_1, x_2, \ldots, x_k \in X$, if $x_i P x_{i+1}$ for $i = 1, \ldots n-1$, then $x_1 R x_n$.

   (a) Show that if $R$ is transitive, it is also acyclic.

   (b) Provide an example to demonstrate that there exist acyclic preferences that are not transitive. Thus, acyclicity is weaker than transitivity.

   (c) Let $M(R, Y)$ (where $Y \subseteq X$) denote the set of $R$-maximal elements in $Y$ i.e.

   $$M(R, Y) = \{x \in Y : \text{for all } y \in Y,\ xRy\}.$$

   $M(R, Y)$ can be thought of as the *choice set* from $Y$. We know that $R$ being transitive is sufficient for choice to be well-defined, i.e. for $M(R, Y)$ to be non-empty. Prove that $M(R, Y) \neq \varnothing$ for all $Y \subseteq X$ *if and only if* $R$ is acyclic.

2. The purpose of this problem is to make sure that you fully understand the basic concepts of utility representation and continuous preferences.

   (a) Is the statement "if both $U$ and $V$ represent $\succsim$ then there is a *strictly* monotonic function $f : \mathbb{R} \to \mathbb{R}$ such that $V(x) = f(U(x))$" correct?

   (b) Can a continuous preference relation be represented by a discontinuous function?

   (c) Show that in the case of $X = \mathbb{R}$, the preference relation that is represented by the discontinuous utility function $u(x) = \lfloor x \rfloor$ (the largest integer $n$ such that $x \geq n$) is not a continuous relation.

   (d) Show that the following definition of a continuous preference relation is equivalent to the standard definition given in MWG (Definition 3.C.1, page 46):

   **Definition 1** *For any $x \in X$, the upper and lower contour sets $\{y \mid y \succsim x\}$ and $\{y \mid x \succsim y\}$ are closed sets in $X$.*

3. A consumer has *lexicographic* preferences over $X \in \mathbb{R}^2_+$ if the relation $\succsim$ satisfies $(x_1, x_2) \succsim (y_1, y_2)$ whenever $x_1 > y_1$, or $x_1 = y_1$ and $x_2 \geq y_2$.

   (a) Sketch an indifference map for these preferences.

   (b) Can these preferences be represented by a continuous utility function? Why or why not?

4. Suppose $u(x_1, x_2)$ and $v(x_1, x_2)$ are utility functions.

   (a) Prove that if $u(x_1, x_2)$ and $v(x_1, x_2)$ are both homogeneous of degree $r$, then $s(x_1, x_2) \equiv u(x_1, x_2) + v(x_1, x_2)$ is also homogeneous of degree $r$.

   (b) Prove that if $u(x_1, x_2)$ and $v(x_1, x_2)$ are both quasiconcave, then $m(x_1, x_2) \equiv \min\{(x_1, x_2), v(x_1, x_2)\}$ is also quasiconcave.