**ECON 1905: Problem Set 3**

Due 4/17/08 in class

1. Suppose a group of citizens are faced with the following public goods contribution problem: the benefit to each citizen if the good is provided is 1; the cost of contributing to the good is $\frac{3}{8}$; and the good will be provided if at least 2 citizens contribute. Citizens choose simultaneously whether to contribute or not, and costs are not refunded even if the good is not provided.

   (a) Suppose there are just 3 citizens. There are two symmetric (mixed-strategy) Nash equilibria of the game. Find them, and compute the probability that the good is provided in each case.

   (b) Repeat part (a) for the case where there are 4 citizens.

2. Consider Palfrey & Rosenthal's (ASPR 1984) model of voting. There are 2 voters who prefer candidate 1 and two voters who prefer candidate 2. Each voters' utility is given by $p - c$ if she votes, or $p$ if she doesn't, where $p$ is the probability that her favorite candidate wins.

   (a) Suppose $c = \frac{1}{2}$ for all voters, and this is common knowledge. Find the symmetric equilibria of the model.

   (b) Now suppose that each voter's cost $c$ is drawn from a uniform distribution on $[0, 1]$, and she knows only her own value of $c$. Suppose each voter employs the same cut-off strategy $c^*$, voting if and only if $c \leq c^*$. Find the equilibrium value of $c^*$, and hence the probability that a given agent votes.

3. In the context of Cox's (AJPS 1987) model, suppose four candidates simultaneously choose their positions in the policy space $[0, 1]$, and each attempts to maximize her vote share; voters' ideal points are uniformly distributed along the policy space, and each voter votes for the candidate who is closest to his ideal point. Find all of the pure strategy Nash equilibrium policy choices of the candidates when the electoral system is:

   (a) plurality rule.

   (b) Borda count.

   (c) negative voting.

4. Crawford and Sobel's (Ecta. 1982) cheap talk game can be used to model communication between a legislative committee and Congress, with the committee playing the role of the sender and Congress playing the role of the receiver. Suppose the sender's type $t$ is uniformly distributed between 0 and 1; the action space is the interval from 0 to 1; and payoffs are given by $U_R = -(t - a)^2$ and $U_S = -(t + b - a)^2$.

   (a) If the sender's bias $b = \frac{1}{20}$, what equilibrium outcomes are possible? Compute the receiver's expected payoff in each case.

   (b) Suppose that, instead of choosing the action himself after observing the sender's message, the receiver delegates decision-making power to the sender. What action would she choose, and what would be the resulting expected payoff for the receiver?

   (c) For what values of $b$ does the receiver prefer delegation to the best outcome under cheap talk?