ECON 1905: Problem Set 2

Due 2/19/08 in class

1. Suppose there are two voters, 1 and 2, trying to choose between three alternatives, x, y, and z. If we assume preferences are strict, we can represent a voting rule in a 6×6 grid, describing the outcome of the voting rule for each of the 36 pairs of strict preference orderings. The Gibbard-Satterthwaite theorem tells us that if a voting rule is strategy proof, and each outcome is picked for at least one pair of preference orderings, it must be a dictatorship.

   (a) Consider the following voting rule, which picks the first alphabetically of the Pareto-optimal alternatives.

	xyz	xzy	yxz	yzx	zxy	zyx
xyz	x	x	x	x	x	x
xzy	x	x	x	x	x	x
yxz	x	x	y	y	x	y
yzx	x	x	y	y	y	y
zxy	x	x	x	y	z	z
zyx	x	x	y	y	z	z

   This voting rule is manipulable at just four pairs of orderings. Where, and by whom?

   (b) Construct a voting rule that is manipulable at just two pairs of orderings.

   (c) Is there a voting rule which is manipulable at just one pair of orderings? (Hard)

1. Consider a simple voting model, where voters have identical but lack information, and vote for the correct alternative with probability p. We use the random variable X_i to represent voter i's decision, where

$$X_i = \begin{cases} 1 \text{ if } i \text{ votes correctly} \\ 0 \text{ if } i \text{ votes incorrectly} \end{cases},$$

   so $p = \Pr(X_i = 1)$. With n voters, let $S_n = X_1 + \ldots + X_n$ be the number of correct votes (assume that n is odd). Let $P_n = \Pr\left(S_n \geq \frac{n+1}{2}\right)$ be the probability that a majority of voters is correct. Prove the following theorem:

   Theorem 1 *Suppose votes are statistically independent, and $p > \frac{1}{2}$. Then $P_n > p$ and $P_n \to 1$ as $n \to \infty$.*