# Lecture 24: Chapter 10, Section 1 Inference for Quantitative Variable; Confidence Intervals

- Inference for Means vs. Proportions
- Population Standard Deviation Known or Unknown
- Constructing CI for Mean (S.D. Known)
- Checking Normality
- Details of Confidence Interval for Mean

# Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical (discussed in Lectures 21-23)
    - 1 quantitative: confidence intervals, hypothesis tests
    - categorical and quantitative
    - 2 categorical
    - 2 quantitative

# Inference for Proportions or Means: Similarities

- 3 forms of inference (point estimate, CI, test)
- Point est. unbiased estimator for parameter if…?
- **Confidence Interval**: estimate ± margin of error

  = sample stat ± multiplier × s.d. of sample stat
  - Sample stat must be unbiased
  - Sample must be large enough so multiplier is correct

    Note: higher confidence→larger multiplier→wider interval
  - Pop at least 10*n* so s.d. is correct

    Note: larger sample→smaller s.d.→narrower interval

  Correct interpretation of interval; interval related to test.

# Inference for Proportions or Means: Similarities

- **Hypothesis Test**: Does parameter = proposed value?
  - 3 forms of alternative (greater, less, not equal)
  - 4-steps follow 4 processes of statistics
    1. Data production: sample unbiased? $n$ large? pop $\geq 10n$?
    2. Find sample statistic and standardize; is it "large"?
    3. Find $P$-value=prob of sample stat this extreme; is it "small"?
    4. Draw conclusions: reject null hypothesis if $P$-value is small
  - $P$-value for 2-sided alternative twice that for 1-sided
  - Cut-off level $\alpha$ (often 0.05) is probability of Type I Error (false positive)
  - Rejection: if sample stat far from proposed parameter, or $n$ large, or spread small
  - Type II Error (false negative) also possible, especially for small $n$

# Inference for Proportions or Means: Differences

- Different summaries for quantitative variables
  - Population mean $\mu$
  - Sample mean $\bar{x}$
  - Population standard deviation $\sigma$
  - Sample standard deviation $s$

  (For proportions, s.d. could be calculated from $n$ and $p$)
- Standardized statistic not always "$z$"
- No easy Rule of Thumb for what $n$ is large enough to ensure normality; must examine shape of sample data.

# Behavior of Sample Mean *(Review)*

For random sample of size *n* from population with mean $\mu$, sample mean $\bar{X}$ has

- mean $\mu$

  → $\bar{X}$ is *unbiased estimator* of $\mu$

*(sample must be random)*

# Example: *Checking if Estimator is Unbiased*

- **Background**: Anonymous on-line survey of intro stat students (various ages, majors) at a university produced sample mean earnings.
- **Questions:**
  - Is the sample representative of all students at that university? Does it represent *all* college students?
  - Were the values of the variable (earnings) recorded without bias?
- **Responses:**
  - Various ages, majors →

    Socio-economic conditions depend on school
    →
  - Anonymous online survey →

# **Example:** *Point Estimate for $\mu$*

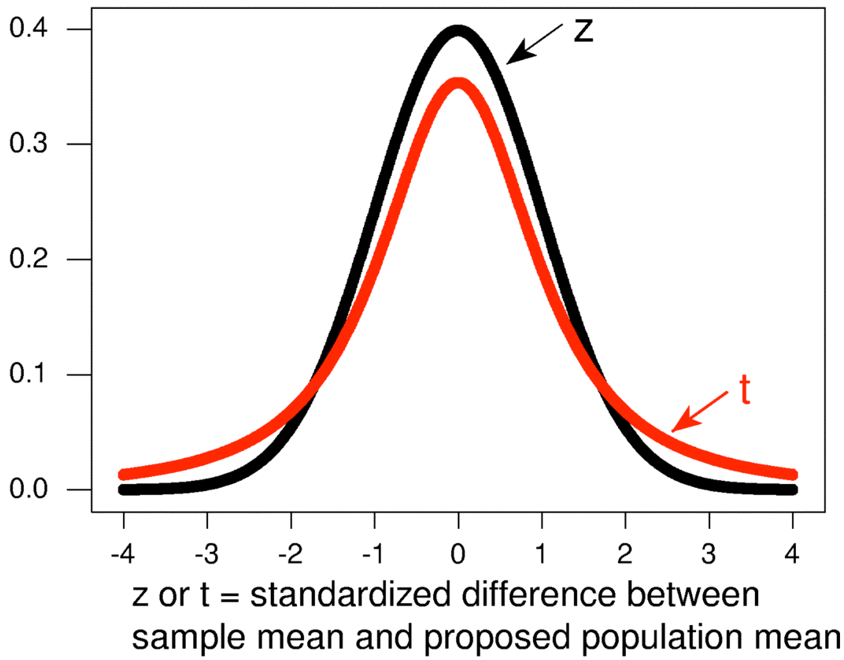
- **Background**: In a representative sample of students at a university, mean earnings were $3,776.
- **Question:** What is our best guess for mean earnings of all students at that university?
- **Response:** $\bar{X}$ is an unbiased estimator for $\mu$ so is our best guess.

***Looking Ahead:*** *For point estimate we don't need to know s.d. For confidence intervals and hypothesis tests, to quantify how good our point estimate is, we must know sigma or estimate it with s. This makes an important difference in procedure. We also need n.*

# Inference About Mean Based on $z$ or $t$

- $\sigma$ known→ standardized $\bar{x}$ is $z$
- $\sigma$ unknown→ standardized $\bar{x}$ is $t$

(may use $z$ if $\sigma$ unknown but $n$ large)




*Inference with t discussed after inference with z*

# Behavior of Sample Mean *(Review)*

For random sample of size $n$ from population with mean $\mu$, standard deviation $\sigma$, sample mean $\bar{x}$ has

- mean $\mu$
- standard deviation $\frac{\sigma}{\sqrt{n}}$
- shape approximately normal for large enough $n$

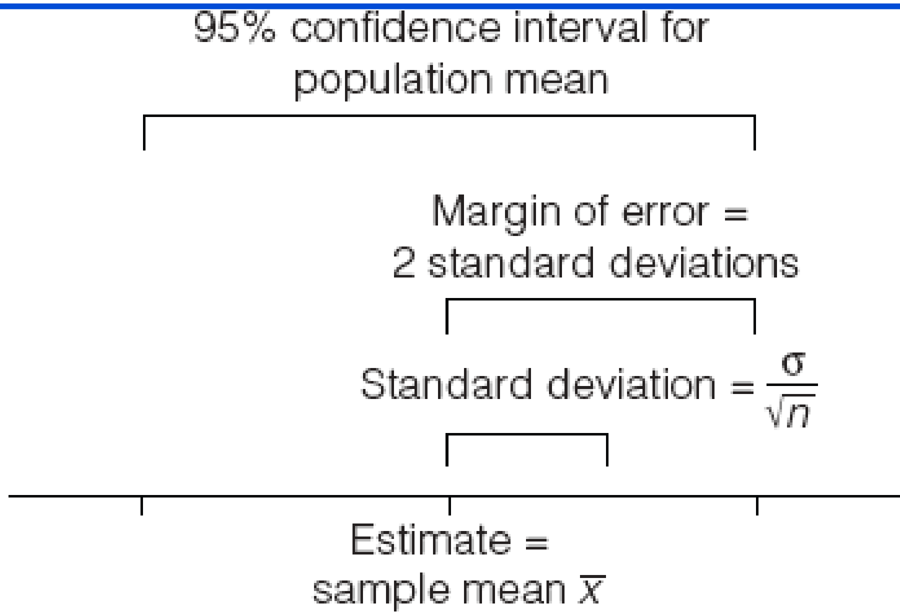
→Probability is 0.95 that $\bar{x}$ is within $2\frac{\sigma}{\sqrt{n}}$ of $\mu$

***Looking Ahead: Probability** results lead to **confidence** interval for* $\mu$.

# Confidence Interval for Population Mean

95% confidence interval for $\mu$ is $\bar{x} \pm 2\frac{\sigma}{\sqrt{n}}$

- Sample must be unbiased
- Population size must be at least 10*n*
- *n* must be large enough to justify multiplier 2 from normal distribution



# Guidelines for Sample Mean Approx. Normal

Can assume shape of $\bar{X}$ for random samples of size *n* is approximately normal if

- Graph of sample data appears normal; or
- Graph of sample data fairly symmetric and *n* at least 15; or
- Graph of sample data moderately skewed and *n* at least 30; or
- Graph of sample data very skewed and *n* much larger than 30

***A Closer Look:*** *Besides examining display, consider what shape we'd expect to see for the variable's distribution.*

# **Example:** *Revisiting Original Question*

- **Background**: Mean yearly earnings for 446 students at a particular university was $3,776. Assume population standard deviation $6,500.
- **Question:** Assuming sample is representative, what interval should contain population mean earnings?
- **Response:** 95% C.I. for $\mu$ is $\bar{x} \pm 2\frac{\sigma}{\sqrt{n}} =$

*A Closer Look: 446 is large enough to offset right skewness.*





Practice: 10.9b-c p.478

## **Example:** *C.I. as Range of Plausible Values*

- **Background**: Mean yearly earnings for 446 students at a particular university was \$3,776. Assume population standard deviation \$6,500. 95% confidence interval for $\mu$ is (3160, 4392).
- **Question:** Is \$5,000 a plausible value for population mean earnings?
- **Response:**

***Looking Ahead:*** *This kind of decision is addressed more formally and precisely with a hypothesis test.*



Practice: 10.9d p.478

# **Example:** *Role of Sample Size in C.I.*

- **Background**: Mean yearly earnings for 446 students at a particular university was \$3,776. Assume population standard deviation \$6,500. 95% confidence interval for $\mu$ is 3,776 ± 616.
- **Question:** What would happen to the C.I. if *n* were one fourth the size (111 instead of 446)?
- **Response:** Divide *n* by 4 →

$$\bar{x} \pm 2\frac{\sigma}{\sqrt{n}} =$$



Practice: 10.9e p.478

# **Example:** *Other Levels of Confidence*

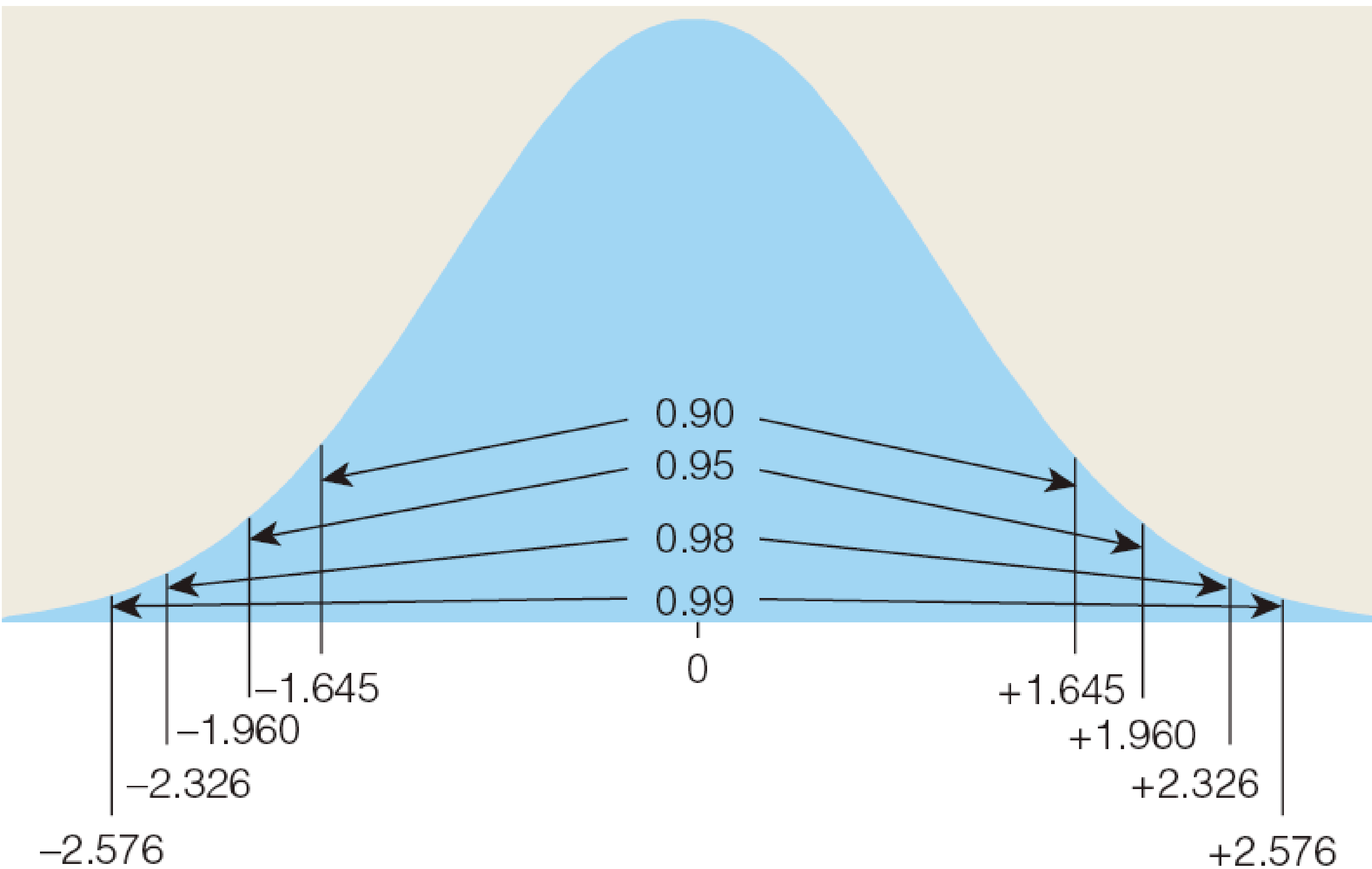
- **Background**: Mean yearly earnings for 446 students at a particular university was \$3,776. Assume population standard deviation \$6,500. A 95% confidence interval for $\mu$

$$= 3776 \pm 2\frac{6500}{\sqrt{446}} = 3776 \pm 616 = (3160, 4392)$$

- **Question:** How would we construct intervals at 90% or 99% confidence?
- **Response:**

# Other Levels of Confidence

"Inside" probabilities correspond to various multipliers.



# Other Levels of Confidence *(Review)*

Confidence level 95% uses multiplier 2. Other levels use other multipliers, based on normal curve.

More precise multiplier for 95% is 1.96 instead of 2.

| Level | Multiplier |
|---|---|
| 90% | 1.645 |
| 95% | 1.960 |
| 98% | 2.326 |
| 99% | 2.576 |

# **Example:** *Other Levels of Confidence*

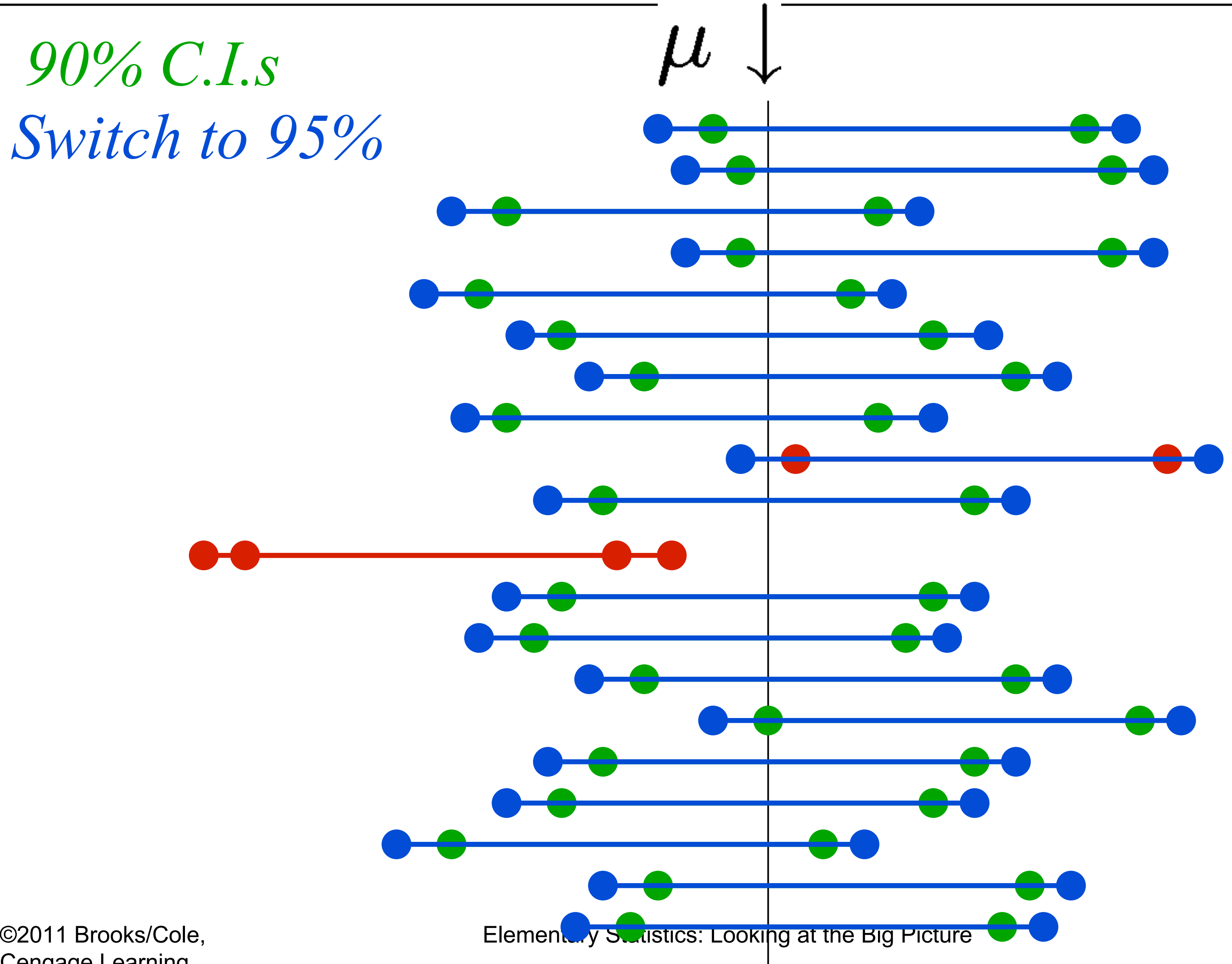
- **Background**: Mean yearly earnings for 446 students at a particular university was \$3,776. Assume population standard deviation \$6,500.
- **Question:** What are 90% and 99% confidence intervals for population mean earnings?
- **Response:** Interval is $3776 \pm \text{multiplier}\frac{6500}{\sqrt{446}}$
  - **90% C.I.** $= (3270, 4282)$
  - **99% C.I.** $= (2983, 4569)$

Tradeoff: higher level of confidence → precise interval

# Wider Intervals ←→ More Confidence

- Consider illustration of many 90% confidence intervals in the long run: 18 in 20 should contain population parameter.
- If they were widened to 95% intervals (multiply s.d. by 2 instead of 1.645), then they'd have a higher probability (19 in 20) of capturing population parameter.

# Wider Intervals ←→ More Confidence



# Interpretation of XX% Confidence Interval

- We are XX% confident that the interval contains the unknown parameter.
- XX% intervals' long-run probability of capturing the unknown parameter is XX%.

# **Example:** *Interpreting Confidence Interval*

- **Background**: A 95% confidence interval for mean U.S. household size $\mu$ is (2.166, 2.714).
- **Question:** Which of the following are true?
  - Probability is 95% that $\mu$ is in the interval (2.166, 2.714).
  - 95% of household sizes are in the interval (2.166, 2.714).
  - Probability is 95% that $\bar{x}$ is in the interval (2.166, 2.714).
  - We're 95% confident that $\bar{x}$ is in interval (2.166, 2.714).
  - We're 95% confident that $\mu$ is in interval (2.166, 2.714).
  - The probability is 95% that our sample produces an interval which contains $\mu$.
- **Response:**





Practice: 10.10 p.479

# Lecture Summary
*(Inference for Means: Confidence Interval)*

- Inference for means vs. proportions
  - Similarities (many)
  - Differences: population s.d. may be unknown
- Constructing CI for mean with $z$ (pop. s.d. known)
- Checking assumption of normality
- Role of sample size
- Other levels of confidence
- Interpreting the confidence interval