Lecture 33: Chapter 12, Section 2
Two Categorical Variables
More About Chi-Square

- Hypotheses about Variables or Parameters
- Computing Chi-square Statistic
- Details of Chi-square Test
- Confounding Variables

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical (discussed in Lectures (21-23)
    - 1 quantitative (discussed in Lectures (24-27)
    - cat and quan: paired, 2-sample, several-sample (Lectures 28-31)
    - 2 categorical
    - 2 quantitative

H_0 and H_a for 2 Cat. Variables *(Review)*

- In terms of variables
  - $\boxed{H_0}$: two categorical variables are $\boxed{\text{not}}$ related
  - H_a: two categorical variables are related
- In terms of parameters
  - H_0: population proportions in response of interest are equal for various explanatory groups
  - $\boxed{H_a}$: population proportions in response of interest are $\boxed{\text{not}}$ equal for various explanatory group

Word "not" appears in Ho about variables, Ha about parameters.

Chi-Square Statistic

- Compute table of counts expected if H_0 true: each is

$$\text{Expected} = \frac{\text{Column total} \times \text{Row total}}{\text{Table total}}$$

  - Same as counts for which proportions in response categories are equal for various explanatory groups
- Compute **chi-square** test statistic χ^2

$$\text{chi-square} = \text{sum of } \frac{(\text{observed - expected})^2}{\text{expected}}$$

"Observed" and "Expected"

Expressions "observed" and "expected" commonly used for chi-square hypothesis tests.

More generally, "observed" is our sample statistic, "expected" is what happens on average in the population when H_0 is true, and there is no difference from claimed value, or no relationship.

Variable(s)	Observed	Expected
1 Categorical	$\hat{p}$	p_o
1 Quantitative	$\bar{x}$	μ_o
1 Cat & 1 Quan	$\bar{x}_d$ $\bar{x}_1 - \bar{x}_2$	0 0
2 Categorical	Observed Counts	Expected Counts

Example: *2 Categorical Variables: Data*

- **Background**: We're interested in the relationship between gender and lenswear.

	contacts	glasses	none	All
female	121	32	129	282
	42.91%	11.35%	45.74%	100.00%
male	42	37	85	164
	25.61%	22.56%	51.83%	100.00%
All	163	69	214	446

- **Question:** What do data show about sample relationship?
- **Response:** Females wear contacts more (______ vs. ______); males wear glasses more (vs.); proportions with none are close (vs.).

Example: *Table of Expected Counts*

- **Background**: We're interested in the relationship between gender and lenswear.

Expected	Contacts	Glasses	None	Total
Female				282
Male				164
Total	163	69	214	446

- **Question:** What counts are expected if gender and lenswear are not related?
- **Response:** Calculate each expected count as

Practice: 12.42b p.623

Example: *"Eyeballing" Obs. and Exp. Tables*

- **Background**: We're interested in the relationship between gender & lenswear.

Chi-square procedure: Compare counts observed to counts expected if null hypothesis were true

Observed	Contacts	Glasses	None	Total
Female	121	32	129	282
Male	42	37	85	164
Total	163	69	214	446

Expected	Contacts	Glasses	None	Total
Female	103	44	135	282
Male	60	25	79	164
Total	163	69	214	446

- **Question:** Do observed and expected counts seem very different?
- **Response:**

Example: *Components for Comparison*

- **Background**: Observed and expected tables:

Observed	Contacts	Glasses	None	Total
Female	121	32	129	282
Male	42	37	85	164
Total	163	69	214	446

Expected	Contacts	Glasses	None	Total
Female	103	44	135	282
Male	60	25	79	164
Total	163	69	214	446

- **Question:** What are the components of chi-square?
- **Response:** Calculate each

Practice: 12.13g p.610

Example: *Components for Comparison*

- **Background**: Components of chi-square are

$$\frac{(121-103)^2}{103}=3.1 \qquad \frac{(32-44)^2}{44}=3.3 \qquad \frac{(129-135)^2}{135}=0.3$$

$$\frac{(42-60)^2}{60}=5.4 \qquad \frac{(37-25)^2}{25}=5.8 \qquad \frac{(85-79)^2}{79}=0.5$$

- **Questions:** Which contribute most and least to the chi-square statistic? What is chi-square? Is it large?
- **Responses:**
  - largest: most impact from
  - smallest: least impact from

Chi-Square Distribution *(Review)*

chi-square = sum of $\frac{(\text{observed - expected})^2}{\text{expected}}$ follows predictable pattern known as

chi-square distribution with df = (r-1) × (c-1)

- r = number of rows (possible explanatory values)
- c = number of columns (possible response values)

Properties of chi-square:

- Non-negative (based on squares)
- Mean=df [=1 for smallest (2×2) table]
- $\boxed{\text{Spread depends on df}}$
- Skewed right

Example: *Chi-Square Degrees of Freedom*

- **Background**: Table for gender and lenswear:

Observed	Contacts	Glasses	None	Total
Female	121	32	129	282
Male	42	37	85	164
Total	163	69	214	446

- **Question:** How many degrees of freedom apply?
- **Response:** row variable (male or female) has $r =$ ___, column variable (contacts, glasses, none) has $c =$ ___.

df =

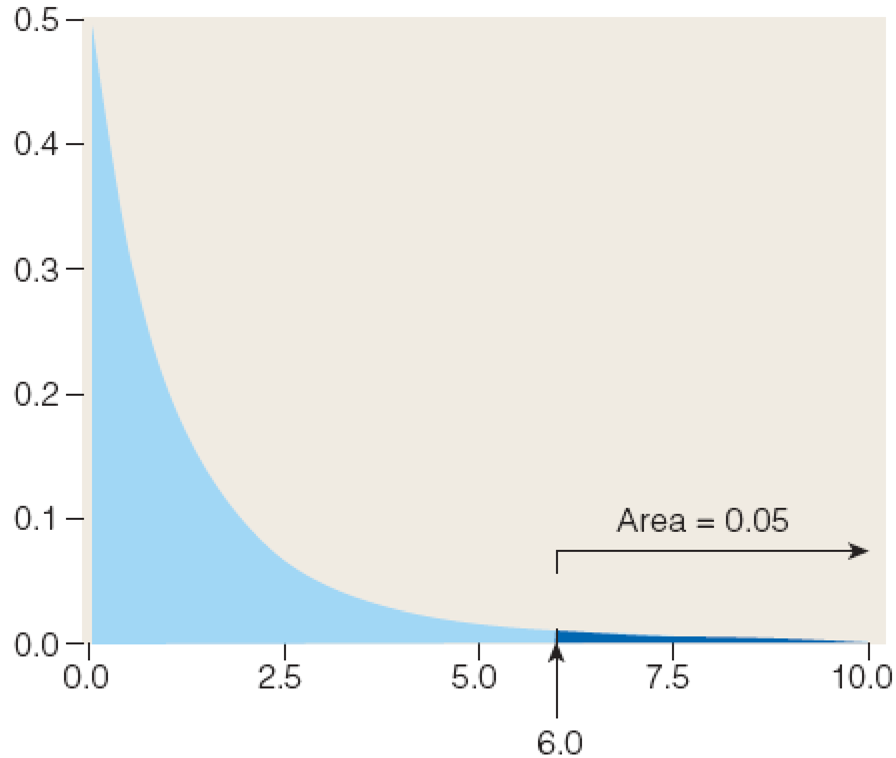
A Closer Look: *Degrees of freedom tell us how many unknowns can vary freely before the rest are "locked in."*

Practice: 12.42a p.623

Chi-Square Density Curve

For chi-square with 2 df, $P(\chi^2 \geq 6) = 0.05$

→ If χ^2 is more than 6, *P*-value is less than 0.05.



Chi-square with 2 df (for 2-by-3 table)

Example: *Assessing Chi-Square*

- **Background**: In testing for relationship between gender and lenswear in 2×3 table, found $\chi^2 = 18.4$.
- **Question:** Is there evidence of a relationship in general between gender and lenswear (not just in the sample)?
- **Response:** For df = (2-1)×(3-1) = 2, chi-square is considered "large" if greater than 6. Is 18.6 large?
  _ Is the *P*-value small?
  Is there statistically significant evidence of a relationship between gender and lenswear?

Example: *Checking Assumptions*

- **Background**: We produced table of expected counts below right:

Observed	Contacts	Glasses	None	Total
Female	121	32	129	282
Male	42	37	85	164
Total	163	69	214	446

Expected	Contacts	Glasses	None	Total
Female	103	44	135	282
Male	60	25	79	164
Total	163	69	214	446

- **Question:** Are samples large enough to guarantee the individual distributions to be approximately normal, so the sum of standardized components follows a χ^2 distribution?
- **Response:**

Example: *Chi-Square with Software*

- **Background**: Some subjects injected under arm with Botox, others with placebo. After a month, reported if sweating had decreased.

```
Expected counts are printed below observed counts
     Decreased NotDecreased Total
Botox       121         40       161
          80.50      80.50
Placebo      40        121       161
          80.50      80.50
Total       161        161       322
Chi-Sq = 20.376 + 20.376 +
         20.376 + 20.376 = 81.503
DF = 1, P-Value = 0.000
```

- **Question:** What do we conclude?
- **Response:** Sample sizes large enough? Proportions with reduced sweating Seem different? *P*-value = ➔diff significant? Conclude Botox reduces sweating?

Guidelines for Use of Chi-Square *(Review)*

- Need random samples taken independently from two or more populations.
- Confounding variables should be separated out.
- Sample sizes must be large enough to offset non-normality of distributions.
- Need populations at least 10 times sample sizes.

Example: *Confounding Variables*

- **Background**: Students of all years: $\chi^2 = 13.6, p = 0.000$

	On Campus	Off Campus	Total	Rate On Campus
Undecided	124	81	205	124/205=60%
Decided	96	129	225	96/225=43%

Underclassmen: $\chi^2 = 0.025, p = 0.873$

Underclassmen	On Campus	Off Campus	Total	Rate On Campus
Undecided	117	55	172	117/172=68%
Decided	82	37	119	82/119=69%

Upperclassmen: $\chi^2 = 1.26, p = 0.262$

Upperclassmen	On Campus	Off Campus	Total	Rate On Campus
Undecided	7	26	33	7/33=21%
Decided	14	92	106	14/106=13%

- **Question:** Are major (dec or not) and living situation related?
- **Response:**

 Practice: 12.49 p.626

Lecture Summary

(Inference for Cat→Cat; More Chi-Square)

- Hypotheses about variables or parameters
- Computing chi-square statistic
  - Observed and expected counts
- Chi-square test
  - Calculations
  - Degrees of freedom
  - Chi-square density curve
  - Checking assumptions
  - Testing with software
- Confounding variables