

Lecture 18: more Chapter 7, Section 3 Continuous Random Variables; Tails of the Normal Curve

- Preview Two Forms of Inference
- 68-95-99.7 Rule; Rule for Tails (90-95-98-99)
- Standard Normal Tail-Probability Problems
- Non-standard Tail-Probability Problems

Looking Back: Review

- 4 Stages of Statistics
 - Data Production (discussed in Lectures 1-4)
 - Displaying and Summarizing (Lectures 5-12)
 - Probability
 - Finding Probabilities (discussed in Lectures 13-14)
 - Random Variables (introduced in Lecture 15)
 - Binomial (discussed in Lecture 16)
 - Normal
 - Sampling Distributions
 - Statistical Inference

Tails of Normal Curve in Inference

- **Goal:** Perform inference in 2 forms about unknown population proportion or mean:
 - Produce interval that has high probability (such as 90%, 95%, or 99%) of containing unknown population parameter
 - Test if proposed value of population proportion or mean is implausible (low probability---1% or 5%---of sample data)
- **Strategy:** Focus on tails of normal curve, in the vicinity of $Z=+2$ or $Z=-2$.

68-95-99.7 Rule for Z (Review)

For standard normal Z , the probability is

- 68% that Z takes a value in interval $(-1, +1)$
 - 95% that Z takes a value in interval $(-2, +2)$
 - 99.7% that Z takes a value in interval $(-3, +3)$
- Need to fine-tune information for probability at or near 95%.

90-95-98-99 Rule for Standard Normal Z

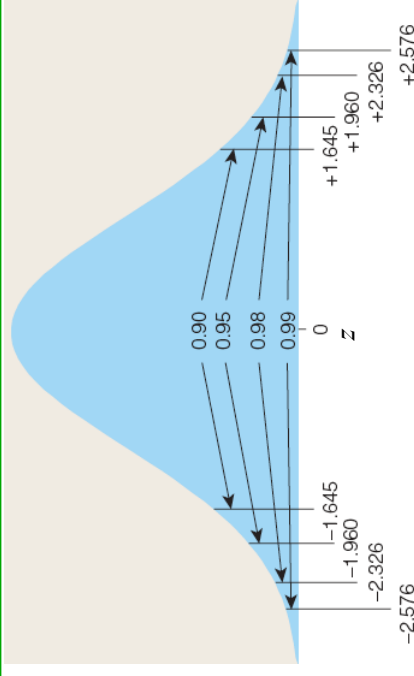
For standard normal Z, the probability is

- 0.90 that Z takes a value in interval (-1.645, +1.645)
- 0.95 that Z takes a value in interval (-1.960, +1.960)
- 0.98 that Z takes a value in interval (-2.326, +2.326)
- 0.99 that Z takes a value in interval (-2.576, +2.576)

Looking Back: The 68-95-99.7 Rule rounded 0.9544 for 2 s.d.s to 0.95. For exactly 95%, need 1.96 s.d.s.

90-95-98-99 Rule: “Inside” Probabilities

Looking Ahead: This will be useful for “confidence intervals”.



90-95-98-99 Rule: “Outside” Probabilities

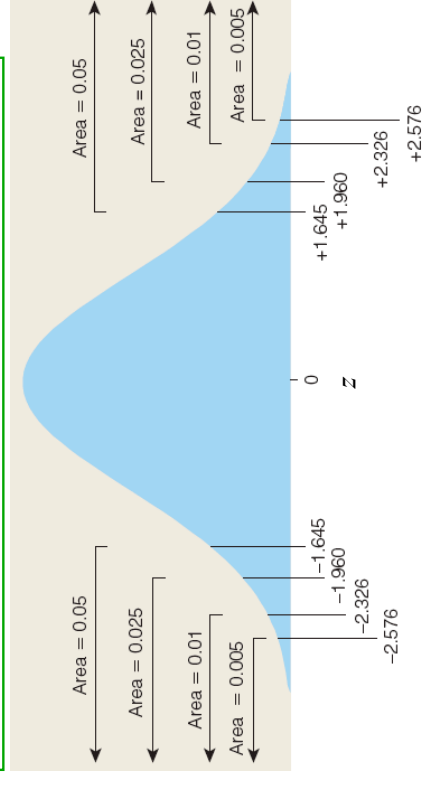
For standard normal Z, the probability is

- 0.05 that $Z < -1.645$ and 0.05 that $Z > +1.645$
- 0.025 that $Z < -1.96$ and 0.025 that $Z > +1.96$
- 0.01 that $Z < -2.326$ and 0.01 that $Z > +2.326$
- 0.005 that $Z < -2.576$ and 0.005 that $Z > +2.576$

Looking Back: These follow from the inside probabilities, using the fact that the normal curve is symmetric with total area 1.

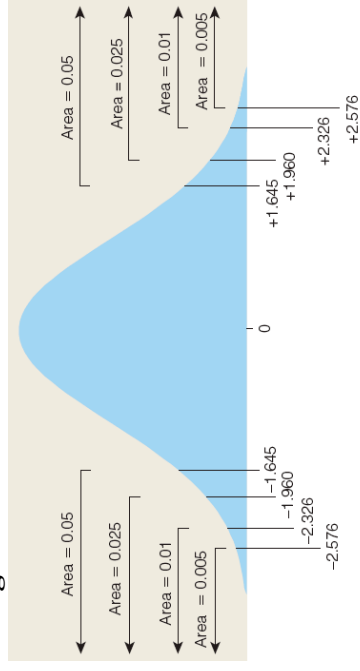
90-95-98-99 Rule: “Outside” Probabilities

Looking Ahead: This will be useful for “hypothesis tests”.



Example: Finding Tail Probabilities

- Background: Refer to sketch.



- Question: What is $P(Z > +2.326)$?

- Response:

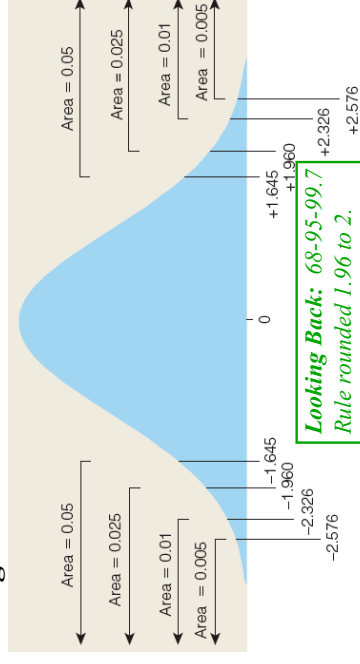
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Elementary Statistics: Looking at the Big Picture

Example: Finding Tail Probabilities

- Background: Refer to sketch.



- Question: What is $P(Z < -1.96)$?

- Response:

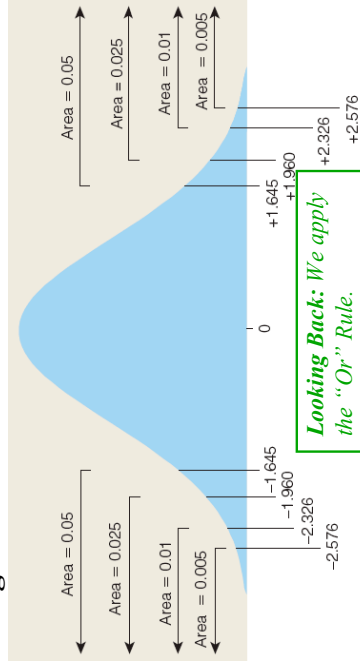
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Elementary Statistics: Looking at the Big Picture

Example: Finding Tail Probabilities

- Background: Refer to sketch.



- Question: What is $P(|Z| > 1.96)$?

- Response:

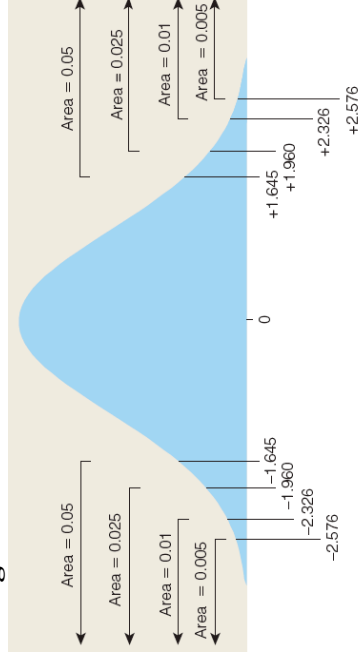
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Elementary Statistics: Looking at the Big Picture

Example: Given Probability, Find z

- Background: Refer to sketch.



- Question: $0.05 = P(Z < ?)$

- Response:

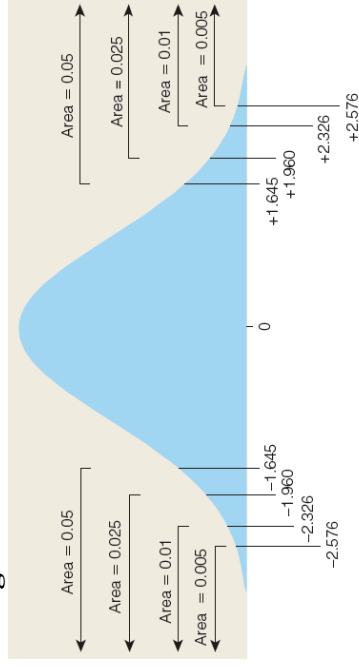
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Elementary Statistics: Looking at the Big Picture

Example: Given Probability, Find z

- **Background:** Refer to sketch.



- **Question:** $0.005 = P(Z > ?)$

- **Response:**

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Non-Standard Normal Problems (Review)

To find probability, given non-standard normal x , first standardize: $z = \frac{x - \mu}{\sigma}$ then find probability (area under z curve).

To find non-standard x , given probability, find z then unstandardize: $x = \mu + z\sigma$

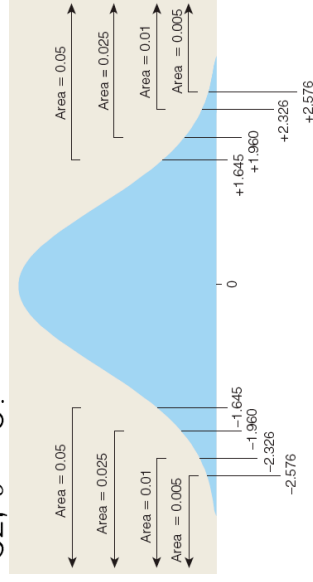
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Example: Given x , Find Probability

- **Background:** Women's waist circumference X (in.) normal; $\mu = 32$, $\sigma = 5$.



- **Question:** What is $P(X > 43)$?

- **Response:** $z =$ _____, between _____ and _____ so $P(X > 43)$ is between _____ and _____.

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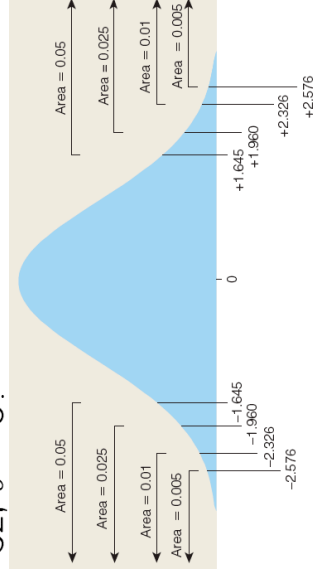
Elementary Statistics: Looking at the Big Picture

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Practice: 7.63 p.333

Example: Given x , Find Probability

- **Background:** Women's waist circumference X (in.) normal; $\mu = 32$, $\sigma = 5$.



- **Question:** What is $P(X < 23)$?

- **Response:** $z =$ _____, between _____ and _____ so $P(X < 23)$ is between _____ and _____.

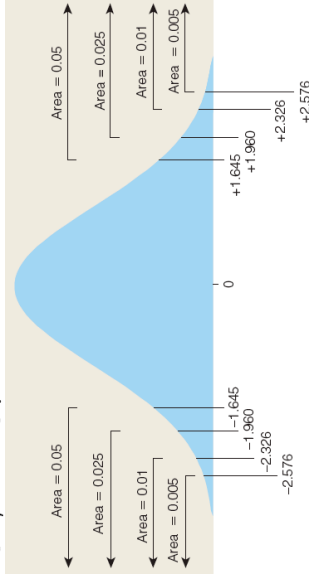
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Example: Given x , Find Probability

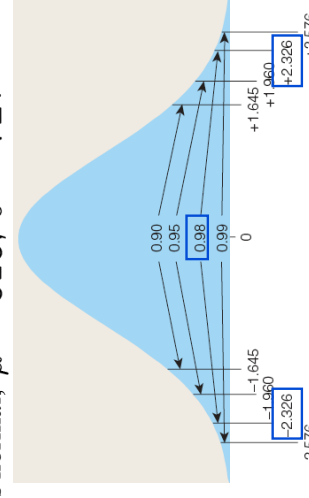
- **Background:** Women's waist circumference X (in.) normal; $\mu = 32$, $\sigma = 5$.



- **Question:** What is $P(X > 39)$?
- **Response:** $z =$ _____
- so $P(X > 39)$ is _____

Example: Given Probability, Find x

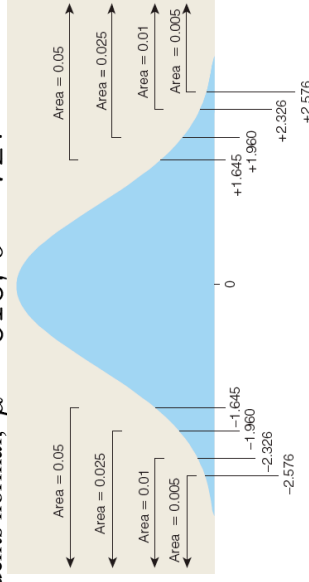
- **Background:** Math SAT score X for population of college students normal; $\mu = 610$, $\sigma = 72$.



- **Question:** 0.98 is probability of X in what interval?
- **Response:** Prob. 0.98 has z from _____ to _____ so x is from _____ to _____

Example: Given Probability, Find x

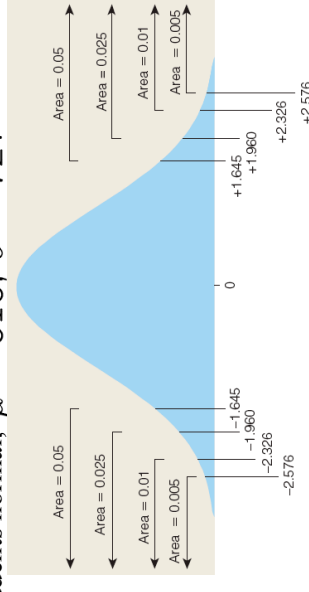
- **Background:** Math SAT score X for population of college students normal; $\mu = 610$, $\sigma = 72$.



- **Question:** Bottom 5% are below what score?
- **Response:** Bottom 0.05 has $z =$ _____
- SO $X =$ _____

Example: Given Probability, Find x

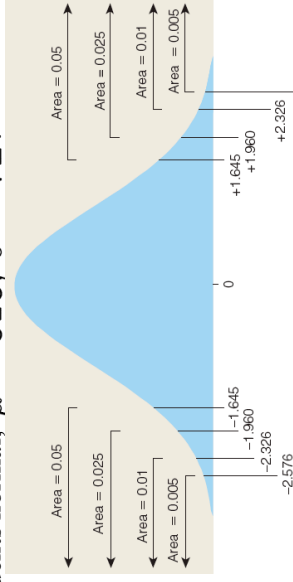
- **Background:** Math SAT score X for population of college students normal; $\mu = 610$, $\sigma = 72$.



- **Question:** Top half a percent were above what score?
- **Response:** Top 0.005 has $z =$ _____
- SO $X =$ _____

Example: Comparing to a Given Probability

- Background: Math SAT score X for population of college students normal; $\mu = 610$, $\sigma = 72$.



- Question: Is $P(X < 480)$ more or less than 0.01?
- Response: 480 has $z =$ _____, prob. is _____.
- Since -1.81 is _____, prob. is _____.

Example: More Comparisons to Given Probability

- Background: $0.01 = P(Z < -2.326) = P(Z > +2.326)$
- Question: Are the following >0.01 or <0.01 ?
 - $P(Z > +2.4)$; $P(Z > +1.9)$; $P(Z < -3.7)$; $P(Z < -0.4)$

Response:

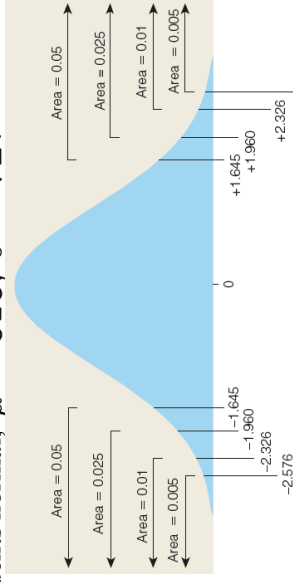
- $P(Z > +2.4)$ _____ 0.01, since +2.4 is _____ extreme than +2.326
- $P(Z > +1.9)$ _____ 0.01, since +1.9 is _____ extreme than +2.326
- $P(Z < -3.7)$ _____ 0.01, since -3.7 is _____ extreme than -2.326
- $P(Z < -0.4)$ _____ 0.01, since -0.4 is _____ extreme than -2.326

A Closer Look: As z gets more extreme, the tail probability gets _____.

Looking Ahead: When we perform inference in Part 4, some key decisions will be based on how a normal probability compares to a set value like 0.01 or 0.05.

Example: Comparing to a Given Probability

- Background: Math SAT score X for population of college students normal; $\mu = 610$, $\sigma = 72$.



- Question: Is $P(X < 480)$ more or less than 0.01?
- Response: 480 has $z =$ _____, prob. is _____.
- Since -1.81 is _____, prob. is _____.

Example: More Comparisons to Given Probability

- Background: $0.01 = P(Z < -2.326) = P(Z > +2.326)$
- Question: Are the following >0.01 or <0.01 ?
 - $P(Z > +2.4)$; $P(Z > +1.9)$; $P(Z < -3.7)$; $P(Z < -0.4)$

Response:

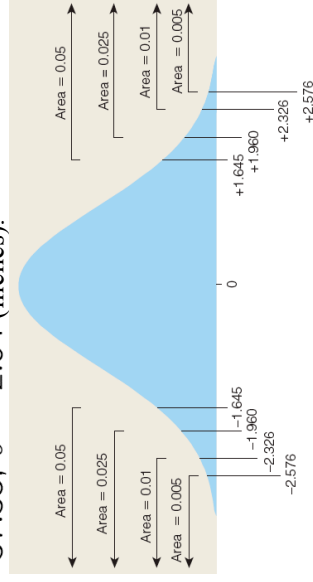
- $P(Z > +2.4)$ _____ 0.01, since +2.4 is _____ extreme than +2.326
- $P(Z > +1.9)$ _____ 0.01, since +1.9 is _____ extreme than +2.326
- $P(Z < -3.7)$ _____ 0.01, since -3.7 is _____ extreme than -2.326
- $P(Z < -0.4)$ _____ 0.01, since -0.4 is _____ extreme than -2.326

A Closer Look: As z gets more extreme, the tail probability gets _____.

Looking Ahead: When we perform inference in Part 4, some key decisions will be based on how a normal probability compares to a set value like 0.01 or 0.05.

Example: Practice with 90-95-98-99 Rule

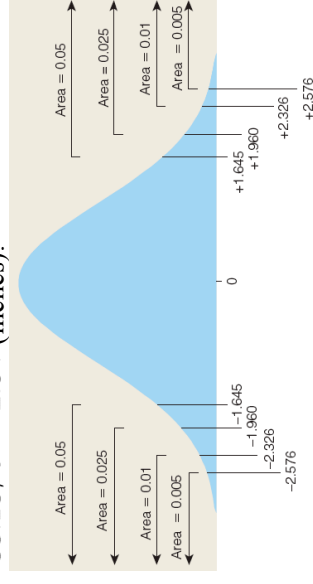
- Background: Male chest sizes X normal; $\mu = 37.35$, $\sigma = 2.64$ (inches).



- Question: $P(X > 45)$ is in what range?
- Response: 45 has $z =$ _____ and _____ so $P(X > 45)$ is between _____ and _____.

Example: More Practice with 90-95-98-99 Rule

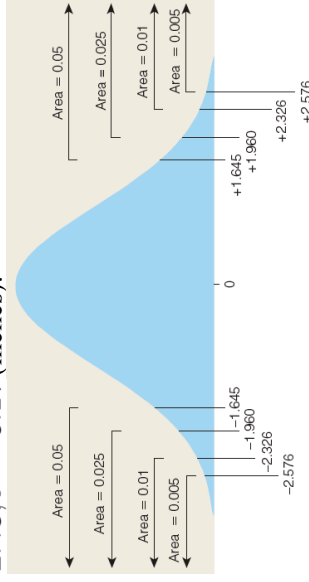
- Background: Female chest sizes X normal; $\mu = 35.15$, $\sigma = 2.64$ (inches).



- Question: $P(X < 28.8)$ is in what range?
- Response: 28.8 has $z =$ _____ and _____ so $P(X < 28.8)$ is between _____ and _____.

Example: 90-95-98-99 Rule, Given Probability

- **Background:** Male ear lengths X normal; $\mu = 2.45$, $\sigma = 0.17$ (inches).



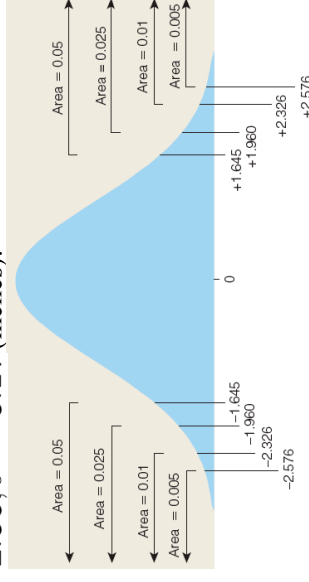
- **Question:** Top 5% are greater than what value?
- **Response:** Top 5% are above $z =$ _____

SO $X =$ _____
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Example: More Use of Rule, Given Probability

- **Background:** Female ear lengths X normal; $\mu = 2.06$, $\sigma = 0.17$ (inches).



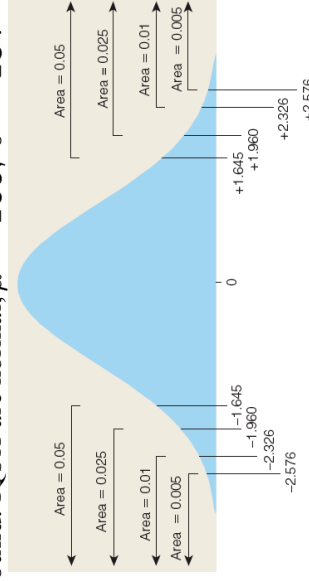
- **Question:** Bottom 2.5% are less than what value?
- **Response:** Bottom 2.5% are below $z =$ _____

SO $X =$ _____
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Example: Sketching Curve with 90-95-98-99 Rule

- **Background:** IQs X are normal; $\mu = 100$, $\sigma = 15$.



- **Question:** What does the Rule tell us about the IQ curve?
- **Response:** _____

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Lecture Summary (Tails of Normal Curve)

- Two forms of inference
 - Interval estimate
 - Test if value is plausible
- 68-95-99.7 Rule and Rule for tails of normal curve
- Reviewing normal probability problems
 - Given x , find probability
 - Given probability, find x
- Focusing on tails of normal curve
 - Standard normal problems
 - Non-standard normal problems

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