# Lecture 28: Chapter 11, Section 1 Categorical & Quantitative Variable Inference in Paired Design

- Inference for Relationships: 2 Approaches
- Cat→Quan Relationship: 3 Designs
- Inference for Paired Design
- Paired vs. Ordinary, *t* vs. *z*



## Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical (discussed in Lectures 21-23)
    - 1 quantitative (discussed in Lectures 24-27)
    - cat and quan: paired, 2-sample, several-sample
    - 2 categorical
    - 2 quantitative



## Inference for Relationships: Two Approaches

- $H_0$ and $H_a$ about variables: not related or related
  - Applies to all three C→Q, C→C, Q→Q
- $H_0$ and $H_a$ about parameters: equality or not
  - C→Q: pop means equal? (mean diff=0? for paired)
  - C→C: pop proportions equal?
  - Q→Q: pop slope equals zero?

Either way, often do *test* before *confidence interval.*

1. Are variables related?
2. If so, quantify: how different are the parameters?



## Example: C→Q *Test Relationship or Parameters*

- **Background**: Research question: "For all students at a university, are their Math SATs related to what year they're in?"
- **Question:** How can we formulate this in terms of parameters?
- **Response:**

***Looking Ahead:*** *This is a several-sample design, to be discussed after paired and two-sample.*

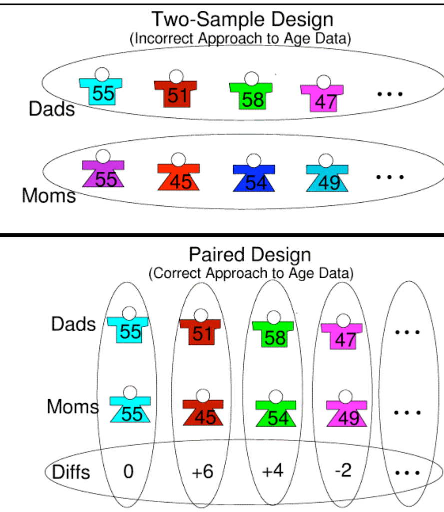
 *Practice: 11.2b p.525* 

## Inference Methods for Cat→Quan Relationship

- Paired: reduces to 1-sample $t$ (already covered)
- Two-Sample: 2-sample $t$ (similar to 1-sample $t$)
- Several-Sample: need new distribution ($F$)



## Paired Data: Incorrect vs. Correct Approach






## **Example:** *Paired vs. Two-Sample Summary*

- **Background**: Research Question: "Are 'age of parent' and 'sex of parent' related for population of students at a university?"
- **Question:** Which output has enough info to do inference?

Descriptive Statistics: DadAge, MomAge

| Variable | N | N* | Mean | Median | TrMean | StDev |
|---|---|---|---|---|---|---|
| DadAge | 431 | 15 | 50.831 | 50.000 | 50.491 | 6.167 |
| MomAge | 441 | 5 | 48.406 | 48.000 | 48.166 | 5.511 |

Descriptive Statistics: AgeDiff

| Variable | N | N* | Mean | Median | TrMean | StDev |
|---|---|---|---|---|---|---|
| AgeDiff | 431 | 15 | 2.448 | 2.000 | 2.171 | 3.877 |

- **Response:**

***Looking Ahead:*** *We will standardize with the StDev of the differences, which cannot be found from the individual StDevs because of dependence.*

*Practice: 11.4b p.526*



## **Example:** *Consider Summaries in Paired Design*

- **Background**: To see if 'age of parent' and 'sex of parent' are related for population of students at a university, took sampled DadAge minus MomAge.

Descriptive Statistics: AgeDiff

| Variable | N | N* | Mean | Median | TrMean | StDev |
|---|---|---|---|---|---|---|
| AgeDiff | 431 | 15 | 2.448 | 2.000 | 2.171 | 3.877 |

- **Question:** Which parent tended to be older in the sample?
- **Response:**

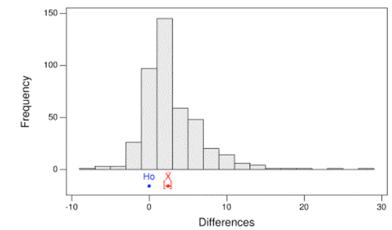
# Example: *Display in Paired Design*

- **Background**: To see if 'age of parent' and 'sex of parent' are related for population of students at a university, took sampled DadAge minus MomAge.
- **Question:** How do we display the data?
- **Response:**



# Example: *Display in Paired Design*

- **Background**: Histogram of age differences:




- **Question:** What does the histogram show?
- **Response:** Age differences have
  - Center: around _____ (dads tend to be about ____ yrs older)
  - Spread: most diffs within ________ yrs or mean)
  - Shape: _____________ (a few dads much older than wife)



# Notation in Paired Study

- Differences have
  - Sample mean $\bar{x}_d$
  - Population mean $\mu_d$
  - Sample standard deviation $s_d$
  - Population standard deviation $\sigma_d$



# Test Statistic in Paired Study

- Start with ordinary 1-sample statistic $t = \frac{\bar{x}-\mu_o}{s/\sqrt{n}}$
- Substitute $\bar{x}_d,\ s_d$ for ordinary summaries $\bar{x},\ s$
- Substitute 0 for $\mu_o$ ($H_0$ will claim $\mu_d = 0$ )
- Result is paired *t* statistic: $t = \frac{\bar{x}_d - 0}{s_d/\sqrt{n}}$

## Example: *Paired t Test*

- **Background**: Paired test on students' parents' ages:

```
Paired T for DadAge - MomAge
                 N      Mean     StDev   SE Mean
DadAge         431    50.831     6.167     0.297
MomAge         431    48.383     5.258     0.253
Difference     431     2.448     3.877     0.187
95% CI for mean difference: (2.081, 2.815)
T-Test of mean difference = 0 (vs not = 0): T-Value = 13.11  P-Value = 0.000
```

- **Question:** What does output tell about formal test?
- **Response:** Testing
  - Unbiased? ____ $n$=431 large? ____ Pop≥10(431)? ____
  - $\bar{x}_d$= ________, $t$ = ________ Large? ______
  - $P$-value = ______ Small? _____
  - Conclude pop mean diff =0? ____Sex and age related? ____



## Example: *One- or Two-Sided $H_a$ in Paired Test*

- **Background**: Paired test on students' parents' ages:

```
Paired T for DadAge - MomAge
                 N      Mean     StDev   SE Mean
DadAge         431    50.831     6.167     0.297
MomAge         431    48.383     5.258     0.253
Difference     431     2.448     3.877     0.187
95% CI for mean difference: (2.081, 2.815)
T-Test of mean difference = 0 (vs not = 0): T-Value = 13.11  P-Value = 0.000
```

- **Response:** Replace $H_a : \mu_d \neq 0$ with __________
  - $P$-value would be ________________
  - Conclude fathers in general are older? ______



## Example: *Paired vs. Ordinary t vs. z*

- **Background**: Paired test on 431 students' parents' ages resulted in paired $t$-statistic +13.11.
- **Question:** What does this tell us about the $P$-value?
- **Response:**

  Paired $t$ same as ordinary $t$ distribution

  → Ordinary $t$ basically same as $z$ for large $n$

  →13.11 sds above mean unusual? ____→$P$-val = ________

  →Evidence that mean age diff is non-zero in pop.? ____

**Note:** for extreme $t$ statistics, software not needed to estimate $P$-value.



## Confidence Interval in Paired Design

Confidence interval for $\mu_d$ is

$$\bar{x}_d \pm \text{multiplier}\frac{s_d}{\sqrt{n}}$$

- Multiplier from $t$ distribution with $n$-1 df
- Multiplier smaller for lower confidence
- Multiplier smaller for larger df

If $n$ is small, diffs need to be approx. normal. (Same guidelines as for 1-sample $t$)

## Guidelines: Sample Mean Diff Approx. Normal

Can assume shape of $\bar{X}_d$ for random samples of $n$ pairs is approximately normal if

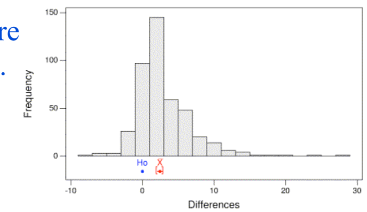
- Graph of sample diffs appears normal; or
- Graph of sample diffs fairly symmetric and $n$ at least 15; or
- Graph of sample diffs moderately skewed and $n$ at least 30; or
- Graph of sample diffs very skewed and $n$ much larger than 30



## **Example:** *Paired Confidence Interval*

- **Background**: Sample of 431 students' parents' age differences have mean +2.45, s.d. 3.88.
- **Question:** What is a 95% confidence interval for population mean age difference?
- **Response:** Since $n$ is so large, $t$ multiplier __________ for 95% confidence. (Also, skewed hist. OK.)

Pretty sure population of fathers are older by about ____ to ____ years.

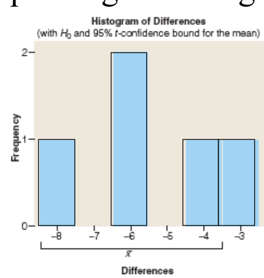





## **Example:** *Checking Conditions for Paired t*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

| City | Highway | MileageDiff |
|---|---|---|
| 20 | 28 | −8 |
| 34 | 40 | −6 |
| 26 | 32 | −6 |
| 30 | 34 | −4 |
| 22 | 25 | −3 |




- **Question:** Is paired $t$ procedure appropriate?
- **Response:** Histogram ____________________



## **Example:** *Paired Test and Confidence Interval*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

```
Paired T for City - Highway
                  N      Mean     StDev   SE Mean
City              5     26.40      5.73      2.56
Highway           5     31.80      5.76      2.58
Difference        5    -5.400     1.949     0.872
95% upper bound for mean difference: -3.541
T-Test of mean difference = 0 (vs < 0): T-Value = -6.19  P-Value = 0.002
95% CI for mean difference: (-7.820, -2.980)
```

- **Question:** What does the output tell us?
- **Response:**
  - $P$-val=0.002→ ____________________
  - C.I.→hway av about ___ to ___ mpg better in pop of cars

## Example: *Paired Confidence Interval by Hand*

- **Background**: Mileage differences for 5 cars, city minus highway, had mean -5.40, s.d. 1.95.
- **Question:** What else is needed to set up a 95% confidence interval by hand for population mean difference?
- **Response:** Need ____________________ (obtained from table before software was available)

  Interval is

Note: $n$ very small→ $t$ multiplier closer to 3 than to 2.



## Example: *Relating Test and Confidence Interval*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

```
Paired T for City - Highway
                 N     Mean    StDev  SE Mean
City             5    26.40     5.73     2.56
Highway          5    31.80     5.76     2.58
Difference       5   -5.400    1.949    0.872
95% upper bound for mean difference: -3.541
T-Test of mean difference = 0 (vs < 0): T-Value = -6.19  P-Value = 0.002
95% CI for mean difference: (-7.820, -2.980)
```

- **Question:** How is $P$-value consistent with C.I.?
- **Response:**
  - Small $P$-value→conclude $H_a$: pop mean of diffs ____
  - Confidence interval shows only________ numbers are plausible values for mean of diffs (entire C.I. ________)



## Example: *Switching Columns in Paired Design*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

```
Paired T for City - Highway
                 N     Mean    StDev  SE Mean
City             5    26.40     5.73     2.56
Highway          5    31.80     5.76     2.58
Difference       5   -5.400    1.949    0.872
95% upper bound for mean difference: -3.541
T-Test of mean difference = 0 (vs < 0): T-Value = -6.19  P-Value = 0.002
95% CI for mean difference: (-7.820, -2.980)
```

- **Question:** What would change if we took highway minus city?
- **Response:** Since we suspect higher on highway,
  - Change to Highway-City and sign in $H_a$ changes to ____
  - Sample mean of diffs would be ______ and $t$ = ______
  - $P$-value still 0.002, reject $H_0$→ ________________
  - Confidence interval would be ________________



## Lecture Summary
*(Inference for Cat→Quan; Paired)*

- Inference for relationships
  - Focus on variables
  - Focus on parameters
- cat→quan relationship: paired, 2- or several-sample
- Inference for paired design
  - Output
  - Display
  - Notation
  - Test statistic
  - Form of alternative
- Paired $t$ vs. ordinary $t$ vs. $z$
- Paired confidence interval vs. hypothesis test