Lecture 21/Chapter 18

When Intuition Differs from Relative Frequency

- Birthday Problem and Coincidences
- Gambler's Fallacy
- Confusion of the Inverse
- Expected Value: Short Run vs. Long Run

Psychological Influences *(Review)*

Certainty effect
Pseudocertainty effect
Availability heuristic
Anchoring
Representativeness heuristic
Conjunction fallacy
Forgotten base rates
Optimism
Conservatism
Overconfidence

These phenomena focused on misguided personal probabilities. Today the focus is on solving for actual probabilities that are counter-intuitive. We'll also discuss *gambler's fallacy* and *"law" of small numbers*.

Example: *Shared Birthdays*

- **Background**: Students were asked "what do you think is the probability that at least 2 people in this room have the same birthday?" and their responses varied all the way from 0% to 100% (average 29%, sd 32%).
- **Question:** Who is right? Why were so many wrong?
- **Response:** Systematically apply Rules 1, 2, and 3 to solve for the probability that
  - At least 2 in a group of 3 share the same birthday[_________]
  - At least 2 in a group of 10 share the same birthday
  - At least 2 in a group of 80 share the same birthday [________]

Example: *Why is Birthday Solution Unintuitive?*

- **Background**: Most people drastically underestimate the probability that at least 2 people in a room share the same birthday.
- **Question:** Why?
- **Response:**

Example: *Coincidences*

- **Background**: We say it's a coincidence when two people in the room share the same birthday.
- **Question:** How do we define "coincidence"?
- **Response:** A coincidence is

Example: *A Coincidence Story*

- **Background**: A really surprising coincidence happened to me…
- **Questions:** Should we be surprised by coincidences? Do they defy the laws of probability?
- **Response:**

Example: *"With Love All Things Are Possible"*

- **Background**: A letter describes a variety of coincidences…
- **Question:** Are they surprising?
- **Response:**

Example: *What's Likelier in 6 Coin Tosses?*

- **Background**: Students were asked: In 6 tosses of a fair coin, circle which is the more likely outcome: (a) HHTHTT (b) HHHHHH (c) equally likely
- **Question:** Why did 29/82=35% pick (a) and only 1/82=1% pick (b)? [Correct answer is (c): even though 3 heads and 3 tails in *any* order is likelier than all 6 heads, the probability of each specific sequence of heads and tails is $\left(\frac{1}{2}\right)^6$.]
- **Response:** (________________) People believe random events should be self-correcting; (_____________________) People believe that a population proportion must hold for small samples.

Example: *Boy or Girl Next Time?*

- **Background**: A couple has had 3 boys so far.
- **Question:** What is the probability that their next child is a girl…

  (a) if the genetic probability of a girl each time is 0.5?

  (b) if the couple has been picking children from a cabbage patch of 8 babies, which started with half each boys and girls?
- **Response:** (a) ______ (b) ________

Example: *Boy or Girl Next Time?*

- **Background**: People are inclined to believe that (a) since 50% of all births are girls, close to 50% in a given family should be girls; and (b) if a couple has had many boys, they are "due" for a girl.
- **Questions:** Which of these describes the gambler's fallacy (you're more likely to win after many losses)? Which is belief in the "law" of small numbers (small samples should be highly representative of the larger population)?
- **Response:** (__) is the gambler's fallacy; (__) is belief in the "law" of small numbers.

Example: *Comparing Poker Probabilities*

- **Background**: Poker hands dealt from separate decks:

Case #1: I tell you that a student has an ace in his poker hand of 5 cards.

Case #2: I tell you that a student has the ace of spades in her poker hand of 5 cards.

- **Question:** In which case is the probability of another ace higher (or are they the same)?
- **Response:** ________

probability of one ace plus at least one more / probability of at least one ace

probability of ace of spades plus at least one more ace / probability of the ace of spades

Example: *Comparing Poker Probabilities*

- **Background**: Poker hands dealt from separate decks:

Case #1: I tell you that a student has an ace in his poker hand of 5 cards.

Case #2: I tell you that a student has the ace of spades in her poker hand of 5 cards.

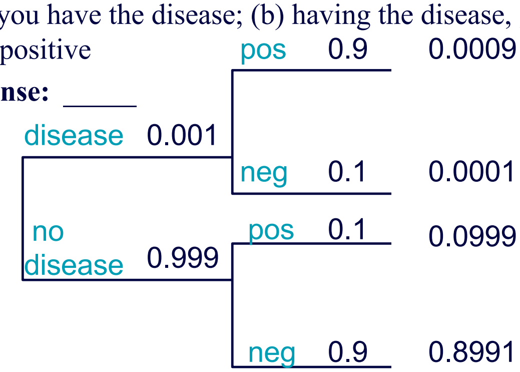
- **Question:** Why do people think the probability of another ace is higher in Case 1?
- **Response:**

Example: *Chance of Disease If Tested Positive?*

- **Background**: Suppose 1 in 1000 people have a certain disease. The chance of correctly testing positive when a person actually has the disease is 90%. The chance of incorrectly testing positive when a person does not have the disease is 10%. If someone tests positive for the disease, what is the chance of actually having it?
- **Question:** Were students' estimates (av 74%) close?
- **Response:** Used a tree diagram to show…
  - Prob of diseased= 0.001
    - Given diseased, prob of testing positive=0.90
    - Given diseased, prob of testing negative=0.10
  - Prob of not diseased= 0.999
    - Given not diseased, prob of testing positive=0.10
    - Given not diseased, prob of testing negative=0.90

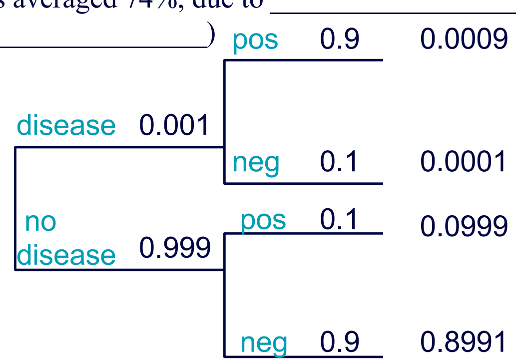
Example: *Probability of A given B or B given A*

- **Background**: The probability of disease is 0.001; probability of testing positive is 0.9 if you have the disease, 0.1 if you don't.
- **Question:** Are these probabilities the same? (a) testing positive, given you have the disease; (b) having the disease, given you tested positive
- **Response:** _____



Example: *Probability of A given B or B given A*

- **Response**: (a) The probability of testing positive, given you have the disease, is 0.9 (b) The probability of having the disease, given you tested positive, is 0.0009/(0.0009+0.0999)=0.0089. (Class guesses averaged 74%, due to ____________________ or ____________________)



Definitions

base rate: probability of having the disease

sensitivity: probability of a correct positive test

specificity: probability of a correct negative test

Example: *Sensitivity and Specificity*

- **Background**: In a certain population, the probability of HIV is 0.001. The probability of testing positive is 0.98 if you have HIV, 0.05 if you don't.
- **Questions:** What is the sensitivity of the test? What is the specificity?
- **Responses:**

  Sensitivity is probability of correct positive: ______.
  Specificity is probability of correct negative: ______.

Example: *Expected Value, Short Run & Long Run*

- **Background**: Respondents were asked to choose between (a) and (b) in each case:

#1. (a) guaranteed gift of $240
 (b) 25% chance to win $1000 and 75% chance to win $0
#2. (a) sure loss of $740
 (b) 75% chance to lose $1000 and 25% chance to lose $0
#3. (a) a 1 in 1000 chance to win $5000
 (b) A sure gain of $6
#4. (a) a 1 in 1000 chance of losing $5000
 (b) A sure loss of $6

- **Question:** Which choices are preferred?
- **Response:** #1(__), #2(__), #3(__), #4 (__)

Example: *Expected Value, Short Run & Long Run*

- **Background**: Respondents were asked to choose between (a) and (b) in each case:

#1. (a) guaranteed gift of $240
 (b) 25% chance to win $1000 and 75% chance to win $0

- **Questions:** Which has a higher expected value? Why was the other choice preferred?
- **Response:** (b) ______________________ is higher than $240 but 75/84=89% of students preferred (a) because

  ____

Example: *Expected Value, Short Run & Long Run*

- **Background**: Respondents were asked to choose between (a) and (b) in each case:

#2. (a) sure loss of $740
 (b) 75% chance to lose $1000 and 25% chance to lose $0

- **Questions:** Which has a higher expected value? Why was the other choice preferred?
- **Response:**(a) -$740 is more than ______________________ but 61/84=73% of students preferred (b) because of

  ____

Example: *Expected Value, Short Run & Long Run*

- **Background**: Respondents were asked to choose between (a) and (b) in each case:

#3. (a) a 1 in 1000 chance to win $5000
(b) A sure gain of $6

- **Questions:** Which has a higher expected value? Why was the other choice preferred?
- **Response:** (b) $6 is more than ____________________ but 60/84=71% of students preferred (a) because ____________________

Example: *Expected Value, Short Run & Long Run*

- **Background**: Respondents were asked to choose between (a) and (b) in each case:

#4. (a) a 1 in 1000 chance of losing $5000
(b) A sure loss of $6

- **Questions:** Which has a higher expected value? Why was the other choice preferred?
- **Response:** (a) ____________________ is more than -$6 but 56/84=79% of students preferred (b) because

Example: *Expected Value, Short Run & Long Run*

- **Background**: Respondents were asked to choose between (a) and (b) in each case:

#1. (a) guaranteed gift of $240
(b) 25% chance to win $1000 and 75% chance to win $0
#2. (a) sure loss of $740
(b) 75% chance to lose $1000 and 25% chance to lose $0
#3. (a) a 1 in 1000 chance to win $5000
(b) A sure gain of $6
#4. (a) a 1 in 1000 chance of losing $5000
(b) A sure loss of $6

- **Question:** Which has a higher expected value?
- **Response:** #1(__), #2(__), #3(__), #4 (__)

Example: *Expected Value, Short Run & Long Run*

- **Background**: Consider each pair of options.

#1. (a) guaranteed gift of $240
(b) 25% chance to win $1000 and 75% chance to win $0
#2. (a) sure loss of $740
(b) 75% chance to lose $1000 and 25% chance to lose $0
#3. (a) a 1 in 1000 chance to win $5000
(b) A sure gain of $6
#4. (a) a 1 in 1000 chance of losing $5000
(b) A sure loss of $6

- **Question:** Which option is better if the offer is made (A) once? (B) every day?
- **Response:** (A) #1(__)#2(__)#3(__)#4(__)
- (B) #1(__)#2(__)#3(__)#4(__)