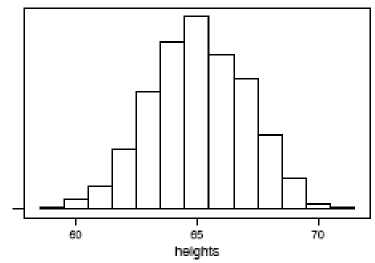
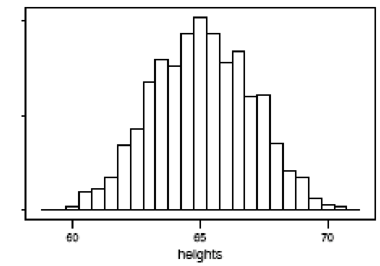
Lecture 10/Chapter 8
Bell-Shaped Curves & Other Shapes

- From a Histogram to a Frequency Curve
- Standard Score
- Using Normal Table
- Empirical Rule

From Histogram to Normal Curve

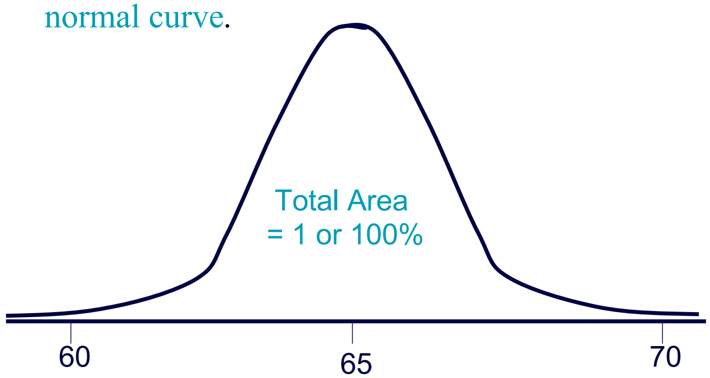
- Start: sample of female hts to nearest inch (left)
- Fine-tune: sampled hts to nearest 1/2-inch (right)





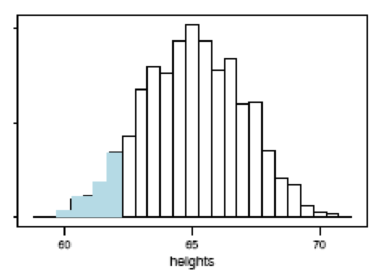
From Histogram to Normal Curve

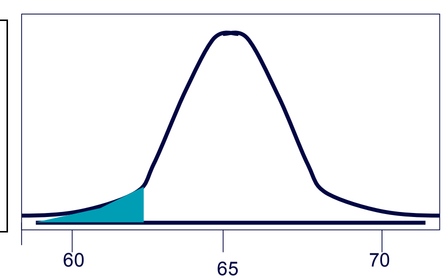
- Idealize: Population of infinitely many hts over continuous range of possibilities modeled with normal curve.



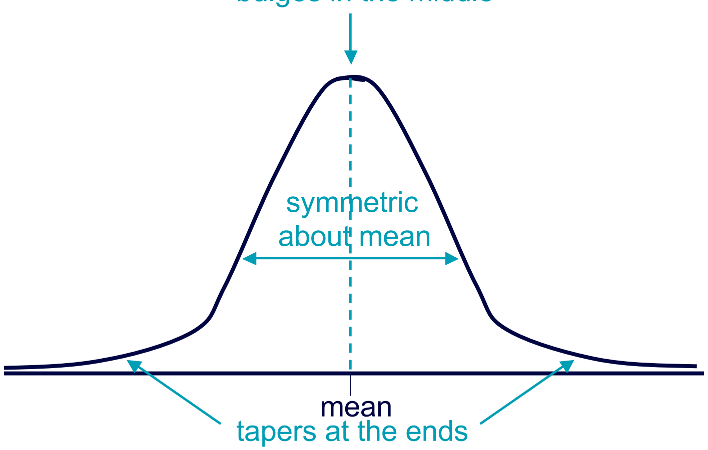
How Areas Show Proportions

- Area of histogram bars to the left of 62 shows proportion of sampled heights below 62 inches.
- Area under curve to the left of 62 shows proportion of all heights in population below 62 inches.





Properties of Normal Curve



Background of Normal Curve

Karl Friedrich Gaus (1777-1855) was one of the first to explore normal distributions.

Many distributions--such as test scores, physical characteristics, measurement errors, etc.--naturally follow this particular pattern.

If we know the shape is normal, and the value of the mean and standard deviation, we know exactly how the distribution behaves.

There are infinitely many normal curves possible.

Standardizing Values of Normal Distribution

Put a value of a normal distribution into perspective by standardizing to its *z*-score:

$$z = \frac{\text{observed value - mean}}{\text{standard deviation}}$$

Example: *Sign of z*

- **Background**: A person's *z*-score for height is found; its sign is negative.
- **Question:** What do we know about the person's height?
- **Response:**

Example: *What z Tells Us*

- **Background**: Heights of women (in inches) have mean 65, standard deviation 2.5. Heights of men have mean 70, standard deviation 3.
- **Question:** Who is taller relative to others of their sex: Jane at 71 inches or Joe at 76 inches?
- **Response:** Jane has z=_________________
  Joe has z=_______________

Example: *More about What z Tells Us*

- **Background**: Jane's z-score for height is +2.4 and Joe's is +2.0.
- **Question:** How do their heights relate to the averages, respectively, for women and men?
- **Response:** Jane's height is

Joe's height is

Example: *Finding a Proportion, Given z*

- **Background**: Jane's z-score for height is +2.4 and Joe's is +2.0, so the proportion of women shorter than Jane is more than the proportion of men shorter than Joe.
- **Question:** What are the proportions? Sketch #1
- **Response:** *(See table p. 157.)* The proportion below z=+2.4 is about ____; the proportion below z=+2.0 is about ____.

(Jane is in the ___th percentile; Joe is in the___th.)

Example: *Finding %, Given Original Value*

- **Background**: Verbal SAT scores for college-bound students are approximately normal with mean 500, standard deviation 100.
- **Question:** If a student scored 450, what percentage scored less than she did? Sketch #2
- **Response:** z=(value-mean)/sd =
  = _____

[450 is ___ stan. deviation below mean]

Table shows ____% are below this.

Example: *Finding Percentage Above*

- **Background**: Verbal SAT scores for college-bound students are approximately normal with mean 500, standard deviation 100.
- **Question:** If a student scored 400, what percentage scored *more* than he did? Sketch #3
- **Response:** z=(value-mean)/sd =____________ = ___ [400 is ___ stan. deviation below mean]

Table shows ____% are *below* this so _____________% are *above* this.

Example: *Finding z, Given Percentile*

- **Background**: Verbal SAT scores for college-bound students are approximately normal with mean 500, standard deviation 100.
- **Question:** A student scored in the 90th percentile; what was her score? Sketch #4
- **Response:** Table shows 90th percentile has z=____: her score is ___ sds above the mean, or ___________________

Example: *Finding z, Given Percentile*

- **Background**: Verbal SAT scores for college-bound students are approximately normal with mean 500, standard deviation 100.
- **Question:** What is the cutoff for top 5%? Sketch #5
- **Response:** Proportion above = 0.05 ➔ proportion below = ____ ➔ z=_____ ➔ the value is _____stan. deviations above mean ➔ the value is ___________________.

Example: *Finding Proportion between Scores*

- **Background**: Verbal SAT scores for college-bound students are approximately normal with mean 500, standard deviation 100.
- **Question:** What proportion scored between 425 and 633? Sketch #6
- **Response:** 425 has z=____; prop. below =____
  633 has z=____; proportion below =_____

Prop. with z bet.-0.75 and +1.33 is_____________

Example: *Proportion within 1 sd of Mean*

- **Background**: *Table 8.1* p. 157 Sketch #7
- **Question:** What proportion of normal values are within 1 standard deviation of the mean?
- **Response:** Proportion below -1 is ____; proportion below +1 is ____, so______________ are between -1 and +1.

Example: *Proportion within 2 sds of Mean*

- **Background**: *Table 8.1* p. 157 Sketch #8
- **Question:** What proportion of normal values are within 2 standard deviations of the mean?
- **Response:** Proportion below -2 is ______; proportion below +2 is ____ ➔ ____________ are between -2 and +2.

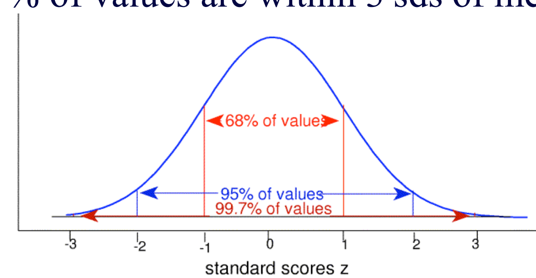
Example: *Proportion within 3 sds of Mean*

- **Background**: *Table 8.1* p. 157 Sketch #9
- **Question:** What proportion of normal values are within 3 standard deviations of the mean?
- **Response:** Proportion below -3 is_______ proportion below +3 is ______ ➔ ___________________ are between -3 and +3.

Empirical Rule (68-95-99.7 Rule)

For any normal curve, approximately

- 68% of values are within 1 sd of mean
- 95% of values are within 2 sds of mean
- 99.7% of values are within 3 sds of mean



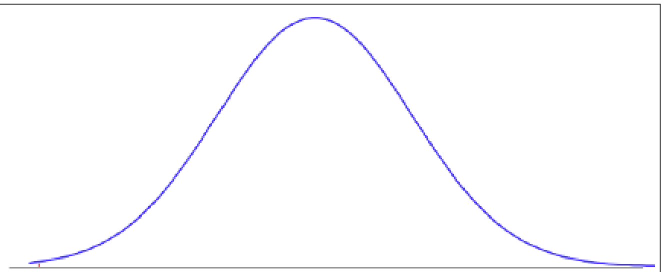
Example: *Applying Empirical Rule*

- **Background**: IQ scores normal with mean 100, standard deviation 15.
- **Question:** What does Empirical Rule tell us?
- **Response:**
  - 68% of IQ scores are between ____ and ____
  - 95% of IQ scores are between ____ and ____
  - 99.7% of IQ scores are between ____ and ____

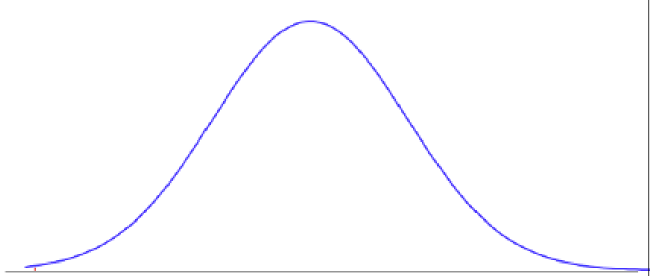
Example: *Applying Empirical Rule?*

- **Background**: Earnings for a large group of students had mean \$4000, stan. dev. \$6000.
- **Question:** What does Empirical Rule tell us?
- **Response:**
  - 68% of earnings are between -\$2000 and \$10,000?
  - 95% of earnings are between -\$8000 and \$16,000?
  - 99.7% of earnings between -\$14,000 and \$22,000?

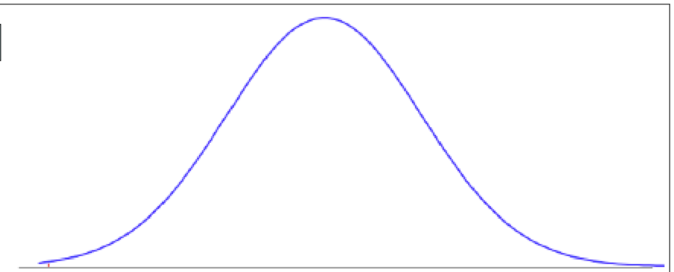
Sketch #1



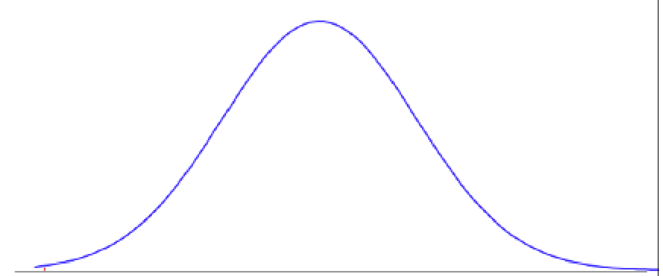
Sketch #2



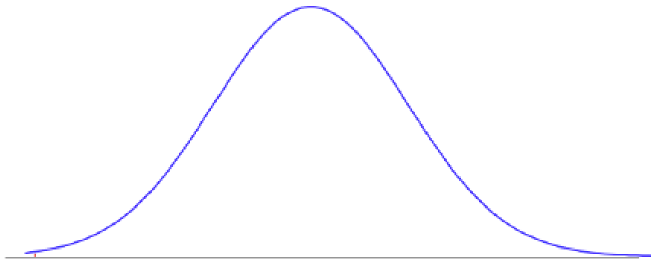
Sketch #3



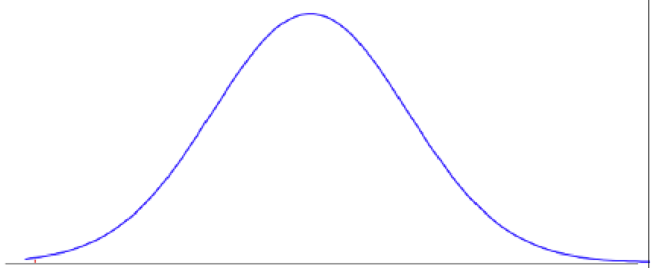
Sketch #4



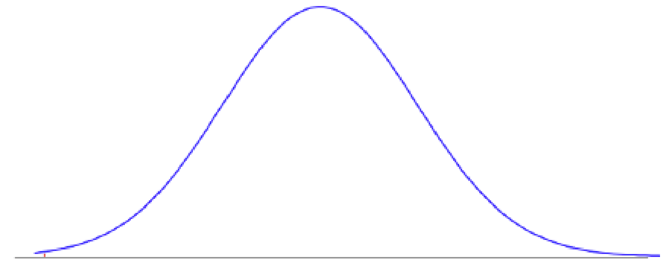
Sketch #5



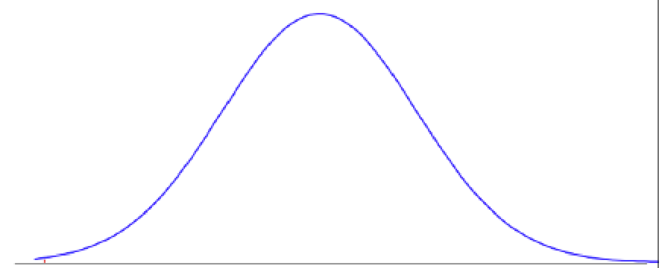
Sketch #6



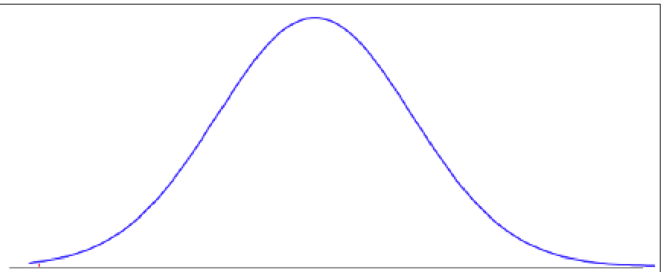
Sketch #7



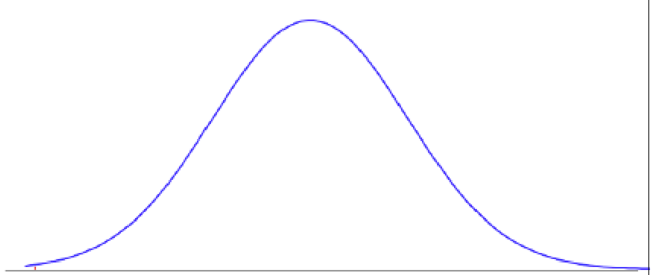
Sketch #8



Sketch #9



Sketch #10



Normal Practice Exercises

Try all the exercises in Lecture 11 before next class; we'll discuss the solutions in lecture.