Lecture 26/Chapter 22
Hypothesis Tests for Proportions

- Null and Alternative Hypotheses
- Standardizing Sample Proportion
- *P*-value, Conclusions
- Examples

Two Forms of Inference

Confidence interval: Set up a range of plausible values for the unknown population proportion (if variable of interest is categorical) or mean (if variable of interest is quantitative).

Hypothesis test: Decide if a particular proposed value is plausible for the unknown population proportion (if variable of interest is categorical) or mean (if variable of interest is quantitative).

Example: *Revisiting the Wording of Questions*

- **Background**: A Pew poll asked if people supported *civil unions* for gays; some were asked **before** a question about whether they supported *marriage* for gays; others **after**. Of 735 people asked **before** the marriage question, 55% opposed civil unions. Of 780 asked **after** the marriage question, 47% opposed.
- **Question:** What explains the difference?
- **Response:**

Example: *Testing a Hypothesis about a Majority*

- **Background**: In a Pew poll of 735 people, 0.55 opposed civil unions for gays.
- **Question:** Are we convinced that a majority (more than 0.5) of the population oppose civil unions for gays?
- **Response:** It depends; if the population proportion opposed were only , how improbable would it be for at least in a random sample of 735 people to be opposed?

Example: *Testing a Hypothesis about a Minority*

- **Background**: In a slightly different Pew poll of 780 people, 0.47 opposed civil unions for gays.
- **Question:** Are we convinced that a minority (less than 0.5) of the population oppose civil unions for gays?
- **Response:** It depends; if the population proportion opposed were as high as how improbable would it be for no more than in a random sample of 780 people to be opposed?

Note: In both examples, we test a **hypothesis** about the larger population, and our conclusion hinges on the *probability* of observed behavior occurring in a random sample. This probability is called the ***P*-value**.

Testing Hypotheses About Pop. Value

1. Formulate hypotheses.
2. Summarize/standardize data.
3. Determine the P-value.
4. Make a decision about the unknown population value (proportion or mean).

Null and Alternative Hypotheses

For a test about a single proportion,

- **Null hypothesis:** claim that the population proportion equals a proposed value.
- **Alternative hypothesis:** claim that the population proportion is greater, less, or not equal to a proposed value.

  An alternative formulated with $\neq$ is **two-sided**; with $>$ or $<$ is **one-sided**.

Testing Hypotheses About Pop. Value

1. Formulate hypotheses.
2. Summarize/standardize data.
3. Determine the P-value.
4. Make a decision about the unknown population value (proportion or mean).

Standardizing Normal Values *(Review)*

Put a value of a normal distribution into perspective by standardizing to its z-score:

$$z = \frac{\text{observed value - mean}}{\text{standard deviation}}$$

The observed value that we need to standardize in this context is the **sample proportion**. We've established Rules for its mean and standard deviation, and for when the shape is approximately normal, so that a probability (the *P*-value) can be assessed with the normal table.

Rule for Sample Proportions *(Review)*

- **Center:** The mean of sample proportions equals the true population proportion.
- **Spread:** The standard deviation of sample proportions is standard error =

$$\sqrt{\frac{\text{population proportion} \times (1\text{-population proportion})}{\text{sample size}}}$$

- **Shape:** (Central Limit Theorem) The frequency curve of proportions from the various samples is approximately normal.

Standardized Sample Proportion

- To test a hypothesis about an unknown population proportion, find sample proportion and standardize to

$$z = \frac{\text{sample proportion - population proportion}}{\sqrt{\dfrac{\text{population proportion (1-population proportion)}}{\text{sample size}}}}$$

- *z* is called the **test statistic**.

Note that “sample proportion” is what we’ve observed, “population proportion” is the value proposed in the null hypothesis.

Conditions for Rule of Sample Proportions

1. Randomness [affects **center**]
   - Can't be biased for or against certain values
2. Independence [affects **spread**]
   - If sampling without replacement, sample should be less than 1/10 population size
3. Large enough sample size [affects **shape**]
   - Should sample enough to expect at least 5 each in and out of the category of interest.

If 1st two conditions don't hold, the mean and sd in z are wrong; if 3rd doesn't hold, P-value is wrong.

Testing Hypotheses About Pop. Value

1. Formulate hypotheses.
2. Summarize/standardize data.
3. Determine the *P*-value.
4. Make a decision about the unknown population value (proportion or mean).

P-value in Hypothesis Test about Proportion

The *P*-value is the probability, assuming the null hypothesis is true, of a sample proportion at least as low/high/different as the one we observed. In particular, it depends on whether the alternative hypothesis is formulated with a less than, greater than, or not-equal sign.

Testing Hypotheses About Pop. Value

1. Formulate hypotheses.
2. Summarize/standardize data.
3. Determine the *P*-value.
4. Make a decision about the unknown population value (proportion or mean).

Making a Decision Based on a *P*-value

If the *P*-value in our hypothesis test is small, our sample proportion is improbably low/high/different, assuming the null hypothesis to be true. We conclude it is **not** true: we reject the null hypothesis and believe the alternative.

If the *P*-value is not small, our sample proportion is believable, assuming the null hypothesis to be true. We are willing to believe the null hypothesis.

P-value small ⟶ reject null hypothesis

P-value not small ⟶ don't reject null hypothesis

Hypothesis Test for Proportions: Details

1. null hypothesis: pop proportion = proposed value

   alt hyp: pop proportion < or > or ≠ proposed value
2. Find sample proportion and standardize to z.
3. Find the P-value= probability of sample proportion as low/high/different as the one observed; same as probability of z this far below/above/away from 0.
4. If the P-value is small, conclude alternative is true. In this case, we say the data are **statistically significant** (too extreme to attribute to chance). Otherwise, continue to believe the null hypothesis.

Example: *Testing a Hypothesis about a Majority*

- **Background**: In a Pew poll of 735 people, 0.55 opposed civil unions for gays.
- **Question:** Are we convinced that a majority (more than 0.5) of the population oppose civil unions for gays?
- **Response:**

1. Null: pop proportion Alt: pop proportion
2. Sample proportion= , z =
3. P-value=prob of z this far above 0:
4. Because the P-value is small, we reject null hypothesis. Conclude

Example: *Testing a Hypothesis about a Minority*

- **Background**: In a Pew poll of 780 people, 0.47 opposed civil unions for gays.
- **Question:** Are we convinced that a minority (less than 0.5) of the population oppose civil unions for gays?
- **Response:**

1. Null: pop proportion Alt: pop proportion
2. Sample proportion = $z =$
3. *P*-value=prob of z this far below 0: approximately
4. Because the *P*-value is

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Example: *Testing a Hypothesis about M&Ms*

- **Background**: Population proportion of red M&Ms is unknown. In a random sample, 15/75=0.20 are red.
- **Question:** Are we willing to believe that 1/6 = 0.17 of all M&Ms are red?
- **Response:**

1. Null: pop proportion Alt: pop proportion
2. Sample proportion = , z =
3. *P*-value=prob of z this far away from 0 (either direction)
4. Because the *P*-value isn't too small,