Lecture 23/Chapter 19
Diversity of Sample Means

- Means versus Proportions
- Behavior of Sample Means: Example
- Behavior of Sample Means: Conditions
- Behavior of Sample Means: Rules

Approach to Inference

- **Step 1 (Chapter 19):** Work *forward*---if we happen to know the population mean and standard deviation, what behavior can we expect from sample means for repeated samples of a given size?
- **Step 2:** Work *backward*---if sample mean for a sample of a certain size is observed to take a specified value, what can we conclude about the value of the unknown population mean?

We covered Step 1 for proportions, now we'll cover Step 1 for means.

Proportions then Means, Probability then Inference

Today we'll establish a parallel theory for means, when the variable of interest is quantitative (number on dice instead of color on M&M). After that, we'll

- Perform inference with **confidence intervals**
  - For proportions (Chapter 20)
  - For means (Chapter 21)
- Perform inference with **hypothesis testing**
  - For proportions (Chapters 22&23)
  - For means (Chapters 22&23)

Understanding Sample Mean

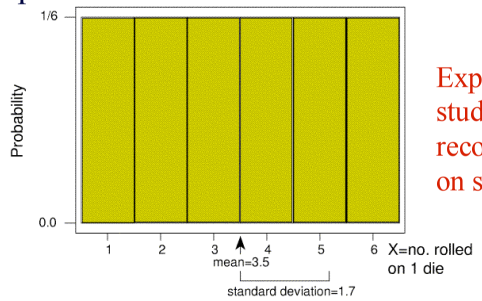
3 Approaches:

1. Intuition
2. Hands-on Experimentation
3. Theoretical Results

We'll find that our **intuition** is consistent with **experimental** results, and both are confirmed by mathematical **theory**.

Example: *Intuit Behavior of Sample Mean*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7.
- **Question:** How should sample mean roll behave for repeated rolls of 2 dice?



Experiment: each student rolls 2 dice, records sample mean on sheet **and** in notes.

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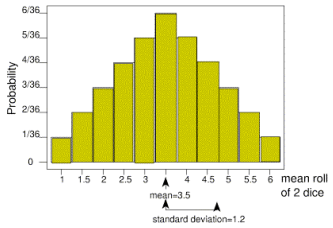
- **Question:** How should sample mean roll behave for repeated rolls of 2 dice?
- **Response:** Summarize by telling
  - **Center:** Some means less than 3.5, others more; altogether, they should average out to____
  - **Spread:** Means for 2 dice easily range from___to___
  - **Shape:**_____ (up from 1 to 3.5, down to 6).

Example: *Intuit Behavior of Sample Mean*



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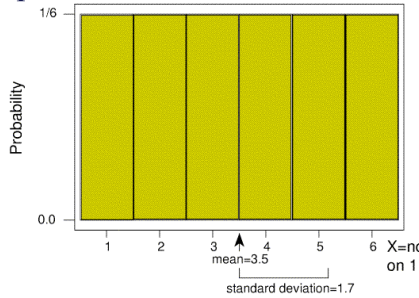
  - **Shape:** _____ (up from 1 to 3.5, down to 6).

Example: *Sample Mean for Larger Samples*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7.
- **Question:** How should sample mean roll behave for repeated rolls of 8 dice?



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Example: *Sample Mean for Larger Samples*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7.
- **Question:** How should sample mean roll behave for repeated rolls of 8 dice?
- **Response:** Summarize by telling
  - **Center:** Altogether they should average out to ___
  - **Spread:** Means for 8 dice rarely as low as 1 or as high as 6: _____spread than for 2 dice.
  - **Shape:** Bulges more near 3.5, tapers more at extremes 1 and 6→shape close to _____

Conditions for Rule of Sample Means

- Randomness [affects center]
- Independence [affects spread]
  - If sampling without replacement, sample should be less than 1/10 population size
- Large enough sample size [affects shape]
  - If population shape is normal, any sample size is OK
  - If population if not normal, a larger sample is needed.

Example: *Checking Conditions for 2 Dice*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7. Repeatedly roll 2 dice and calculate the sample mean roll.
- **Question:** Are the 3 Conditions met?
- **Response:**
  - **Random?** ____
  - **Independent?** _____
  - **Sample large enough?**

    _____

    _____

Example: *Checking Conditions for 8 Dice*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7. Repeatedly roll 8 dice and calculate the sample mean roll.
- **Question:** Are the 3 Conditions met?
- **Response:**
  - **Random?** _____
  - **Independent?** _____
  - **Sample large enough?**

    _____

    _____

Rule for Sample Means (if conditions hold)

- **Center:** The mean of sample means equals the true population mean.
- **Spread:** The standard deviation of sample means is standard error =

$$\frac{\text{population standard deviation}}{\sqrt{\text{sample size}}}$$

- **Shape:** (Central Limit Theorem) The frequency curve will be approximately normal, depending on how well 3rd condition is met.

Example: *Behavior of Sample Mean, 2 Dice*

- **Background**: Population of dice rolls has mean 3.5, sd 1.7. Repeatedly roll 2 dice.
- **Question:** How must sample means behave?
- **Response:** For repeated random samples of size 2, sample mean roll has…
  - **Center:** mean of sample means is _____
  - **Spread:** standard error is
  - **Shape:** _____

Example: *Behavior of Sample Mean, 8 Dice*

- **Background**: Population of dice rolls has mean 3.5, sd 1.7. Repeatedly roll 8 dice.
- **Question:** How must sample means behave?
- **Response:** For repeated random samples of size 8, sample mean roll has…
  - **Center:** mean of sample means is _____
  - **Spread:** standard error is
  - **Shape:** _____

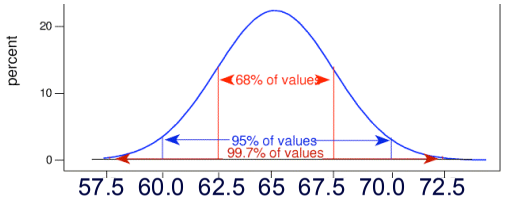
Example: *68-95-99.7 Rule for 8 Dice*

- **Background**: Sample mean roll for 8 dice has mean 3.5, sd 0.6, and shape fairly normal.
- **Question:** What does 68-95-99.7 Rule tell us about behavior of sample mean?
- **Response:** The probability is approximately
  - 0.68 that sample mean is within _____: in (2.9, 4.1)
  - 0.95 that sample mean is within _____: in (2.3, 4.7)
  - 0.997 that sample mean is within _____: in (1.7, 5.3)

Activity: check how class dice rolls conform.

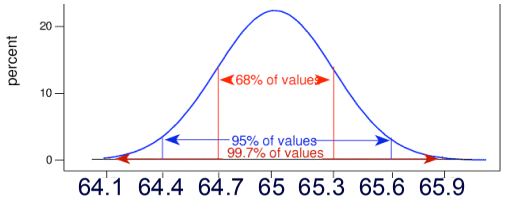
Example: *68-95-99.7 Rule for Single Hts*

- **Background**: Women's hts normal; mean 65, sd 2.5.
- **Question:** What does 68-95-99.7 Rule tell us about the height of a randomly chosen woman?
- **Response:** The probability is
  - 0.68 that her height is within _____: in (62.5, 67.5)
  - 0.95 that her height is within _____: in (60.0, 70.0)
  - 0.997 that her height is within _____: in (57.5, 72.5)



Example: *68-95-99.7 Rule for Mean Ht*

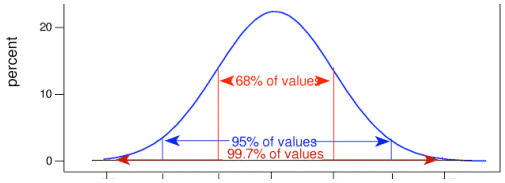
- **Background**: Women's hts normal; mean 65, sd 2.5.
- **Question:** What does 68-95-99.7 Rule tell us about sample mean ht for random samples of 64 women?
- **Response:** Sample means have mean 65, sd_____ and shape normal because population is normal. Probability is
  - 0.68 that sample mean is within _____: in (64.7, 65.3)
  - 0.95 that sample mean is within _____: in (64.4, 65.6)
  - 0.997 that sample mean is within _____: in (64.1,65.9)



Mean of 64 females in class is 64.9.

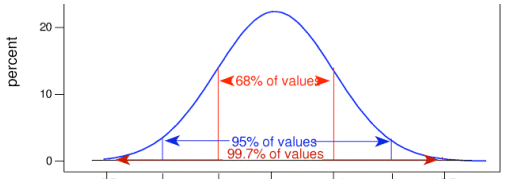
Example: *68-95-99.7 Rule for Male Hts*

- **Background**: Men's hts normal; mean 70, sd 3.
- **Question:** What does 68-95-99.7 Rule tell us about the height of a randomly chosen man?
- **Response:** The probability is
  - 0.68 that his height is within 1(3) of 70: in _____
  - 0.95 that his height is within 2(3) of 70: in _____
  - 0.997 that his height is within 3(3) of 70: in _____



Example: *68-95-99.7 Rule: Mean Male Ht*

- **Background**: Men's hts normal; mean 70, sd 3.
- **Question:** What does 68-95-99.7 Rule tell us about sample mean ht for random samples of 25 men?
- **Response:** Sample means have mean 70, sd _____ and shape normal because population is normal. Probability is
  - 0.68 that sample mean is within 1(0.6) of 70: in _____
  - 0.95 that sample mean is within 2(0.6) of 70: in _____
  - 0.997 that sample mean is within 3(0.6) of 70: in _____



Mean of 25 males in class is 70.5.