Lecture 19/Chapter 16

Probability & Long-Term Expectations

- Expected Value
- More Rules of Probability
- Tree Diagrams

Example: *Intuiting Expected Value*

- **Background**: Historically, Stat 800 grades have

Grade Pts.	4	3	2	1	0
Probability	0.25	0.40	0.20	0.10	0.05

- **Question:** What is the expected grade of a randomly chosen student? (Same as average of all students.)
- **Response:**______________________________

=1.00+1.20+0.40+0.10+0.00=2.70

Definition

- **Expected Value:** If k amounts are possible and amount A1 has probability p1, A2 has probability p2, …Ak has probability pk, then the expected value of the amount is

$A1 \times p1 + A2 \times p2 + \ldots + Ak \times pk$

"expected amount" is the same as "mean amount"

Example: *Calculating Expected Value*

- **Background**: Household size in U.S. has

Size	1	2	3	4	5	6	7
Prob	0.26	0.34	0.16	0.14	0.07	0.02	0.01

- **Question:** What is the expected size of a randomly chosen household?
- **Response:** ______________________________

(Since no household actually has the "expected" size, we might prefer to call it the mean instead.)

Example: *Calculating Expected Value*

- **Background**: Suppose you play a game in which there is a 25% chance to win $1000 and a 75% chance to win nothing.
- **Question:** What is your expected gain?
- **Response:** ____________________

Note: Nevertheless, 89% of surveyed students said they'd prefer a guaranteed gift of $240. In Chapter 18, we'll discuss this and other psychological influences.

Example: *Calculating Expected Value*

- **Background**: Suppose a raffle ticket costs $5, and there is a 1% chance of winning $400.
- **Question:** What is your expected gain?
- **Response:** ____________________

Basic Probability Rules *(Review)*

We established rules for

0. What probabilities values are *permissible*
1. The probability of *not* happening
2. The probability of one *or* the other of two mutually exclusive events occurring
3. The probability of one *and* the other of two independent events occurring
4. How probabilities compare if one event is the subset of another

We need more general "or" and "and" rules.

Example: *Parts of Table Showing "Or" and "And"*

- **Background**: Professor notes gender (female or male) and grade (A or not A) for students in class.
- **Questions:** What part of a two-way table shows…
  - Students who are female *and* get an A?
  - Students who are female *or* get an A?

	A	not A	Total
Femae	0.15	0.45	0.60
Male	0.10	0.30	0.40
Total	0.25	0.75	1.00

Example: *Parts of Table Showing "Or" and "And"*

- **Background**: Professor notes gender (female or male) and grade (A or not A) for students in class.
- **Responses:**
  - Students who are female *and* get an A: table on ____
  - Students who are female *or* get an A: table on _____

	A	not A	Total
Female			
Male			
Total			

	A	not A	Total
Female			
Male			
Total			

Example: *Intuiting Rule 5*

- **Background**: Professor says: probability of being a female is 0.60; probability of getting an A is 0.25. Probability of both is 0.15.
- **Question:** What is the probability of being a female *or* getting an A?
- **Response:**

Example: *Intuiting Rule 5*

- **Response:** Illustration with two-way tables:

	A	not A	Total
Female			
Male			
Total			

+

	A	not A	Total
Female			
Male			
Total			

−

	A	not A	Total
Female			
Male			
Total			

=

	A	not A	Total
Female			
Male			
Total			

Rule 5 (General "Or" Rule)

For any two events, the probability of one *or* the other happening is the sum of their individual probabilities, minus the probability that both occur.

***Note:** The word "or" still entails addition.*

Example: *Applying Rule 5*

- **Background**: In a list of potential roommates, the probability of being a smoker is 0.20. The probability of being a non-student is 0.10. The probability of both is 0.03.
- **Question:** What's the probability of being a smoker or a non-student?
- **Response:**

Example: *When Probabilities Can't Simply Be Multiplied (Review)*

- **Background**: In a child's pocket are 2 quarters and 2 nickels. He randomly picks a coin, does *not* replace it, and picks another.
- **Question:** What is the probability of the first and the second coins both being quarters?
- **Response:** Can't take 2/4 × 2/4 because after first coin is picked, probabilities change.

Definitions *(Review)*

For some pairs of events, whether or not one occurs impacts the probability of the other occurring, and vice versa: the events are said to be **dependent**.

If two events are **independent**, they do not influence each other; whether or not one occurs has no effect on the probability of the other occurring.

Rule 3 (Independent "And" Rule) *(Review)*

For any two independent events, the probability of one *and* the other happening is the *product* of their individual probabilities.

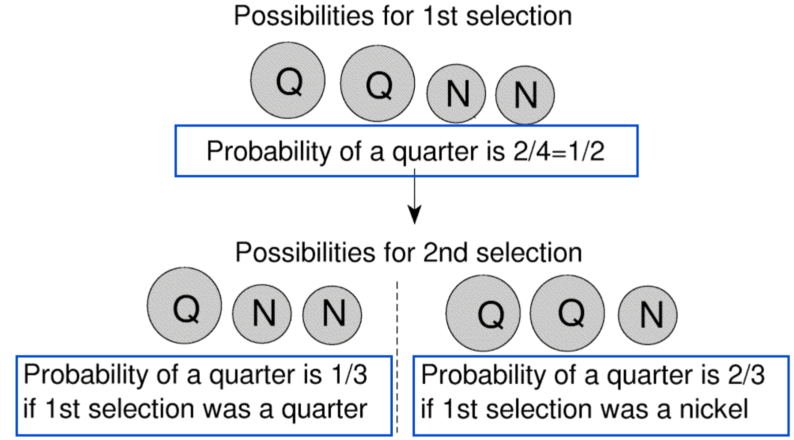
We need a rule that works even if two events are *dependent*.

- Sampling *with replacement* is associated with events being *independent*.
- Sampling *without replacement* is associated with events being *dependent*.

Example: *When Probabilities Can't Simply be Multiplied (Review)*

- **Background**: In a child's pocket are 2 quarters and 2 nickels. He randomly picks a coin, does *not* replace it, and picks another.
- **Response:** To find the probability of the first *and* the second coin being quarters, we can't multiply 0.5 by 0.5 because after the first coin has been removed, the probability of the second coin being a quarter is *not* 0.5: it is 1/3 if the first coin was a quarter, 2/3 if the first was a nickel.

Example: *When Probabilities Can't Simply be Multiplied*



Rule 6 (General "And" Rule)

The conditional probability of a second event, given a first event, is the probability of the second event occurring, assuming that the first event has occurred.

The probability of one event *and* another occurring is the *product* of the first and the (conditional) probability of the second, given that the first has occurred.

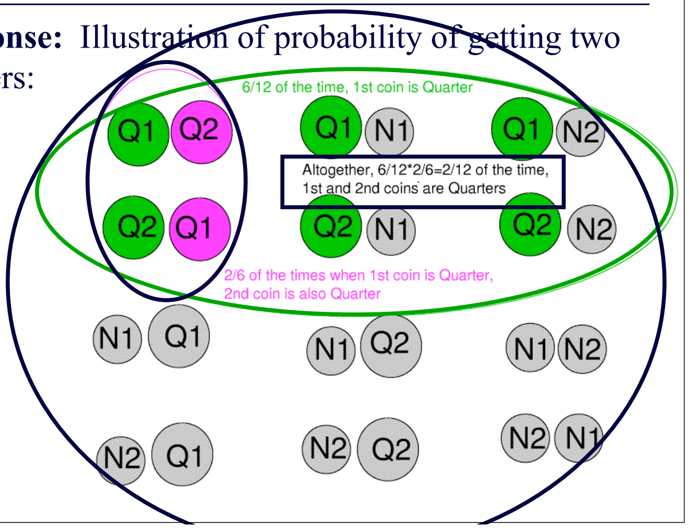
Example: *Intuiting the General "And" Rule*

- **Background**: In a child's pocket are 2 quarters and 2 nickels. He randomly picks a coin, does *not* replace it, and picks another.
- **Question:** What is the probability that the first *and* the second coin are quarters?
- **Response:** probability of first a quarter (___), times (conditional) probability that second is a quarter, **given** first was a quarter (___):

____

Example: *Intuiting the General "And" Rule*

- **Response:** Illustration of probability of getting two quarters:



Rule 6 (alternate formulation)

The conditional probability of a second event, given a first event, is the probability of both happening, divided by the probability of the first event.

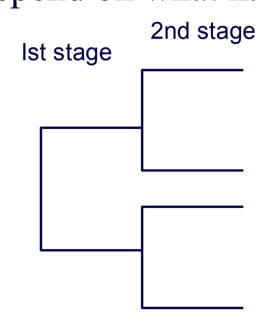
Example: *Applying Rule for Conditional Probability*

- **Background**: In a list of potential roommates, the probability of being both a smoker and a non-student is 0.03. The probability of being a non-student is 0.10.
- **Question:** What's the probability of being a smoker, given that a potential roommate is a non-student?
- **Response:** ________________

[Note that the probability of being a smoker is higher if we know a person is not a student.]

Tree Diagrams

These displays are useful for events that occur in stages, when probabilities at the 2nd stage depend on what happened at the 1st stage.

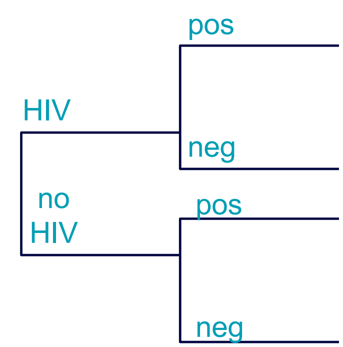


Example: *A Tree Diagram for HIV Test*

- **Background**: In a certain population, the probability of HIV is 0.001. The probability of testing positive is 0.98 if you have HIV, 0.05 if you don't.
- **Questions:** What is the probability of having HIV and testing positive? Overall prob of testing positive? Probability of having HIV, given you test positive?
- **Response:** To complete the tree diagram, note that probability of not having HIV is ______. The probability of testing negative is ______ if you have HIV, ______ if you don't.

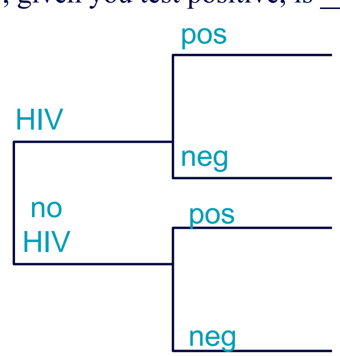
Example: *A Tree Diagram for HIV Test*

- **Background**: The probability of HIV is 0.001; probability of testing positive is 0.98 if you have HIV, 0.05 if you don't. (So probability of not having HIV is ______. The probability of testing negative is _____ if you have HIV, _____ if you don't.)



Example: *A Tree Diagram for HIV Test*

- **Background**: The probability of having HIV and testing positive is __________________. The overall probability of testing positive is _____________________.The probability of having HIV, given you test positive, is ___________________



EXTRA CREDIT (Max. 5 pts.) Choose 2 categorical variables from the survey data (available on the course website) and use a two-way table to display counts in the various category combinations. Report the probability of a student in the class being in one *or* the other of two categories; report the probability of being in one *and* the other of two categories.