Lecture 28: Chapter 11, Section 1 Categorical & Quantitative Variable Inference in Paired Design

- Inference for Relationships: 2 Approaches
- Cat→Quan Relationship: 3 Designs
- Inference for Paired Design
- Paired vs. Ordinary, *t* vs. *z*

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical (discussed in Lectures 21-23)
    - 1 quantitative (discussed in Lectures 24-27)
    - cat and quan: paired, 2-sample, several-sample
    - 2 categorical
    - 2 quantitative

Inference for Relationships: Two Approaches

- H_0 and H_a about variables: not related or related
  - Applies to all three C→Q, C→C, Q→Q
- H_0 and H_a about parameters: equality or not
  - C→Q: pop means equal? (mean diff=0? for paired)
  - C→C: pop proportions equal?
  - Q→Q: pop slope equals zero?

Either way, often do *test* before *confidence interval*.

1. Are variables related?
2. If so, quantify: how different are the parameters?

Example: C→Q *Test Relationship or Parameters*

- **Background**: Research question: "For all students at a university, are their Math SATs related to what year they're in?"
- **Question:** How can we formulate this in terms of parameters?
- **Response:**

Looking Ahead: *This is a several-sample design, to be discussed after paired and two-sample.*

Practice: 11.2b p.525

Inference Methods for Cat→Quan Relationship

- Paired: reduces to 1-sample t (already covered)
- Two-Sample: 2-sample t (similar to 1-sample t)
- Several-Sample: need new distribution (F)

Example: *Paired vs. Two-Sample Data*

- **Background**: Research Question: "Are 'age of parent' and 'sex of parent' related for population of students at a university?"
- **Question:** How can this data set be used to answer the research question?
- **Response:**

DadAge	MomAge
55	55
51	45
58	54
47	49
...	...

 Practice: 11.4a p.526

Example: *Paired vs. Two-Sample Summary*

- **Background**: Research Question: "Are 'age of parent' and 'sex of parent' related for population of students at a university?"
- **Question:** Which output has enough info to do inference?

Descriptive Statistics: DadAge, MomAge

Variable	N	N*	Mean	Median	TrMean	StDev
DadAge	431	15	50.831	50.000	50.491	6.167
MomAge	441	5	48.406	48.000	48.166	5.511

Descriptive Statistics: AgeDiff

Variable	N	N*	Mean	Median	TrMean	StDev
AgeDiff	431	15	2.448	2.000	2.171	3.877

- **Response:**

Looking Ahead: *We will standardize with the StDev of the differences, which cannot be found from the individual StDevs because of dependence.*

Practice: 11.4b p.526

Example: *Consider Summaries in Paired Design*

- **Background**: To see if 'age of parent' and 'sex of parent' are related for population of students at a university, took sampled DadAge minus MomAge.

Descriptive Statistics: AgeDiff

Variable	N	N*	Mean	Median	TrMean	StDev
AgeDiff	431	15	2.448	2.000	2.171	3.877

- **Question:** Which parent tended to be older in the sample?
- **Response:**

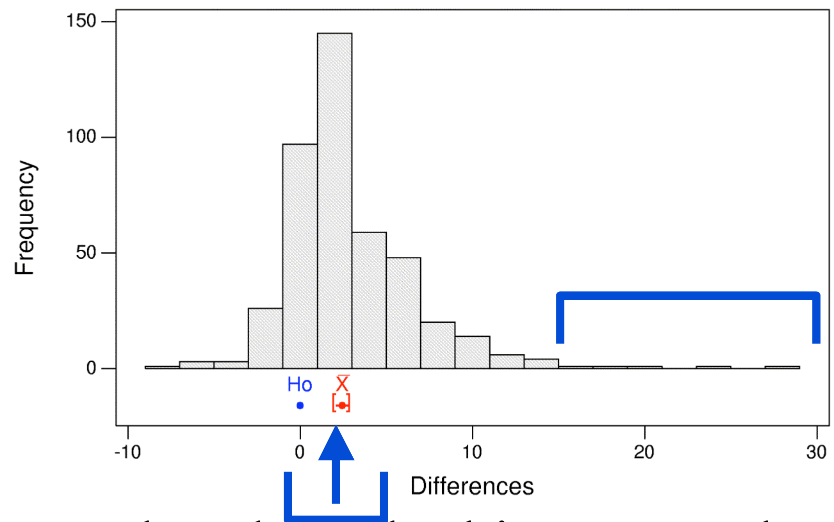
Example: *Display in Paired Design*

- **Background**: To see if 'age of parent' and 'sex of parent' are related for population of students at a university, took sampled DadAge minus MomAge.
- **Question:** How do we display the data?
- **Response:**

Practice: 11.6c p.526

Example: *Display in Paired Design*

- **Background**: Histogram of age differences:



- **Question:** What does the histogram show?
- **Response:** Age differences have
  - Center: around (dads tend to be about yrs older)
  - Spread: most diffs within yrs or mean)
  - Shape: (a few dads much older than wife)

Notation in Paired Study

- Differences have
  - Sample mean $\bar{x}_d$
  - Population mean μ_d
  - Sample standard deviation s_d
  - Population standard deviation σ_d

Test Statistic in Paired Study

- Start with ordinary 1-sample statistic $t = \frac{\bar{x} - \mu_o}{s/\sqrt{n}}$
- Substitute $\bar{x}_d$, s_d for ordinary summaries $\bar{x}$, s
- Substitute 0 for μ_o (H_0 will claim $\mu_d = 0$)
- Result is paired *t* statistic: $t = \frac{\bar{x}_d - 0}{s_d/\sqrt{n}}$

Example: *Paired t Test*

- **Background**: Paired test on students' parents' ages:

```
Paired T for DadAge - MomAge
                    N       Mean      StDev    SE Mean
DadAge            431     50.831      6.167      0.297
MomAge            431     48.383      5.258      0.253
Difference        431      2.448      3.877      0.187
95% CI for mean difference: (2.081, 2.815)
T-Test of mean difference = 0 (vs not = 0): T-Value = 13.11  P-Value = 0.000
```

- **Question:** What does output tell about formal test?
- **Response:** Testing
  - Unbiased? ______ n=431 large? ______ Pop≥10(431)? ______
  - $\bar{x}_d$= ______, t = ______ Large? ______
  - P-value = ______ Small? ______
  - Conclude pop mean diff =0? ______ Sex and age related? ______

Practice: 11.6d-f p.527

Example: *One- or Two-Sided H_a in Paired Test*

- **Background**: Paired test on students' parents' ages:

```
Paired T for DadAge - MomAge
                   N       Mean      StDev    SE Mean
DadAge           431     50.831      6.167      0.297
MomAge           431     48.383      5.258      0.253
Difference       431      2.448      3.877      0.187
95% CI for mean difference: (2.081, 2.815)
T-Test of mean difference = 0 (vs not = 0): T-Value = 13.11  P-Value = 0.000
```

- **Response:** Replace $H_a : \mu_d \neq 0$ with
  - *P*-value would be ______________
  - Conclude fathers in general are older?

Example: *Paired vs. Ordinary t vs. z*

- **Background**: Paired test on 431 students' parents' ages resulted in paired *t*-statistic +13.11.
- **Question:** What does this tell us about the *P*-value?
- **Response:**

  Paired *t* same as ordinary *t* distribution

  → Ordinary *t* basically same as *z* for large *n*

  →13.11 sds above mean unusual? →*P*-val =

  →Evidence that mean age diff is non-zero in pop.? ______

Note: for extreme *t* statistics, software not needed to estimate *P*-value.

Practice: 11.58b p.576

Confidence Interval in Paired Design

Confidence interval for μ_d is

$$\bar{x}_d \pm \text{multiplier} \frac{s_d}{\sqrt{n}}$$

- Multiplier from *t* distribution with *n*-1 df
- Multiplier smaller for lower confidence
- Multiplier smaller for larger df

If *n* is small, diffs need to be approx. normal.

(Same guidelines as for 1-sample *t*)

Guidelines: Sample Mean Diff Approx. Normal

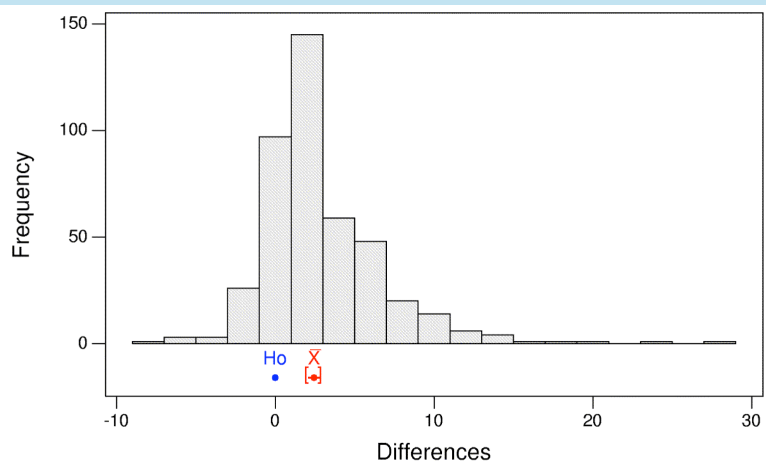
Can assume shape of $\bar{X}_d$ for random samples of *n* pairs is approximately normal if

- Graph of sample diffs appears normal; or
- Graph of sample diffs fairly symmetric and *n* at least 15; or
- Graph of sample diffs moderately skewed and *n* at least 30; or
- Graph of sample diffs very skewed and *n* much larger than 30

Example: *Paired Confidence Interval*

- **Background**: Sample of 431 students' parents' age differences have mean +2.45, s.d. 3.88.
- **Question:** What is a 95% confidence interval for population mean age difference?
- **Response:** Since *n* is so large, *t* multiplier for 95% confidence. (Also, skewed hist. OK.)

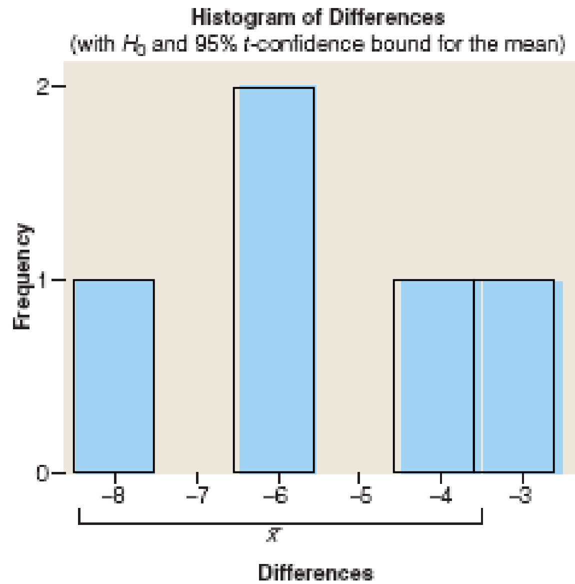
Pretty sure population of fathers are older by about to years.



Example: *Checking Conditions for Paired t*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

City	Highway	MileageDiff
20	28	−8
34	40	−6
26	32	−6
30	34	−4
22	25	−3



- **Question:** Is paired *t* procedure appropriate?
- **Response:** Histogram

Practice: 11.6j p.527

Example: *Paired Test and Confidence Interval*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

```
Paired T for City - Highway
                 N       Mean      StDev   SE Mean
City             5      26.40       5.73      2.56
Highway          5      31.80       5.76      2.58
Difference       5     -5.400      1.949     0.872
95% upper bound for mean difference: -3.541
T-Test of mean difference = 0 (vs < 0): T-Value = -6.19  P-Value = 0.002
 95% CI for mean difference: (-7.820, -2.980)
```

- **Question:** What does the output tell us?
- **Response:**
  - *P*-val=0.002→
  - C.I.→hway av about to mpg better in pop of cars

Example: *Paired Confidence Interval by Hand*

- **Background**: Mileage differences for 5 cars, city minus highway, had mean -5.40, s.d. 1.95.
- **Question:** What else is needed to set up a 95% confidence interval by hand for population mean difference?
- **Response:** Need
  (obtained from table before software was available)
  Interval is

Note: *n* very small→ *t* multiplier closer to 3 than to 2.

Practice: 11.6g p.527

Example: *Relating Test and Confidence Interval*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

```
Paired T for City - Highway
                  N      Mean     StDev   SE Mean
City              5     26.40      5.73      2.56
Highway           5     31.80      5.76      2.58
Difference        5    -5.400     1.949     0.872
95% upper bound for mean difference: -3.541
T-Test of mean difference = 0 (vs < 0): T-Value = -6.19  P-Value = 0.002

95% CI for mean difference: (-7.820, -2.980)
```

- **Question:** How is *P*-value consistent with C.I.?
- **Response:**
  - Small *P*-value→conclude H_a: pop mean of diffs
  - Confidence interval shows only numbers are plausible values for mean of diffs (entire C.I.

Example: *Switching Columns in Paired Design*

- **Background**: Mileages for 5 cars, each tested in city and on highway (suspect higher on highway).

```
Paired T for City - Highway
                  N      Mean     StDev   SE Mean
City              5     26.40      5.73      2.56
Highway           5     31.80      5.76      2.58
Difference        5    -5.400     1.949     0.872
95% upper bound for mean difference: -3.541
T-Test of mean difference = 0 (vs < 0): T-Value = -6.19  P-Value = 0.002
95% CI for mean difference: (-7.820, -2.980)
```

- **Question:** What would change if we took highway minus city?
- **Response:** Since we suspect higher on highway,
  - Change to Highway-City and sign in H_a changes to
  - Sample mean of diffs would be and $t =$
  - P-value still 0.002, reject H_0 →
  - Confidence interval would be

Lecture Summary
(Inference for Cat→Quan; Paired)

- Inference for relationships
  - Focus on variables
  - Focus on parameters
- cat→quan relationship: paired, 2- or several-sample
- Inference for paired design
  - Output
  - Display
  - Notation
  - Test statistic
  - Form of alternative
- Paired t vs. ordinary t vs. z
- Paired confidence interval vs. hypothesis test