Lecture 25: more Chapter 10, Section 1 Inference for Quantitative Variable: Hypothesis Tests

- *z* Test about Population Mean: 4 Steps
- Examples: 1-sided or 2-sided Alternative
- Relating Test and Confidence Interval
- Factors in Rejecting Null Hypothesis
- Inference Based on *t* vs. *z*

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical (discussed in Lectures 21-23)
    - 1 quantitative: confidence intervals, hypothesis tests
    - categorical and quantitative
    - 2 categorical
    - 2 quantitative

Behavior of Sample Mean *(Review)*

For random sample of size n from population with mean μ and standard deviation σ, sample mean $\bar{X}$ has

- mean μ
- standard deviation $\frac{\sigma}{\sqrt{n}}$
- shape approximately normal for large enough n

→If σ is known, standardized $\bar{X}$ follows z (standard normal) distribution

Hypothesis Test About μ (with z)

Problem Statement $H_0 : \mu = \mu_0$ vs. $H_a : \left\{ \begin{array}{c} \mu > \mu_0 \\ \mu < \mu_0 \\ \mu \neq \mu_0 \end{array} \right\}$

1. Consider sampling and study design.
2. Summarize with $\bar{x}$, standardize to $z = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}}$ assuming $H_o : \mu = \mu_o$ is true; is z "large"?
3. Find P-value (prob. of Z this far above/below/away from 0); is it "small"?
4. Based on size of P-value, choose H_0 or H_a.

Hypothesis Test About μ with *z* *(Details)*

1. Consider sampling and study design.
2. Summarize with $\bar{x}$, standardize to $z = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}}$ g is true; is *z* $H_o: \mu = \mu_o$
3. Find prob. of *z* this far above/below/away from 0 (*P*-value); consider if it is "small".
4. Based on size of *P*-value, choose H_0 or H_a.

- If sample is biased, mean of $\bar{X}$ is not μ_o.
- If pop<10*n*, s.d. of $\bar{X}$ is not $\sigma / \sqrt{n}$.
- If *n* is too small, distribution of $\bar{X}$ is not normal, won't standardize to *z*: graph data, see guidelines.

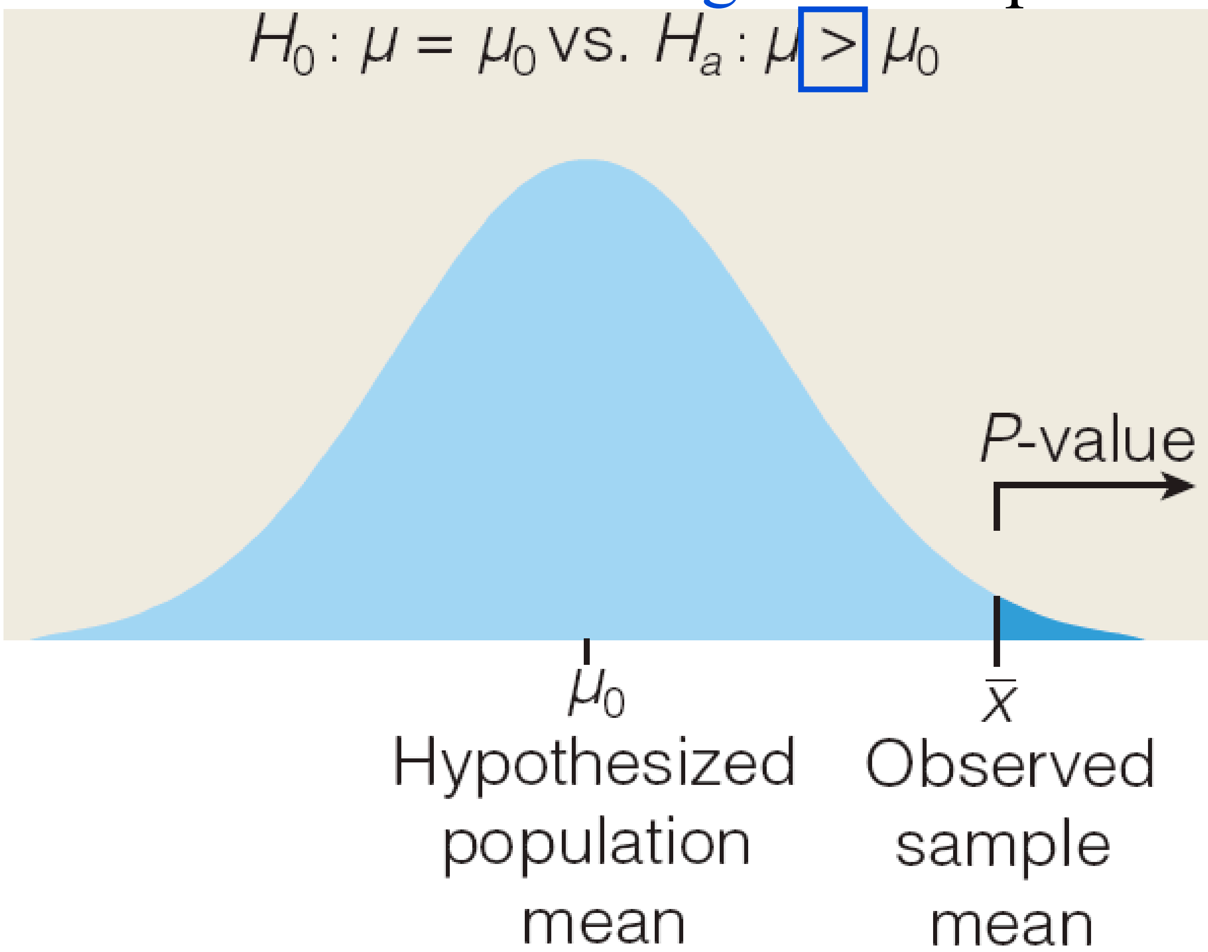
Hypothesis Test About μ with *z (Details)*

1. Consider sampling and study design
2. Summarize with $\bar{x}$, standardize to $z = \frac{\bar{x} - \mu_o}{\sigma/\sqrt{n}}$ assuming $H_o: \mu = \mu_o$ is true; is *z* "large"?
3. Find prob. of *z* this far above/below/away from 0 (*P*-value); consider if it is "small".
4. Based on size of *P*-value, choose H_0 or H_a.

- Assess *P*-value based on form of alternative hypothesis (greater, less, or not equal)

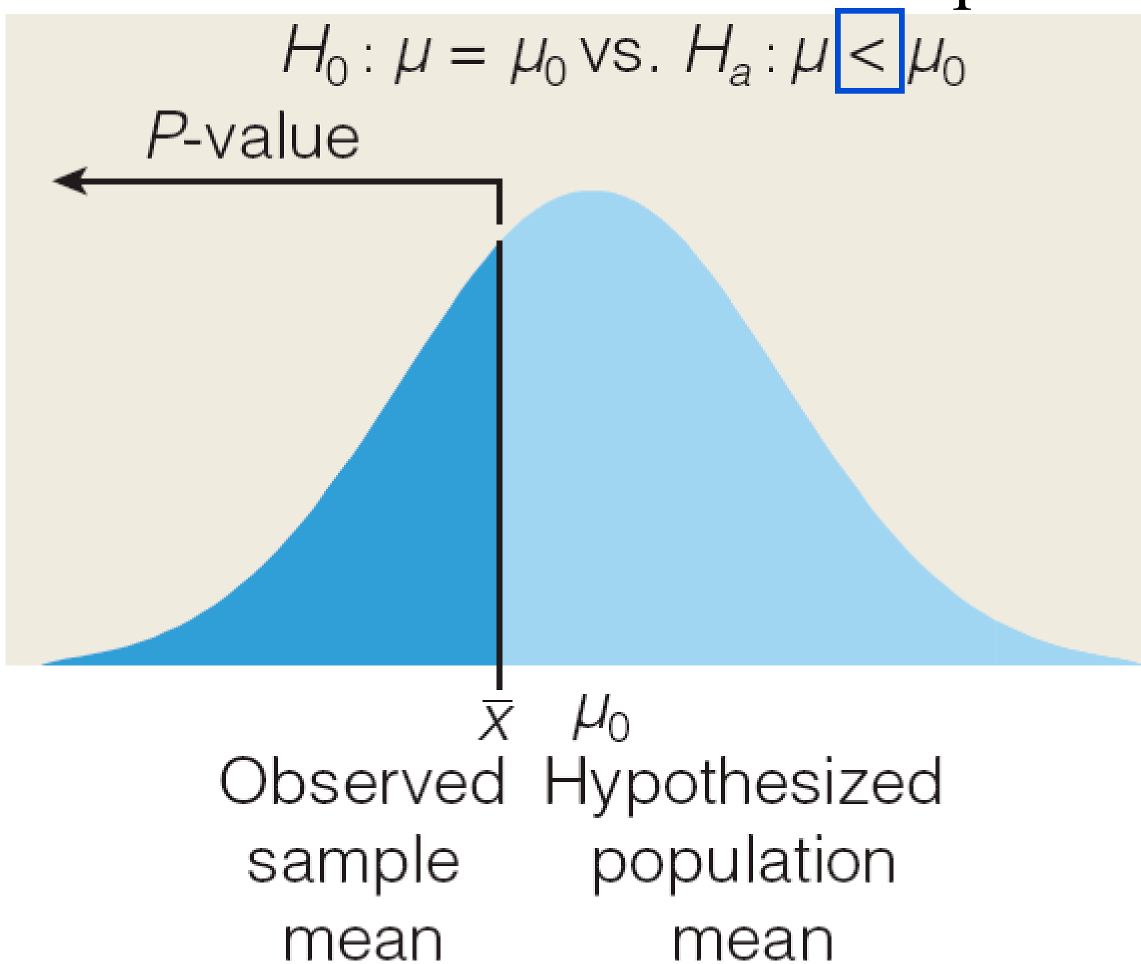
Hypothesis Test About μ with *z (Details)*

Alternative ">": *P*-value is right-tailed probability



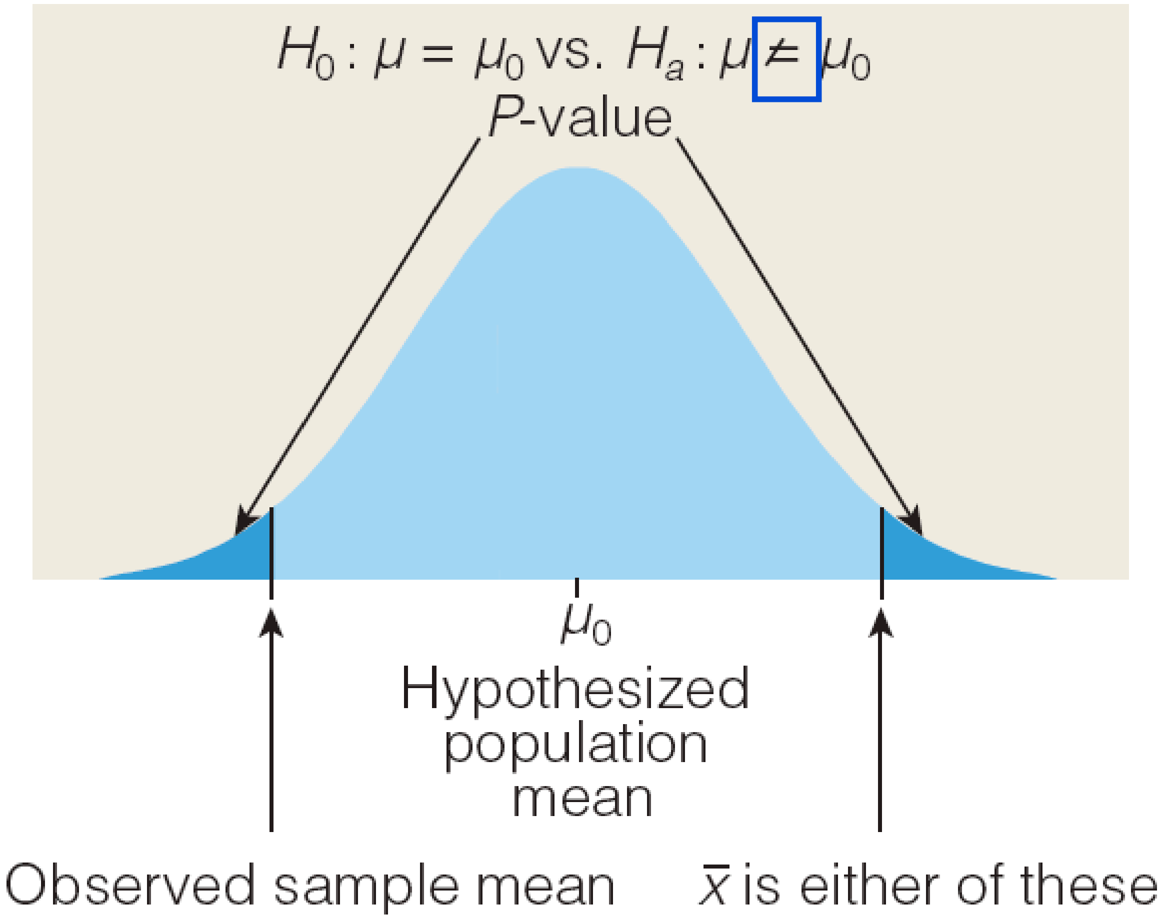
Hypothesis Test About μ with z *(Details)*

Alternative "<": *P*-value is left-tailed probability



Hypothesis Test About μ with z *(Details)*

Alternative "≠": *P*-value is two-tailed probability



Example: *Assumptions for z Test*

- **Background**: Earnings of 446 surveyed university students had mean \$3,776. The mean of earnings for the population of students is unknown. Assume we know population standard deviation is \$6,500.
- **Question:** What aspect of the situation is unrealistic?
- **Response:**

Looking Ahead: *In real-life problems, we rarely know the value of the population standard deviation. Eventually, we'll learn how to proceed when all we know is the sample standard deviation s.*

Example: *Test with One-Sided Alternative*

- **Background**: Earnings of 446 surveyed university students had mean \$3,776. Assume population s.d. \$6,500.
- **Question:** Are we convinced that μ is less than \$5,000?
- **Response:** State H_o : vs. H_a :

```
One-Sample Z: Earned
Test of mu = 5 vs mu < 5
The assumed sigma = 6.5
Variable          N      Mean     StDev   SE Mean
Earned          446     3.776     6.503     0.308
Variable      95.0% Upper Bound         Z       P
Earned                    4.282     -3.98   0.000
```

1. Data production issues were discussed for confidence interval.
2. Output shows sample mean _______ and z = Large?
3. P-value = . Small?
4. Conclude?

Example: *Notation*

- **Background**: Want to test if mean of all male shoe sizes could be 11.0, based on a sample mean 11.222 from 9 male students. Assume pop. s.d. 1.5.
- **Question:** How do we denote the numbers given?
- **Response:**
  - 11.0 is proposed value of population mean
  - 11.222 is sample mean _____
  - 9 is sample size _____
  - 1.5 is population standard deviation

Practice: 10.4a p.477

Example: *Intuition Before Formal Test*

- **Background**: Want to test if mean of all male shoe sizes could be 11.0, based on a sample mean 11.222 from 9 male students. Assume pop. s.d. 1.5.
- **Question:** What conclusion do we anticipate, by "eye-balling" the data?
- **Response:**

Sample mean (11.222) seems close to proposed μ_o=11.0?

Sample size (9) small→

S.d. (1.5) not very small→ ________________

Anticipate standardized sample mean z large?

→P-value small?

→conclude population mean may be 11.0?

Example: *Test with Two-Sided Alternative*

- **Background:** Want to test if mean of all male shoe sizes could be 11.0, based on a sample mean 11.222 from 9 male students. Assume pop. s.d. 1.5.
- **Question:** What do we conclude from the output?
- **Response:** $z = 0.44$. Large?
P-value (two-tailed) = 0.657. Small?
Conclude population mean may be 11.0?

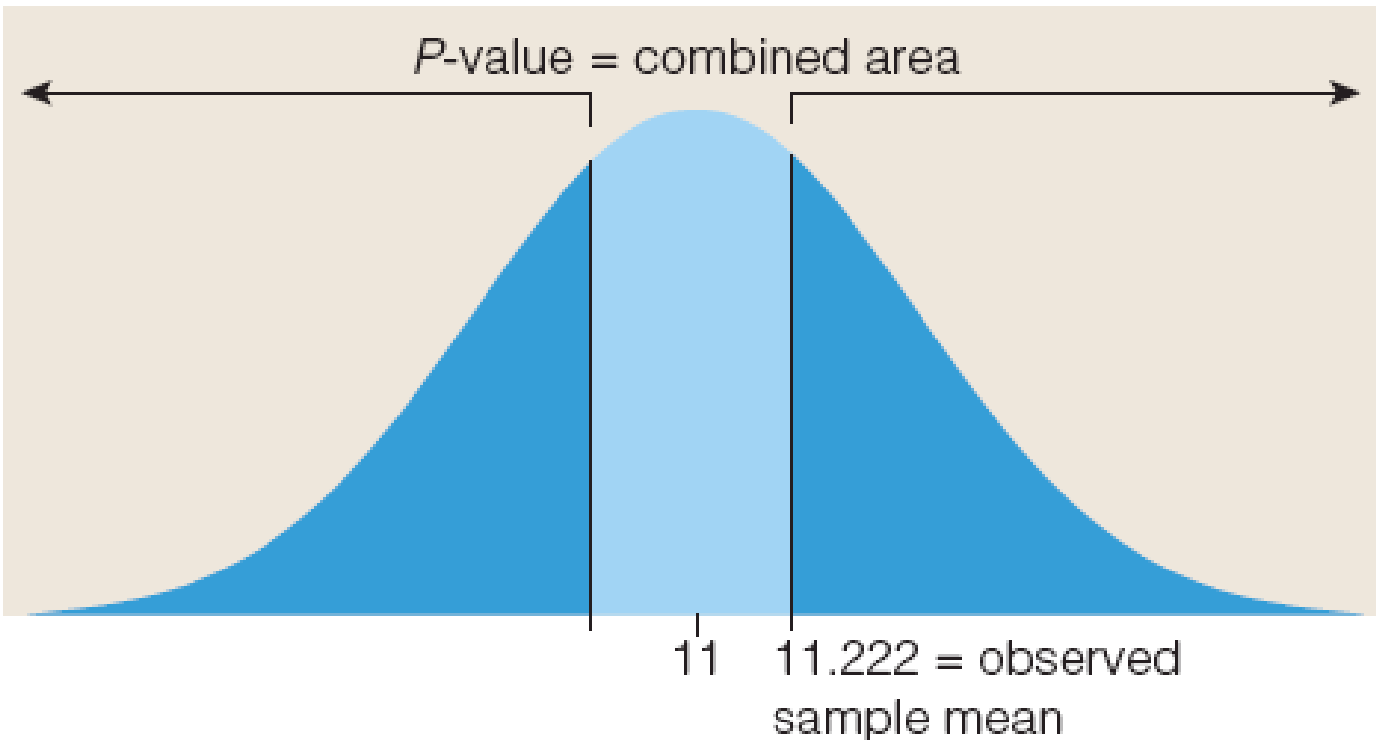
```
One-Sample Z: Shoe
Test of mu = 11 vs mu not = 11
The assumed sigma = 1.5
Variable          N      Mean     StDev   SE Mean
Shoe              9    11.222     1.698     0.500
Variable               95.0% CI            Z       P
Shoe           (  10.242,  12.202)      0.44   0.657
```


Practice: 10.13a p.480

P-value as Nonstandard Normal Probability

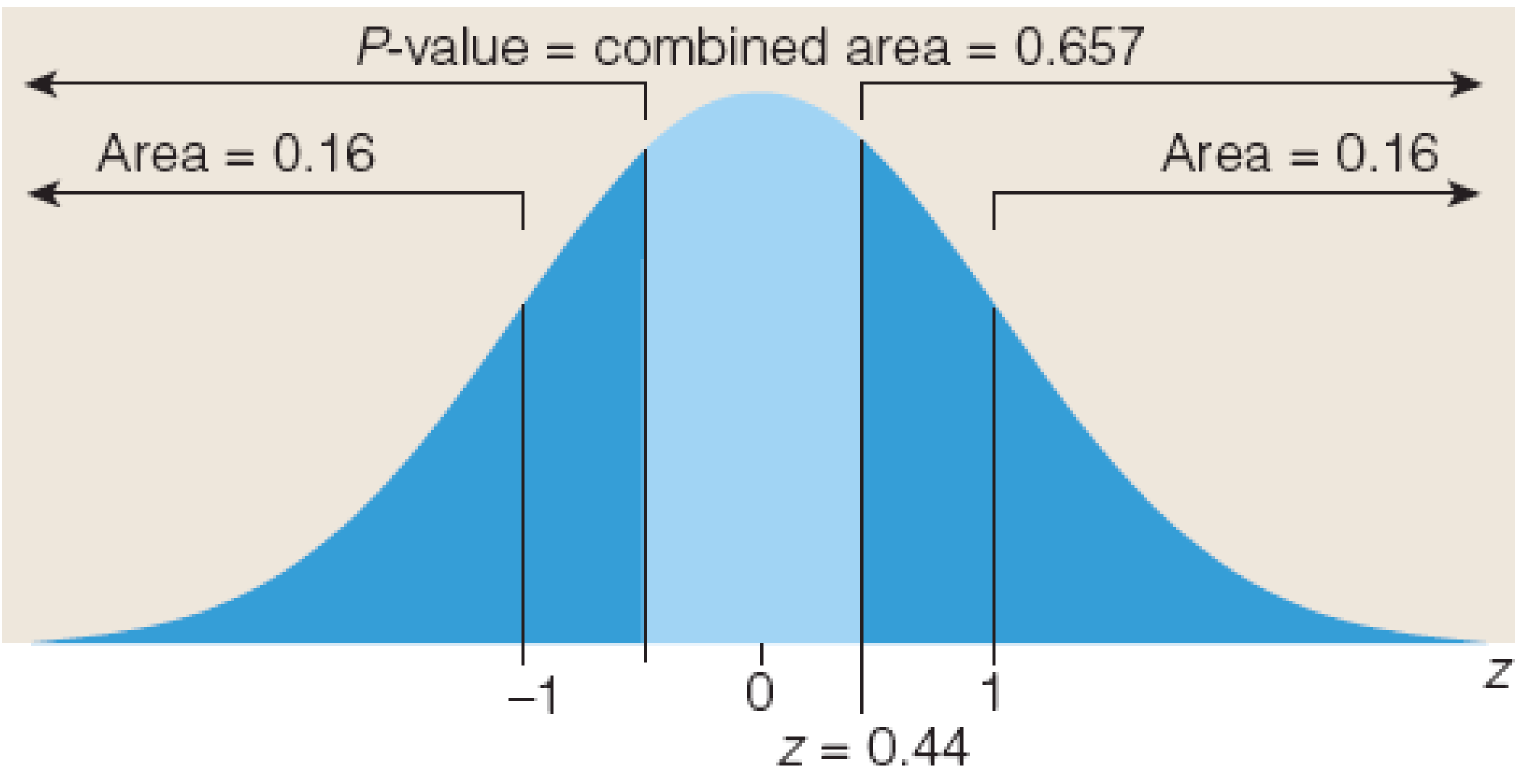
P-value is probability of sample mean as far from 11.0 (in either direction) as 11.222.

H_0: mu = 11.0 vs. H_a: mu ≠ 11.0



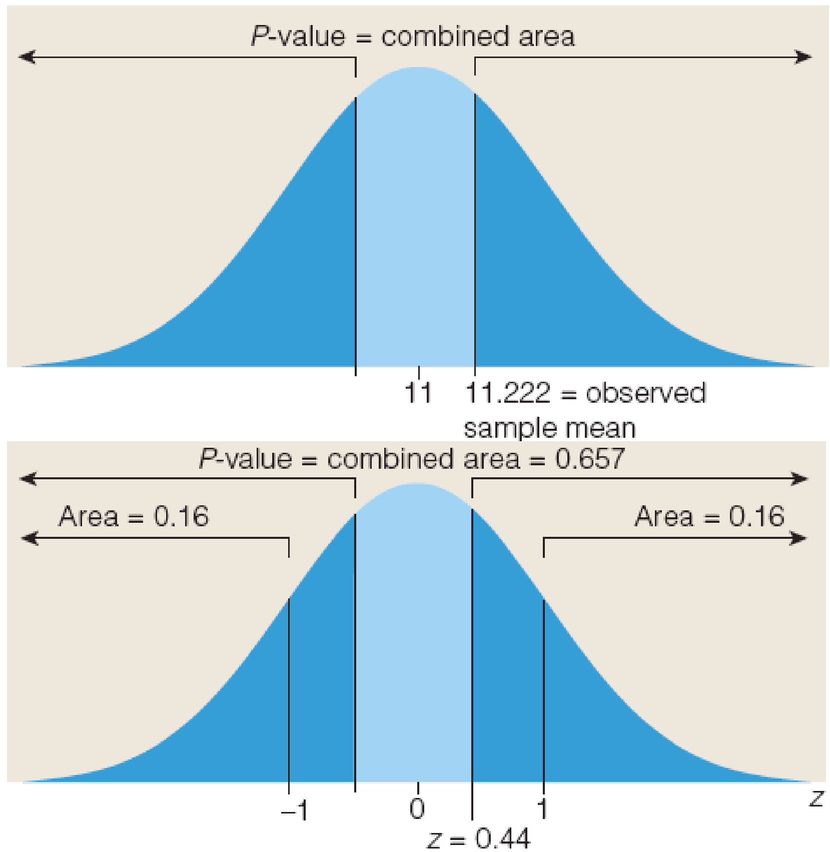
P-value as Standard Normal Probability

P-value as probability of standardized sample mean *z* as far from 0 (in either direction) as 0.44.



Comparing P-value Based on $\bar{x}$ vs. z

Same area under curve, just different scales on horizontal axis due to standardizing (below).



Example: *Test Results and Confidence Interval*

- **Background:** Tested if mean of all male shoe sizes could be 11.0, based on a sample mean 11.222 from 9 male students. Assumed pop. s.d. 1.5. *P*-value was 0.657; didn't reject null.
- **Question:** Would we expect 11.0 to be contained in a confidence interval for μ ?
- **Response:** Test showed 11.0 to be plausible for μ → (In fact, 11.0 is _______ contained in the confidence interval.)

```
One-Sample Z: Shoe
Test of mu = 11 vs mu not = 11
The assumed sigma = 1.5
Variable            N      Mean     StDev   SE Mean
Shoe                9    11.222     1.698     0.500
Variable                95.0% CI               Z        P
Shoe             (  10.242,  12.202)        0.44    0.657
```


Practice: 10.30b p.501

Example: *Test Results and Confidence Interval*

- **Background:** Tested if mean earnings of all students at a university could be \$5,000, based on a sample mean \$3,776 for n=446. Assumed pop. s.d. \$6,500. P-value was 0.000; rejected null hypothesis.
- **Question:** Would 5,000 be contained in the confidence interval for μ?
- **Response:**

Factors That Lead to Rejecting Ho

Statistically significant data produce P-value small enough to reject H_0. z plays a role:

$$z = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}} = \frac{(\bar{x} - \mu_o)\sqrt{n}}{\sigma}$$

Reject H_0 if P-value small; if $|z|$ large; if…

- Sample mean far from μ_o
- Sample size n large
- Standard deviation σ small

Role of Sample Size *n*

- **Large *n***: may reject H_0 even if sample mean is not far from proposed population mean, from a practical standpoint.

Very small *P*-value→strong evidence against Ho but $\overline{x}$ not necessarily very far from μ_o.

- **Small *n***: may fail to reject H_0 even though it is false.

Failing to reject false Ho is 2nd type of error.

Definition *(Review)*

- **Type I Error:** reject null hypothesis even though it is true (false positive)
- **Type II Error:** fail to reject null hypothesis even though it's false (false negative)

Test conclusions determine possible error:

- Reject H_0: correct or Type I
- Do not reject H_0: correct or Type II

Example: *Errors in a Medical Context*

- **Background:** A medical test is carried out for a disease (HIV).
- **Questions:**
  - What does the null hypothesis claim?
  - What are the implications of a Type I Error?
  - What are the implications of a Type II Error?
  - Which type of error is more worrisome?

Responses:

- Null hypothesis:
- False ____: conclude
- False ____: conclude
- Type ____ is more worrisome.

Example: *Errors in a Legal Context*

- **Background:** A defendant is on trial.
- **Questions:**
  - What does H_0 claim?
  - What does a Type I Error imply?
  - What does a Type II Error imply?
  - Which type is more worrisome?

- **Responses:**
  - H_0 :
  - Type I: Conclude
  - Type II: Conclude
  - Type is more worrisome.

Sample Mean Standardizing to z

→If σ is known, standardized $\bar{X}$ follows z (standard normal) distribution:

$$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = z$$

If σ is unknown and n is large enough (20 or 30), then $s \approx \sigma$ and $\frac{\bar{x}-\mu}{s/\sqrt{n}} \approx z$

Can use z if σ is known or n is large.

What if σ is unknown **and** n is small?

Sample mean standardizing to *t*

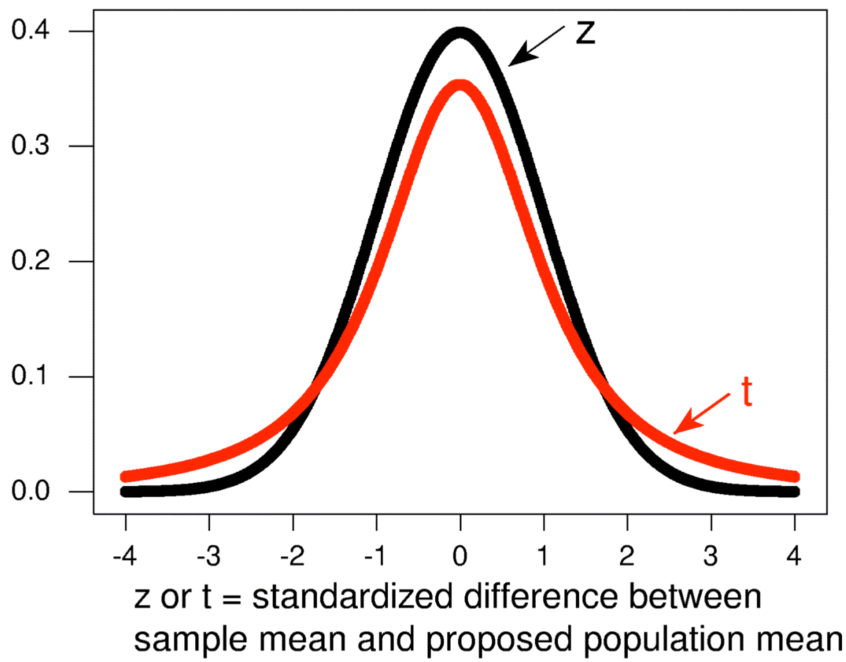
For σ unknown and *n* small, $\frac{\bar{x}-\mu}{s/\sqrt{n}} = t$

- *t* (like *z*) centered at 0 since $\bar{X}$ centered at μ
- *t* (like *z*) symmetric and bell-shaped if $\bar{X}$ normal
- *t* more spread than *z* (s.d.>1) [*s* gives less info]

t has "*n*-1 degrees of freedom"(spread depends on *n*)

Inference About Mean Based on z or t

- σ known→ standardized $\bar{x}$ is z
  (may use z if σ unknown but n large)
- σ unknown→ standardized $\bar{x}$ is t



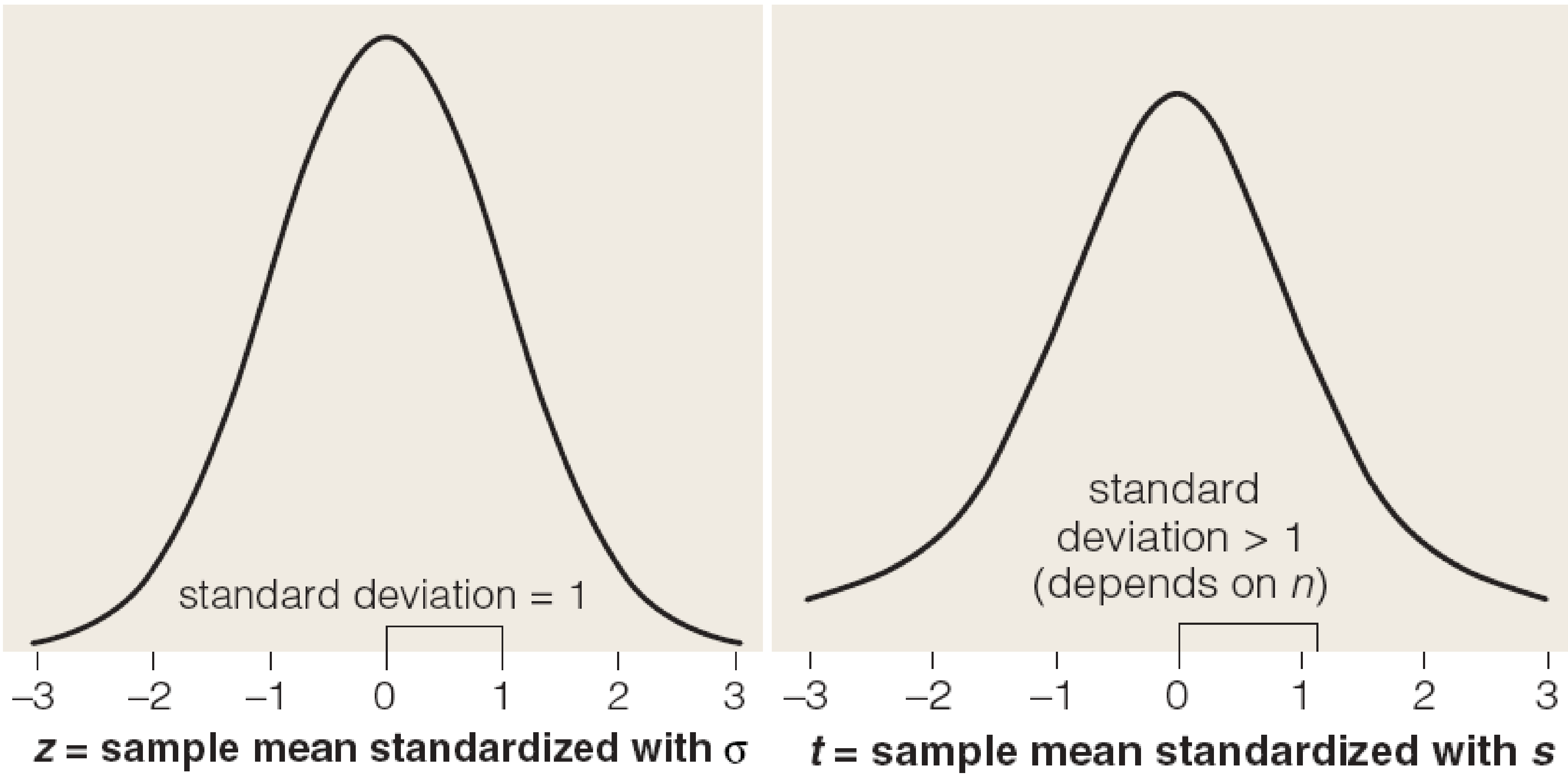
Inference *by Hand* Based on z or t

	σ known	σ unknown
small sample $(n < 30)$	$\frac{x-\mu}{\sigma/\sqrt{n}} = z$	$\frac{x-\mu}{s/\sqrt{n}} = t$
large sample $(n \geq 30)$	$\frac{x-\mu}{\sigma/\sqrt{n}} = z$	$\frac{x-\mu}{s/\sqrt{n}} \approx z$

z used if σ known **or** n large

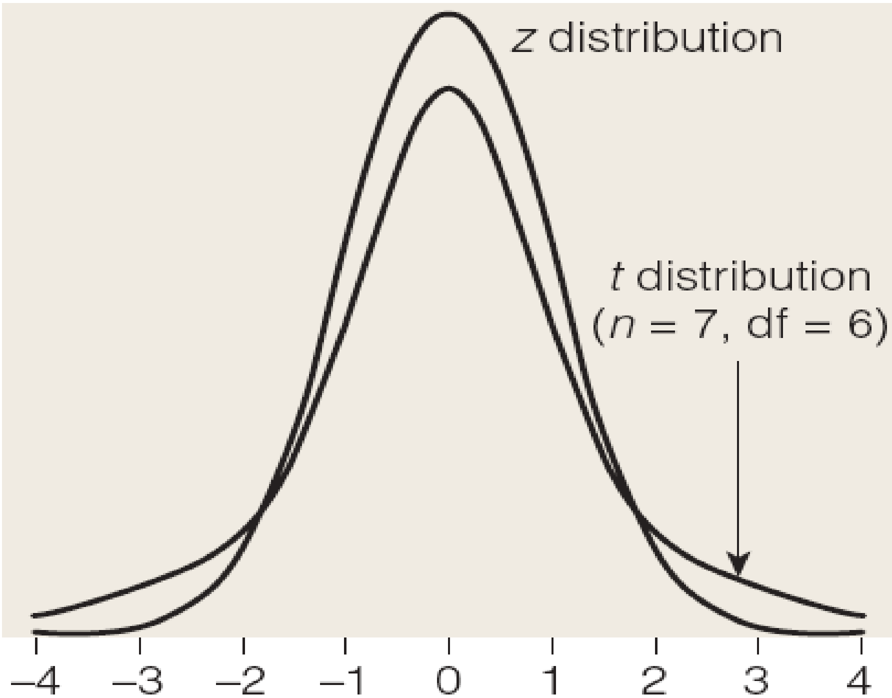
t used if σ unknown **and** n small

z vs. t: How the Sample Mean is Standardized



Example: *Distribution of t* (6 df) *vs. z*

- **Background**: For n=7, $\frac{\bar{x}-\mu}{s/\sqrt{n}} = t$ has 6 df.



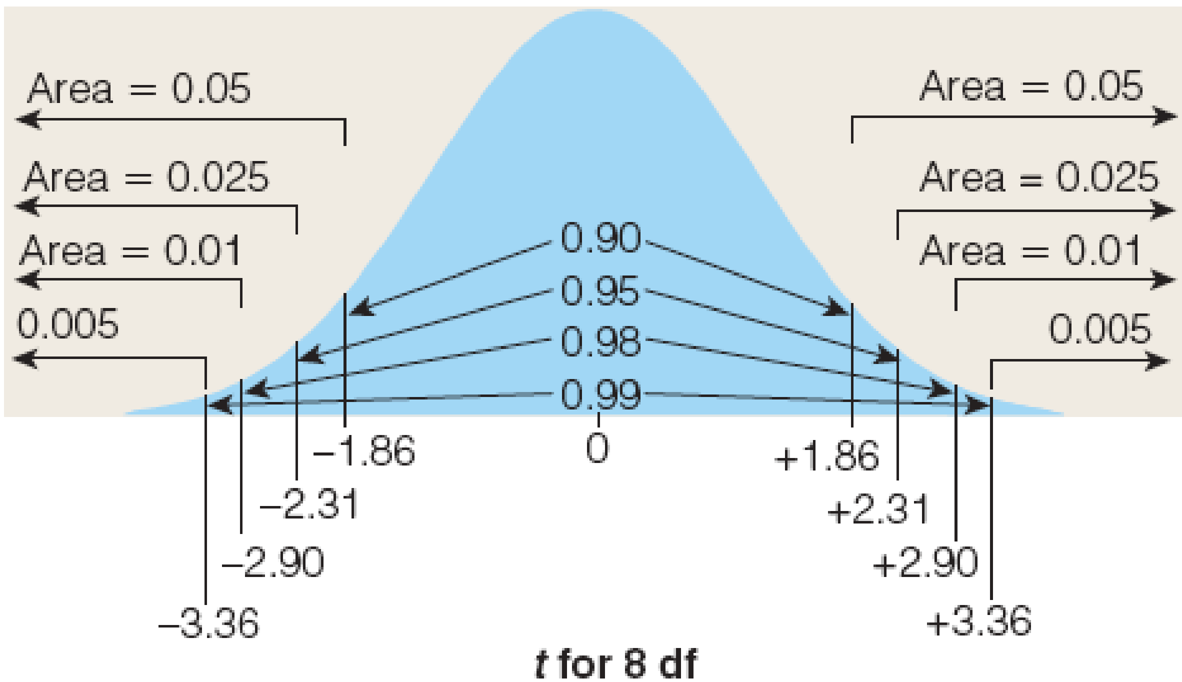
***A Closer Look:** In fact,*
P(t > 2) is about 0.05;
P(z > 2) is about 0.025.

- **Question:** How does P($\boldsymbol{t}$ > 2) compare to P($\boldsymbol{z}$ > 2)?
- **Response:** P($\boldsymbol{t}$ > 2) ________ P($\boldsymbol{z}$ > 2).

Practice: 10.14 p.489

Example: *Distribution of t* (8 df) *vs. z*

- **Background**: According to 90-95-98-99 Rule for z, P(z > 2) is between 0.01 and 0.025 because 2 is between 1.96 and 2.576. Consider the t curve for 8 df.



- **Question:** What is a range for P(t > 2) when t has 8 df?
- **Response:** P(t > 2) is between and

Lecture Summary
(Inference for Means: Hypothesis Tests; t Dist.)

- *z* test about population mean: 4 steps
- Examples: 1-sided and 2-sided alternatives
- Relating test and confidence interval
- Factors in rejecting null hypothesis
  - Sample mean far from proposed population mean
  - Sample size large
  - Standard deviation small
- Inference based on *z* or *t*
  - Population sd known; standardize to *z*
  - Population sd unknown; standardize to *t*
- Comparing *z* and *t* distributions