DENOISING OF MAGNETIC RESONANCE IMAGES USING MINIMUM DESCRIPTION LENGTH PRINCIPLE

Satashu Goel, Jerry Tsai

Department of ECE
Carnegie Mellon University
Pittsburgh, PA 15213

Narayan Krishnamurthy

Department of ECE
University of Pittsburgh
Pittsburgh, PA 15261

ABSTRACT

Magnetic resonance (MR) images are routinely used for medical diagnosis. Denoising of these images to enhance their clinical usability has been an active area of research. This paper considers the thresholding based appproaches for image denoising. The paper investigates the use of appropriate basis for image representation and the choice of an appropriate threshold. The use of wavelet packets and Minimum Desciption Length (MDL) based thresholding is proposed.

1. INTRODUCTION

MR images are typically corrupted with noise, which hinder the medical diagnosis based on these images. There has been substantial interest in the problem of denoising of images in general. Tools from traditional image processing field have been applied to denoise MR images [1]. However, the process of noise suppression must not appreciably degrade the useful features in an image. In particular, edges are important features for MR images and thus the denoising must be balanced with edge preservation.

Wavelets are popular for such image denoising and enhancement applications because they have good localization properties both in space and frequency. Further, use of wavelet packets allows adaptive representation for a given signal. A brief survey of representative techniques for image denoising is now presented. Lee and Tsai discuss the use of wavelets for image enhancement in [2]. Zadeh et al compare various filters (ratio, log ratio and angle image filters) to enhance MR images in [1]. In [3], the authors have looked at noise suppression in MR images using Fourier spectral methods. In [4], the authors used FIR filters along with wavelet decomposition for image enhancement, specifically edge enhancement and edge detection. Recently, in [2] the authors have used wavelets to enhance MR images. They used a mapping function to manipulate the transform coefficients before reconstruction. The mapping function was chosen such that the low frequency coefficients are not affected which prevents distortion. The coefficients with larger absolute values contain more information while the high frequency coefficients contain important edge information. Hence, coefficients belonging to either of these classes were heavily weighted compared to other coefficients. In [5], the author discusses the use of soft-thresholding for image denoising. More recently, denoising using MDL based thresholding was introduced in [6].

The typical flow for wavelet based image denoising is as follows. First, the given image is decoposed using either the Discrete Wavelet Transform (DWT) or a wavelet packet (an adapted wavelet tree). After representing the image using a wavelet basis, the wavelet coefficients are thresholded. The intuition is that the signal of interest is well represented by the wavelet basis whereas the noise is not and thus the small coefficients mainly correspond to noise. The estimated image is then reconstructed using the resulting coefficients.

It is proposed that wavelet packets be used instead of DWT for image representation and MDL based method be used for denoising. The intuition is that before the coefficients are thresholded, the signal of interest must be well represented. This will ensure that the thresholding process only removes the noise but does not distort the underlying signal. Various methods have been proposed to compute the threshold values. The performance of two such algorithms is compared and the use of MDL based method is proposed.

Section 2 provides an overview of the principles of MRI and the acquisition process, wavelet packets and the MDL principle. Section 3 explains the proposed approach. Section 4 discusses the results and section 5 concludes the paper.

2. NECESSARY BACKGROUND

2.1. MR Imaging Background

The basic principle in MR imaging is the detection of *precessing* dipoles by an external coil. H^1 proton the most abundant element in water and lipids of living tissue has a nuclear spin and hence a dipole associated with the spin. The spin and the dipoles in the absence of an external field are random and tend to cancel each other, but align with net moment in the presence of an external field this phenomenon is called *Magnetization* (See Figure 1). In addition to alligning the dipoles the external field exerts a force on the dipoles in a direction perpendicular to the plane containing the dipole and the external field. This causes the dipoles to rotate or *precess* about the external field given by the Larmor equation(1). The external field are chosen to vary with with position in the x, y and z directions using gradients G_x, G_y and G_z that induces spatial distribution of Larmor frequencies given by equation (2)

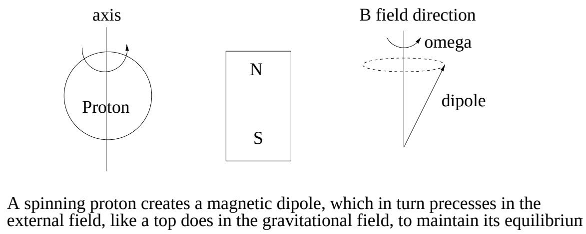

Fig. 1. NMR active dipole

$$\omega = \gamma * B \tag{1}$$

$$\omega(x) = \gamma * B_0 + \gamma * G_x * x \tag{2}$$

γ is the gyromagnetic ratio($\frac{charge}{mass}$) of the nucleus

A typical MR Imaging process is indicated in figure (2), involves the application of a rotating external RF pulse(whose frequency matches the precessing dipoles) followed by the gradients G_z -which isolates a slice, G_x -that identifies a frequency, and repetitions of different G_y -that identifies the phase corresponding to the chosen frequency. Multiple repetitions are used to reconstruct the 2-D image.

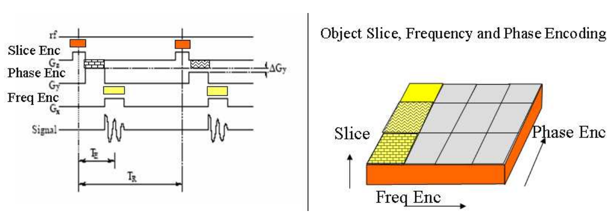

Fig. 2. MR Image Acquisition

2.2. MDL Principle

The MDL principle looks at the problem of model selection and attempts to balance the complexity of the model with the goodness-of-fit of the data. Given a finite amount of data, the classical ML (maximum likelihood) like methods choose a complex model to maximize the goodness-of-fit. However, the generalization error for these models may be very high.

Let $\mathcal{H}^{(1)}$, $\mathcal{H}^{(2)}$, ... denote the candidate models. According to MDL, the best hypothesis $H \in \mathcal{H}^{(1)} \cup \mathcal{H}^{(2)} \cup \ldots$ that explains data D is the one for which $L(H)+L(D|H)$ is the minimum [7]. $L(H)$ and $L(D|H)$ are the description lengths of the hypothesis and the data given a particular hypothesis H is chosen, respectively.

Rissanen formulated the denoising problem using MDL as a metric [6]. It was shown that even with this metric the denoising problem reduces to that of thresholding. Also, the asymptotic behavior of the threshold was analyzed and compared with the threshold obtained by Donoho and Johnstone [5].

3. PROPOSED METHOD

The paper proposes that the image be first represented using a wavelet packet, which is obtained from a full tree by pruning, followed by thresholding of wavelet coefficients for denoising. The the use of entropy metric is suggested to perform tree pruning. The intuition behind using the entropy metric is the following. The thresholding step removes coefficients which are below a certain threshold. Suppose the representation turns out to be such that the wavelet coefficients have very low variance, meaning that most of the coefficients have similar values. Then, thresholding step will either remove too many coefficients (for large threshold value) leading to excess distortion in the reconstructed image or it will remove too few coefficients (for small threshold value) leading to ineffectual denoising. It is clear that the thresholding step cannot effectively denoise the image if it is not represented appropriately in the first place. The paper thus proposes the use of wavelet packet instead of the standard Discrete Wavelet Transform.

Further, the paper proposes the use of MDL for determining the appropriate threshold value. It has been shown in [6] that the threshold value provided by Donoho and Johnstone formula [5] leads to removal of too many coefficients, leading to much smoother signal.

4. EXPERIMENTAL RESULTS

The MR images for the experiments were obtained from [8] The images were zero padded to form 256 x 256 grayscale images. White Gaussian noise was then added to the images. The original image is shown in Figure 3 and Figure 4 shows the image with a noise level of $\sigma = 20$, where σ^2 is the noise variance. Figure 5 shows the estimated image obtained by MDL denoising when using DWT for representation while Figure 6 shows the estimated image when wavelet packet is used. A comparison of the two figures shows that better denoising is done when a wavelet packet

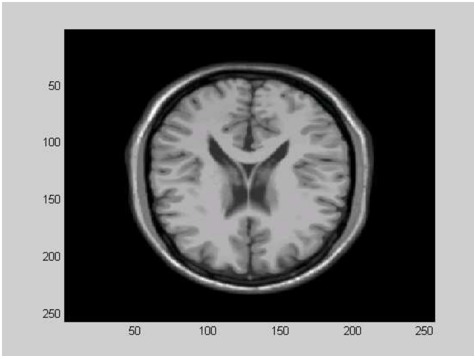

Fig. 3. Original Image

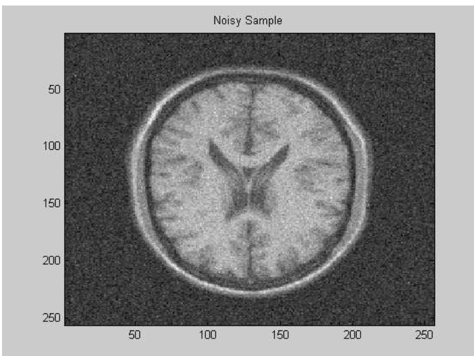

Fig. 4. Image with noise (σ = 20)

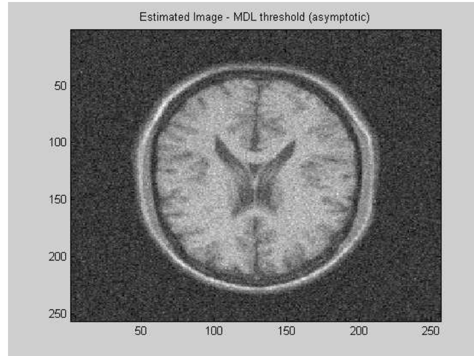

Fig. 5. Estimated Image: MDL asymptotic (using DWT)

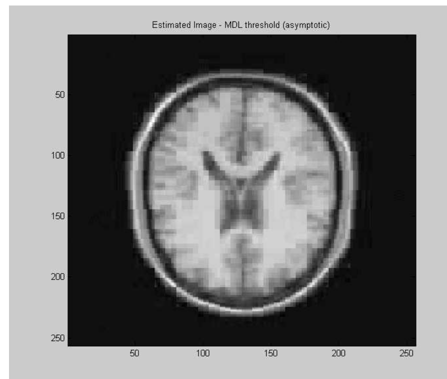

Fig. 6. Estimated Image: MDL asymptotic

is used, as compared to the case when DWT is used. However, the estimated image seems more smoothed out when wavelet packet is used. This can be explained as follows. The threshold values obtained using the Donoho and Johnstone formula and the asymptotic MDL formula [6] depend only on the noise variance and the number of data points (in this case, the number of pixels). Thus, these threshold values remain the same regardless of whether we use a wavelet packet or the standard DWT tree for decomposition. However, when a wavelet packet is obtained using the entropy metric, the distribution of energy is as compacted as possible. These two facts imply that more coefficients are thresholded when a wavelet packet is used along with standard formulae for threshold calculation.

All the other results are for denoising using wavelet packets. On comparing figures 6, 7 and 8, it was observed that the MDL threshold (using the asymptotic approximation based formula) provides better denoising than the Donoho and Johnstone threshold. The square of the threshold value as given by MDL is half that of the value provided by Donoho and Johnstone formula. This means that fewer coefficients are thresholded by MDL and thus the signal is not unnecessarily smoothed. Also, it was observed that the exact MDL denoising [6] performs better than the approximate method where the asymptotic value of the threshold is used. This was expected because the exact MDL threshold is data dependent and it should be able to provide a better threshold value. This is further confirmed by examining the threshold values obtained by different methods. The threshold values obtained from Donoho and Johnstone formula, MDL approximation formula and the exact MDL algorithm were 94.20, 66.61 and 45.13 respectively. The goal of the thresholding step is to find a threshold value such that noise is eliminated while the important features in the image, like edges are not smoothed out. By choosing a small enough threhold value, the MDL algorithm still denoises the image while preserving image features. On the other hand, Donoho and Johnstone's formula seems to over-estimate the value of the threshold, thereby smoothing out the signal in the process of noise removal. It should be pointed out that the exact MDL approach is computationally expensive because the threshold is data dependent. For large data sizes the asymptotic approximation can be used instead. Finally, the results with MDL based denoising were compared to the results obtained by using Matlab's wavelet packet denoising. Again, it is found that the MDL based method performs better in terms of denoising and edge-preservation.

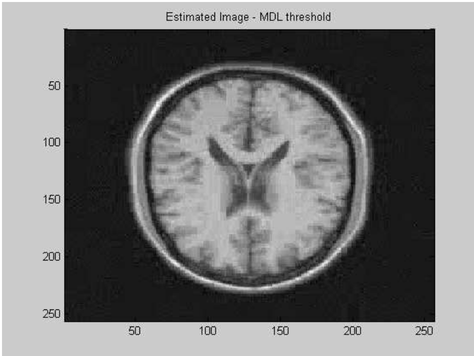

Fig. 7. Estimated Image: MDL exact

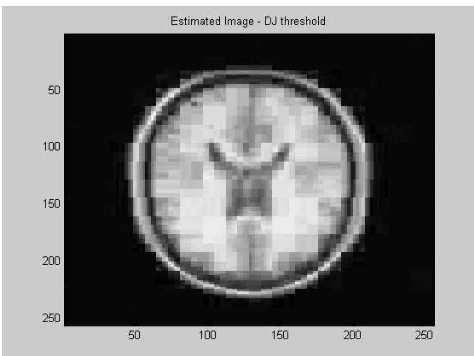

Fig. 8. Estimated Image: DJ

5. CONCLUSIONS AND FUTURE WORK

The paper proposed the use of wavelet packets in place of DWT for image representation. The use of entropy metric was suggested to perform tree pruning. Finally, the use of MDL based methods was proposed to determine the appropriate threshold values for denoising. It was found that the use of wavelet packets for representation improves the performance in terms of denoising. However, the use of standard data independent formulae for determining the threshold value leads to elimination of too many coefficients, leading to smoothing out of edges. It was found that if the data size is not large, it is beneficial to use the exact MDL algorithm to determine the threshold value. This method was shown to outperform other denoising techniques based on thresholding.

6. ACKNOWLEDGMENTS

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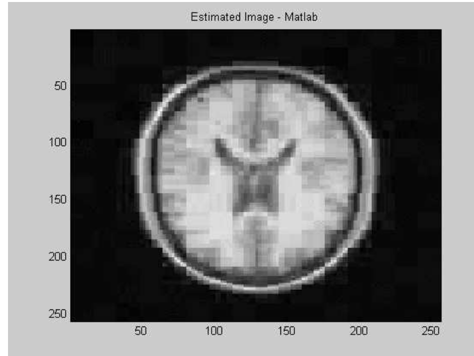

Fig. 9. Estimated Image: Matlab

for providing us with the Matlab code for display of wavelet trees.

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