

Noisy addicts? Estimation of optimal consumption choice with habit formation and measurement error*

Wayne-Roy Gayle,[†] Natalia Khorunzhina[‡]

February 2008

Abstract

In this paper we investigate empirically the presence of internal habit formation in household food consumption using data from the Panel Study of Income Dynamics (PSID). Habit formation in preferences is specified in the multiplicative form. We assume classical measurement error in observed food consumption. We exploit the resulting structure of the Euler equation to develop a nonlinear generalized method of moments (GMM) estimator of all structural parameters of interest: time-discount factor, utility curvature parameter, habit formation parameter, and the variance of measurement error in consumption. The results support the existence of a habit in food consumption. The variance of measurement error is estimated to be about 25%. Not accounting for measurement error results in underestimation of the utility curvature parameter.

JEL C13, C33, D12, D91, E21

*The authors are grateful to Jean-François Richard, David DeJong, and James Feigenbaum for discussions and insightful comments. We also thank participants at seminars at the University of Pittsburgh and the University of Virginia. All errors are our own.

[†]Department of Economics, University of Virginia, 2015 Ivy Road, Room 312, Charlottesville, VA 22904-4182, E-mail: wg4b@virginia.edu.

[‡]Department of Economics, University of Pittsburgh, 230 S. Bouquet St., 4900 WW Posvar Hall, Pittsburgh, PA 15232, E-mail: nak52@pitt.edu.

1 Introduction

In this paper we investigate empirically the presence of internal habit formation in household food consumption using data from the Panel Study on Income Dynamics (PSID). We assume that habit formation takes the multiplicative or ratio form, and that observed consumption is measured with error (i.e., we assume classical measurement error). We exploit the resulting structure of the Euler equation to develop a nonlinear generalized method of moments (GMM) estimator of all the structural parameters of interest: time-discount factor, utility curvature parameter, habit formation parameter, and the variance of measurement errors in consumption.

Time nonseparabilities in preferences have been used to explain a wide variety of macroeconomic and financial phenomena. Constantinides (1991), Abel (1990), and Campbell and Cochrane (1999) sought to resolve the equity premium puzzle of Mehra and Prescott (1985) using time-nonseparable preferences in consumption. Kydland and Prescott (1982) explained the smoothness of wages relative to hours worked, and Barro and King (1984) rationalized the observed procyclicality of hours using nonseparabilities in leisure preference. In the context of a representative agent model, empirical evidence in favor of habit formation in consumption is supported by the studies of Ferson and Constantinides (1991), Heaton (1995), Fuhrer (2000), Chen and Ludvigson (2008), and Smith and Zhang (2007), among others.

However, the evidence for habit formation in micro data is inconclusive. Hayashi (1985) tests nonseparability in preferences over consumption of several commodity groups. He points to a high degree of durability in services and semidurables but does not find the presence of habit or durability in food consumption. Dynan (2000) investigates the presence of habit formation in food consumption and confirms the result of Hayashi (1985). Carrasco, Labeaga, and Lopez-Salido (2005) estimate a model with the direct translog utility function of Meghir and Weber (1996), allowing for fixed-effects heterogeneity in the growth rates of individual consumption. In contrast with the above studies, Carrasco, Labeaga, and Lopez-Salido find habit formation in food consumption and services. Browning and Collado (2007) find evidence of habit formation in several categories of consumption, including food outside the home. In general, comparisons between the above models are made difficult by the differences in specification of preferences as well as the data used in estimation.

The approach of this paper relates closely to the prominent work of Dynan (2000) in that both

papers (i) assume isoelastic preferences in consumption services, (ii) employ panel data from the PSID to estimate the parameters of interest, and (iii) account for measurement error in observed consumption. However, there are important differences between our approach and that of Dynan. Dynan considers a difference model of habit formation, a widely used specification of habit formation in the representative agent literature.¹ However, because individual consumption data are more volatile than aggregate consumption data, this specification is less attractive for studying micro data. The difference specification imposes a nonnegativity constraint on the flow of consumption services that typically can be satisfied only for low values of the habit formation parameter. For example, if a household's consumption increases threefold in a year, the habit formation parameter must be less than one third to satisfy this constraint. This difficulty suggests the need for an alternative specification that is less restricted by the volatility of consumption found in micro data but that still admits tests of the existence of habit formation. The multiplicative habit specification introduced by Abel (1990) is such an alternative, and it is the one we employ in this paper.² The nonnegativity constraint on consumption services in the multiplicative model is satisfied for all values of consumption without restricting the habit formation parameter, while maintaining the interpretational ability of this model.

To estimate the model, Dynan (2000) employs a log-linear approximation of the Euler equation, followed by another approximation of consumption services suggested by Muellbauer (1988). This latter approximation takes the form $\Delta \ln(c_{it} - \alpha c_{it-1}) \approx \Delta \ln c_{it} - \alpha \Delta \ln c_{it-1}$, where the difference is taken over time. Again, this approximation is exact only for the case in which there is no habit ($\alpha = 0$) and appropriate only for low values of habit. Furthermore, this mean value approximation implies that the resulting mean value varies by household and is not a constant as required by the estimation strategy of Dynan (2000). This approximation, however, allows Dynan to account for measurement error in observed consumption without imposing parametric assumptions on the distribution of measurement error.

In contrast, the approach that we take in this paper avoids approximations by deriving the estimator from the exact representation of the Euler equation. We exploit the resulting structure

¹The difference model of habit formation is capable of delivering the attractive asset-pricing implications in the representative agent framework. See Constantinides (1991), Heaton (1995), Campbell and Cochrane (1999), Chen and Ludvigson (2008). Chen and Ludvigson (2008) is an excellent source of references for the difference model of habit formation.

²See Carroll (2000), Alonso-Carrera, Caballe, and Raurich (2005), and Hiraguchi (2008) for theoretical results on preferences with multiplicative habits, and Fuhrer (2000) and Smith and Zhang (2007) for the empirical investigation in the representative agent framework.

of the Euler equation with multiplicative habits to develop a nonlinear GMM estimator. In order to identify the time-discount factor, we impose a semiparametric assumption on the distribution of the measurement error as in Ventura (1994) and Alan, Attanasio, and Browning (2008).

Several issues are addressed in this paper. We test for the existence and the strength of habit formation in individual food consumption. The results, which are robust across different empirical specifications, support the existence of a strong habit in food consumption. The estimated strength of habit is similar to what is found using aggregate data, as in Fuhrer (2000) and Smith and Zhang (2007). Also, we test for the existence of measurement error in the food consumption data. In line with the existing body of literature that uses consumption data from the PSID, we find that measurement errors are substantial. Our results suggest that measurement errors account for approximately 25% of the variance of the observed log-consumption. Our results also show that ignoring measurement errors in consumption significantly biases the parameter estimates. Specifically, not accounting for measurement errors results in underestimation of the utility curvature parameter. Using the parameter estimates from our model, we compute the implied household- and time-specific Intertemporal Elasticity of Substitution (IES). The sample mean of the IES is 0.80, which is consistent with earlier findings for bondholders (see Vissing-Jorgensen, 2002). We also find that the IES increases earlier in household life, peaks around age 40, and declines afterwards.

2 Theoretical Framework

Household i chooses a sequence of consumption $\{c_{is}, s = t, \dots, T\}$ to maximize its expected lifetime utility function, given by

$$\mathbb{E}_{it} \sum_{s=t}^T \beta^{s-t} \phi_{is} \frac{\tilde{c}_{is}^{1-\gamma} - 1}{1-\gamma}, \quad (2.1)$$

where the expectation is taken conditional on all information for household i at time t , $\beta \in (0, 1)$ is the time-discount factor, and $0 \leq \alpha \leq 1$ measures the strength of habits; \tilde{c}_{it} is consumption services in period t . Following Abel (1990) and subsequent body of the literature (Caroll, Overland, and Weil, 2000; Caroll, 2000; Smith and Zhang, 2007), consumption service is defined as the ratio between current consumption expenditures and past consumption expenditures geometrically

weighted:

$$\tilde{c}_{i,t} = \frac{c_{it}}{c_{it-1}^\alpha}. \quad (2.2)$$

Household-specific “taste shifters” ϕ_{it} are given by

$$\phi_{it} = \exp(\delta' w_{it} + \omega_i), \quad (2.3)$$

where w_{it} is a vector of exogenous time-varying observed household characteristics and ω_i is a household fixed effect. We assume that household i is not subject to liquidity constraints and has rational expectations. The first-order necessary condition for the household’s optimization problem is

$$\mathbb{E}_{it} [\beta(1 + r_{i,t+1})MU_{i,t+1} - MU_{i,t} | \mathbb{Z}_{it}] = 0, \quad (2.4)$$

where $r_{i,t+1}$ is the rate of return to savings available to household i between periods t and $t + 1$, \mathbb{Z}_{it} denotes the set of all information that is available to household i at time t , and $MU_{i,t}$ represents household i ’s marginal utility of consumption in period t :

$$MU_{i,t} = \frac{\phi_{it}}{c_{it}} \left(\frac{c_{it}}{c_{it-1}^\alpha} \right)^{1-\gamma} - \alpha\beta \frac{\phi_{it+1}}{c_{it}} \left(\frac{c_{it+1}}{c_{it}^\alpha} \right)^{1-\gamma}. \quad (2.5)$$

Hence we rewrite Equation (2.4) as the following moment condition:

$$\mathbb{E}_{it} \left\{ \beta(1+r_{i,t+1}) \left(\frac{\phi_{it+1}}{c_{it+1}} \left(\frac{c_{it+1}}{c_{it}^\alpha} \right)^{1-\gamma} - \alpha\beta \frac{\phi_{it+2}}{c_{it+1}} \left(\frac{c_{it+2}}{c_{it+1}^\alpha} \right)^{1-\gamma} \right) - \frac{\phi_{it}}{c_{it}} \left(\frac{c_{it}}{c_{it-1}^\alpha} \right)^{1-\gamma} + \alpha\beta \frac{\phi_{it+1}}{c_{it}} \left(\frac{c_{it+1}}{c_{it}^\alpha} \right)^{1-\gamma} \middle| \mathbb{Z}_{it} \right\} = 0 \quad (2.6)$$

Similar to the difference models of habit formation with positive α , current consumption services (Equation 2.2) are positively related to the current level of consumption and negatively related to levels of past consumption. In the multiplicative model this condition is not enough for habit formation to exist. While many studies that consider multiplicative internal habit formation models do not impose restrictions on γ (other than being nonnegative, as in Carroll, 2000, and Smith and Zhang, 2007), for the multiplicative model to exhibit the habit formation property, γ has to be greater than 1. As long as both $\alpha > 0$ and $\gamma > 1$, the household’s marginal utility of consumption in period t is an increasing function of consumption in period $t - 1$ and there is a complementarity effect of consumption over time (see, e.g., Heaton, 1995, and Kocherlakota, 1996). Alonso-Carrera, Caballe, and Raurich (2005) and Hiraguchi (2008) also note that $0 < \alpha < 1$ and $\gamma > 1$ must be

imposed for an interior solution to exist. These arguments must be taken into consideration when we proceed with the testing of the model of multiplicative internal habit formation.

3 The Estimator

In order to derive an exact nonlinear GMM estimator of the parameter vector of interest from Equation (2.6), key issues need to be addressed: nonstationarity of consumption and measurement errors in consumption. We discuss these issues in turn.

3.1 Consumption growth

The estimator derived in this paper implements panel data from the PSID, where the number of households N is large relative to the number of time periods T . The asymptotic properties of the estimator are derived assuming that N goes to infinity and T is fixed. We assume that the (time) vector of household observations is drawn independently from a common distribution. This is in contrast to the implementation of time series data, where the typical sufficient condition is that of stationarity of the variables. However, the resulting estimator may be subject to small sample instability problems in the panel data context if the data are not stationary. We therefore follow the literature and transform the moment equation (Equation 2.6) into one that is expressed in terms of the growth rates of consumption. Let $\mathbb{C}_{it} = (c_{it-1}, c_{it}, c_{it+1}, c_{it+2}, r_{it+1})$, $g_{it} = c_{it}/c_{it-1}$, and $\varphi_{it} = \phi_{it}/\phi_{it-1}$. Then Equation (2.6) can be written as

$$\mathbb{E}_{\mathbb{C}_{it}|\mathbb{Z}_{it}} \left\{ \beta(1+r_{it+1}) \frac{\varphi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it}} \right)^{1-\gamma} \left(1 - \alpha\beta\varphi_{it+2} \left(\frac{g_{it+2}}{g_{it+1}} \right)^{1-\gamma} \right) - \left(1 - \alpha\beta\varphi_{it+1} \left(\frac{g_{it+1}}{g_{it}} \right)^{1-\gamma} \right) \right\} = 0. \quad (3.1)$$

Another advantage of converting Equation (2.6) in terms of growth in consumption is that the unobserved household fixed effects ω_i are eliminated, giving $\varphi_{it} = \exp(\delta' \Delta w_{it})$.

3.2 Measurement error

Given a set of appropriate instruments and the absence of measurement errors, nonlinear GMM estimation based on the moment condition (Equation 3.1) can deliver consistent estimates of the

parameters of interest: α , β , γ , and δ' . However, the estimation of nonlinear rational expectation models with or without habit formation using micro data is complicated by the existence of measurement errors in consumption, which, if ignored, will likely result in biased and inconsistent estimation of the key parameters of interest.

Estimates for measurement errors in food consumption data reported in the PSID vary considerably in the literature. Runkle's (1991) panel data estimation of the linearized consumption Euler equation suggests that measurement errors are responsible for up to 76% of the variance of the consumption growth rate that is not explained by family-specific real interest rates. In a model setting similar to Runkle, Lahiri (1993) finds that measurement errors constitute nearly 23% of the measured consumption. Alan and Browning (2003) estimate it in the range 4% to 28%, and Alan, Attanasio, and Browning (2008) report an estimate of the variance of measurement errors that corresponds to 62% of the variation in consumption growth being noise.

Log-linearization of the Euler equation has the advantage of remaining tractable when accounting for measurement errors in consumption in both time-separable and nonseparable models.³ However, as discussed in Carroll (2001), a log-linear approximation can result in severe bias of the parameter estimates. The linear approximation used in the difference models of Muellbauer (1988) and Dynan (2000) (i.e., $\Delta \ln[c_{it} - \alpha c_{it-1}] \approx \Delta \ln c_{it} - \alpha \Delta \ln c_{it-1}$, where the difference is taken over time) can be argued to take us far from the original specification, and thus the resulting estimates are likely to be biased.

Nonlinear GMM estimators of the Euler equation provide an alternative to log-linearization, but without additional distributional assumptions the problem of measurement errors remains difficult to account for. Significant progress has been made in accounting for classical measurement error in time-separable models. Ventura (1994) assumes that measurement errors are serially independent and lognormally distributed, while Hong and Tamer (2003) make the assumption that the marginal distributions of the measurement errors are Laplace with zero mean and unknown variance and independent over each other. After re-parametrization, the approaches in Ventura (1994) and Hong and Tamer (2003) (applied to the time-separable Euler equation) yield similar moment conditions for the estimation of the utility curvature parameter subject to a proper set of instruments. However, the time-discount factor remains unidentified. Alan, Attanasio, and Browning (2008) suggest two

³Log-linearization of the Euler equation allows Dynan (2000) to account for measurement errors in consumption expenditures without additional parameterization while testing for nonseparabilities in individual current and past consumption.

exact GMM estimators: One assumes a lognormal distribution for the measurement errors, and the other relaxes this assumption. The advantage of the lognormal assumption is that it allows for point identification of the time-discount factor together with the risk aversion parameter.

Nonseparabilities in preferences bring another layer of difficulty into nonlinear estimation in the presence of classical measurement error. Due to the increasing complexity of the moment conditions with habit formation, measurement errors cannot be easily separated from the observed consumption. Thus, to our knowledge, there are no studies that apply nonlinear estimators to test for nonseparabilities in individual consumption.⁴

Following studies that deal with the exact estimation of Euler equations, we assume that true consumption c_{it} is measured with a multiplicative error η_{it} , so that observed consumption is given by $c_{it}^* = c_{it}\eta_{it}$, where $\eta_{it} > 0$. We further make the following distributional assumptions:

Assumption 3.1. *Measurement errors in consumption are log-normally distributed conditional on unobserved individually specific random effects:*

$$\ln \eta_{it} | \mu_i \sim N(\mu_i, \sigma^2), \quad (3.2)$$

where, given μ_i , the η_{it} s are

1. independent from prediction errors in the optimization problem faced by the household;
2. independent from the consumption for all t ;
3. independent from each other over time; and
4. independent from the interest rate and income stochastic processes.

The parametric specification of the distribution of measurement errors can be relaxed at the cost of not being able to identify the time-discount factor. Parts 1, 2, and 4 of Assumption 3.1 are standard for the classical errors-in-variables model. Part 3 of Assumption 3.1 imposes independence over time of the measurement errors, given an unobserved individual specific effect μ_i . The

⁴Chen and Ludvigson (2006), Fuhrer (2000), and Smith and Zhang (2007) undertake the empirical estimation of the consumption model with habit formation using nonlinear exact estimation. But the authors deal with aggregate consumption data. Hence the measurement error issue is not a significant concern.

individual specific effect results in positive correlation in measurement error over time. This is a generalization of the (standard) classical measurement errors specification.

Let \mathbb{C}_{it}^* be the counterpart of the vector \mathbb{C}_{it} where true consumption is replaced by observed consumption, and let $\mathfrak{S}_{it} = (\eta_{it-1}, \eta_{it}, \eta_{it+1}, \eta_{it+2})$ be the corresponding vector of measurement errors. Define also $g_{it}^* = c_{it}^*/c_{it-1}^*$ and $v_{it} = \eta_{it}/\eta_{it-1}$ so that $g_{it}^* = g_{it}v_{it}$. In order to obtain an expression in terms of the observed consumption, we consider Equation (3.1) piece by piece and substitute in the identity for observed consumption in terms of true consumption and measurement error as stated above. We start with the first term.

$$\begin{aligned} & \mathbb{E}_{\mathbb{C}_{it}, \mathfrak{S}_{it} | Z_{it}} \left[\beta(1+r_{it+1}) \frac{\varphi_{it+1}}{g_{it+1}^*} \left(\frac{g_{it+1}^*}{g_{it}^* \alpha} \right)^{1-\gamma} \right] = \\ & \mathbb{E}_{\mathbb{C}_{it}, \mathfrak{S}_{it} | Z_{it}} \left[\beta(1+r_{it+1}) \frac{\varphi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it} \alpha} \right)^{1-\gamma} \frac{1}{v_{it+1}} \left(\frac{v_{it+1}}{v_{it} \alpha} \right)^{1-\gamma} \right] = \\ & \mathbb{E}_{\mathbb{C}_{it} | Z_{it}} \left[\beta(1+r_{it+1}) \frac{\varphi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it} \alpha} \right)^{1-\gamma} \right] \mathbb{E}_{\mathfrak{S}_{it} | Z_{it}} \left[\frac{1}{v_{it+1}} \left(\frac{v_{it+1}}{v_{it} \alpha} \right)^{1-\gamma} \right] = \\ & \mathbb{E}_{\mathbb{C}_{it} | Z_{it}} \left[\beta(1+r_{i,t+1}) \frac{\varphi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it} \alpha} \right)^{1-\gamma} \right] \mathcal{A}_1, \end{aligned}$$

where

$$\mathcal{A}_1 = \exp\{\sigma^2 (\alpha^2(1-\gamma)^2 + \gamma^2 - \alpha\gamma(1-\gamma))\}.$$

Hence

$$\mathbb{E}_{\mathbb{C}_{it} | Z_{it}} \left[\beta(1+r_{i,t+1}) \frac{\varphi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it} \alpha} \right)^{1-\gamma} \right] = \mathbb{E}_{\mathbb{C}_{it}, \mathfrak{S}_{it} | Z_{it}} \left[\beta \mathcal{A}_1^{-1} (1+r_{it+1}) \frac{\varphi_{it+1}}{g_{it+1}^*} \left(\frac{g_{it+1}^*}{g_{it}^* \alpha} \right)^{1-\gamma} \right].$$

The second and the third terms are transformed in the same way to get

$$\begin{aligned} \mathbb{E}_{\mathbb{C}_{it} | Z_{it}} \left[\beta(1+r_{i,t+1}) \frac{\varphi_{it+1} \varphi_{it+2}}{g_{it+1}} \left(\frac{g_{it+1} g_{it+2}}{(g_{it} g_{it+1}) \alpha} \right)^{1-\gamma} \right] &= \mathbb{E}_{\mathbb{C}_{it}, \mathfrak{S}_{it} | Z_{it}} \left[\alpha \beta^2 \mathcal{A}_2^{-1} (1+r_{it+1}) \frac{\varphi_{it+1} \varphi_{it+2}}{g_{it+1}^*} \left(\frac{g_{it+1}^* g_{it+2}^*}{(g_{it}^* g_{it+1}^*) \alpha} \right)^{1-\gamma} \right] \\ \mathbb{E}_{\mathbb{C}_{it} | Z_{it}} \left[\alpha \beta \varphi_{it+1} \left(\frac{g_{it+1}}{g_{it} \alpha} \right)^{1-\gamma} \right] &= \mathbb{E}_{\mathbb{C}_{it}, \mathfrak{S}_{it} | Z_{it}} \left[\alpha \beta \mathcal{A}_3^{-1} \varphi_{it+1} \left(\frac{g_{it+1}^*}{g_{it}^* \alpha} \right)^{1-\gamma} \right], \end{aligned}$$

where

$$\begin{aligned} \mathcal{A}_2 &= \exp\{\sigma^2 (\alpha^2(1-\gamma)^2 + \gamma^2 + (1-\gamma)(1+\alpha))\}, \\ \mathcal{A}_3 &= \exp\{\sigma^2 ((1+\alpha+\alpha^2)(1-\gamma)^2)\}. \end{aligned}$$

The moment condition (Equation 3.1) with (unobserved) true consumption is therefore transformed to a moment condition composed of observed consumption:

$$\mathbb{E}_{\mathcal{C}_{it}, \mathcal{S}_{it} | \mathbb{Z}_{it}^*} \left[\beta \mathcal{A}_1^{-1} (1+r_{it+1}) \frac{\varphi_{it+1}}{s_{it+1}^*} \left(\frac{s_{it+1}^*}{s_{it}^* \alpha} \right)^{1-\gamma} \left(1 - \alpha \beta \frac{\mathcal{A}_1}{\mathcal{A}_2} \varphi_{it+2} \left(\frac{s_{it+2}^*}{s_{it+1}^* \alpha} \right)^{1-\gamma} \right) - \left(1 - \alpha \beta \mathcal{A}_3^{-1} \varphi_{it+1} \left(\frac{s_{it+1}^*}{s_{it}^* \alpha} \right)^{1-\gamma} \right) \right] = 0 \quad (3.3)$$

where \mathbb{Z}_{it}^* is a q -dimensional observable subset of \mathbb{Z}_{it} that can include current and past interest rates as well as observable consumption growth up to time $t - 2$.

We can estimate the unknown structural parameters of interest using Equation (3.3) as a conditional moment condition that we write more compactly as

$$\mathbb{E} \{ m(x_{it+1}^*, \theta_o) | \mathbb{Z}_{it}^* \} = 0, \quad (3.4)$$

where m is defined as

$$m(x_{it+1}^*, \theta_o) = \beta_o \mathcal{A}_{o1}^{-1} (1+r_{it+1}) \frac{\varphi_{oit+1}}{s_{oit+1}^*} \left(\frac{s_{oit+1}^*}{s_{oit}^* \alpha_o} \right)^{1-\gamma_o} \left(1 - \alpha_o \beta_o \frac{\mathcal{A}_{o1}}{\mathcal{A}_{o2}} \varphi_{oit+2} \left(\frac{s_{oit+2}^*}{s_{oit+1}^* \alpha_o} \right)^{1-\gamma_o} \right) - \left(1 - \alpha_o \beta_o \mathcal{A}_{o3}^{-1} \varphi_{oit+1} \left(\frac{s_{oit+1}^*}{s_{oit}^* \alpha_o} \right)^{1-\gamma_o} \right),$$

$x_{it+1}^* = (g_{i,t+2}^*, g_{i,t+1}^*, g_{i,t}^*, r_{i,t+1}, \mathbf{w}_{it+1})$, $\mathbf{w}_{it+1} = (\Delta w_{t+2}, \Delta w_{t+1})$, the subscript o denotes the true value of the parameters, and $\theta_o = (\alpha_o, \beta_o, \gamma_o, \sigma_o, \delta'_o)$ is the vector of parameters to be estimated.

The assumption of rational expectations implies that the prediction errors made by individuals are zero in expectation. It is important to recognize that the sample counterpart of the moment condition converges to zero in expectation as the number of time periods increases, but not as the number of individuals increases. Violation of this assumption can bias the estimates of the model. Hence an immediate extension of the model is to account for aggregate shocks, since we are dealing with a short panel.

Accounting for aggregate shocks can be done by modifying the moment condition (Equation 3.4) as follows:

$$\mathbb{E} \{ m_1(x_{it+1}^*, \theta_o) - \lambda_t | \mathbb{Z}_{it}^* \} = 0. \quad (3.5)$$

This results in estimation of the additional vector of parameters $\lambda' = (\lambda_1, \dots, \lambda_T)$.

We apply the efficient continuous updating GMM of Hansen, Heaton, and Yaron (1996) to

obtain estimates of the structural parameters, a technique robust to possible weak identification.

4 Data

Data on food consumption, as well as income and demographic characteristics at the level of individuals and households are available from the Panel Study of Income Dynamics (PSID). Although it is the longest panel study, and one of the most comprehensive sources of information for studying life cycle processes in general, and poverty and welfare dynamics in particular, its use for life cycle consumption dynamics bears one drawback: consumption data are available only for food. However, data on consumption of food as a perishable good are particularly suitable for testing whether this category of consumption can be habit-forming. The observed data's annual frequency is also advantageous. As argued in Dynan (2000), if there is any effect of durability in food consumption, it is not likely to last more than a few months.

The main consumption sample that we use in this study consists of consumption data from 1974 through 1987. Consumption of households consists of expenditures on food consumed both at home and away from home, and the value of food stamps. Data on food consumed at home and the value of food stamps are deflated using the consumer price index (CPI) for food at home. Data on food consumed away from home are deflated using the CPI deflator for food away from home. All CPI data are taken from the consumer price index releases of the Bureau of Labor Statistics. As in Dynan (2000), food consumption data are deflated according to the month and the year when the interview occurred, while food stamps and data on income are deflated using the CPI for the end of the year before the interview was conducted. In addition, total consumption expenditures are adjusted by the size of household.

Following Zeldes (1989), Runkle (1991), Dynan (2000), and similar studies, we excluded observations for which the consumption growth rate was higher than 3 and lower than 0.34. It is likely, though, that the higher variations in consumption growth rate that we observe in the untrimmed data are due to measurement errors. Thus, the estimated magnitude of the variance of the measurement errors in consumption is considered a lower bound with this data trimming. We exclude households whose marital status changed and those whose head was younger than 22 or older than 65 over the period of estimation. Household characteristics used in estimation as taste shifters include family size and age of the head of household.

As in Shapiro (1984), Runkle (1991), Dynan (2000), and similar studies, we construct the household-specific real after-tax interest rate as $r_{i,t+1} = R_t(1 - \tau_{i,t+1}) - \pi_{t+1}$, where R_t is the average 12-month Treasury bill for the first half of the preceding year, $\tau_{i,t+1}$ is the household marginal tax rate as reported in the PSID, and π_{t+1} is the CPI deflator for the period of the interview. As an alternative to individual-specific interest rates, we also use common real interest rates that we constructed based on the U.S. 12-month Treasury bill rates and the consumer price index.

The estimation of the moment condition (Equation 3.4) requires data on consumption expenditures for four consecutive years for each orthogonality condition. With the restrictions on data described above, we get an unbalanced panel on 3,644 households covering eight years from 1978 through 1985. Instruments include current individual specific interest rate, past and current family characteristics, lagged hours worked, and a constant.

5 Results

We address several problems while discussing the results obtained from the estimation. First, we test whether individual consumption can be habit-forming. And if it is habit-forming, how strong is the habit? Also, we want to know how crucial it is to account for measurement errors in consumption and what is the magnitude of the measurement errors estimated from the habit formation model. We compare the model estimated with measurement error against one that ignores the measurement error issue.

The results from estimating the model while addressing the above issues are presented in Table 1. Empirical models (1) through (4) represent our basic habit formation specification (Equation 3.4) estimated with noisy individual consumption data. Model (1) is our benchmark model that uses individual-specific interest rates, while model (3) is estimated with common interest rates (12-month Treasury bill). Demographic characteristics as taste shifters are included in most of the specifications that we consider. We omit demographic characteristics in specification (2) in order to demonstrate the sensitivity of the results to taste shifters. Model specification (4) shows the results of the model estimated with aggregate shocks (Equation 3.5). Empirical models (5) through (7) are similar to models (1) through (3) with the difference that in the former we ignore the possible measurement errors in consumption data. In model (8) we set the habit formation parameter α to 0, so that the moment condition (Equation 3.4) collapses to the moment condition based on the

Euler equation for a time-separable model similar to one considered by Ventura (1994) and Alan, Attanasio, and Browning (2008).

In general, the parameters are precisely estimated. In most of the specifications, the habit formation parameter α ranges between 0.90 and 0.99 and is statistically significant, implying the presence of strong habit formation in household consumption. We find the estimates of the habit formation parameter are very similar to their analogs obtained from aggregate data, reported by Fuhrer (2000) and Smith and Zhang (2007). The habit formation parameter ranges between 0.80 and 0.90 in Fuhrer (2000) and between 1.00 and 1.02 in Smith and Zhang (2007). We also find that the habit formation parameter is slightly lower in specification (4) when we account for the presence of aggregate shocks. The results using common interest rates (model [3]) are similar to those using individual-specific interest rates.

The importance of augmenting the individual utility function with individual-specific taste shifters has been widely accepted in the estimation of optimal consumption choices with micro data.⁵ As a robustness check, we remove family characteristics as taste shifters from the main model specification (estimated as model [2]). The habit formation parameter drops to 0.16 but still remains significantly different from zero. The difference in the estimates of the parameter in the models with and without taste shifters can be attributed to the additional source of identification provided by the variation in the demographic characteristics.

The utility curvature parameter γ is estimated to be between 7.8 and 8.5 when we account for measurement error (models [1] through [4]). The estimate of this parameter drops to 0.91 - 1.15 when we ignore the noise in the consumption data (models [5] through [7]) or when we do not allow for habit formation (model [8]). The estimates of the curvature parameter are close to those reported by Fuhrer (2000), who reports the estimate of γ at 6.11. When the measurement errors are ignored, the estimates of the curvature parameter are more similar to the estimates of the same parameter in the macro data - based model of Smith and Zhang (2007), who find it to be slightly greater than 1. When we assume no habit formation (specification [8]), we also obtain a lower value of the utility curvature parameter similar to the estimate of the time-separable Euler equation of Alan, Attanasio, and Browning (2008). When we do not account for measurement errors in

⁵The representation of taste shifters in terms of demographic characteristics is a common practice in the estimation of optimal consumption choices with micro data. Examples of augmenting the preferences with taste shifters can be found in Blundell, Browning and Meghir (1994), Banks, Blundell, and Tanner (1998), Dynan (2000), Alan, Attanasio, and Browning (2008), and similar studies.

the consumption data, in the models (5) and (7) the curvature parameter goes beyond the possible bounds for this parameter ($\gamma > 1$). The time-discount factor β is estimated to be between 0.94 and 0.98 in specifications with measurement errors.

Introduction of the measurement errors largely results in increasing the curvature parameter γ , while the time-discount factor β and the habit formation parameter α tend to be stable among most of the specifications. We notice that the curvature parameter moves with the same pattern in the time-separable model estimated with the same data by Alan, Attanasio, and Browning (2008). Their results show that the introduction of measurement error into the nonlinear estimator results in a significant increase in the curvature parameter, although not as dramatic as ours. This suggests that there is a positive relation between the magnitude of the measurement error variance and the utility curvature parameter. Indeed, this is intuitive, as we would expect a smaller utility curvature parameter with more volatile consumption and a greater utility curvature parameter with smoother consumption. By accounting for the measurement errors, we control for the excess noise in the observed consumption series.

The estimate of σ^2 for the specifications (1) and (3) equals 0.035, which constitutes around 25% of the variance of the observed log of consumption that can be attributed to the measurement errors. This is less than the 62% estimated by Alan, Attanasio, and Browning (2008) but closer to the estimates in the range of 4% - 28% by Lahiri (1993) and Alan and Browning (2003). In all cases the variance of the measurement errors is precisely estimated. The measurement error variance is estimated to be somewhat larger when we drop individual-specific taste shifters (model [2]). Apparently taste shifters add to the explanation of some variation in the observed individual consumption. If taste shifters are omitted from the model, the unexplained variation in the consumption data will possibly add to the measurement error. The estimates of the variance of the measurement errors do not change if we use common interest rates instead of individual-specific ones.

Given the parameter estimates of the model, we are able to directly compute various quantities of interest. The measure that has received the most attention in recent literature is the intertemporal elasticity of substitution (IES). In the Appendix, we show by direct calculation that our model

implies that the reciprocal of the IES is time- and household-specific and is given by

$$\frac{1}{IES_{it}} = \gamma - \frac{\alpha\beta(1-\gamma)\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{g_{it+1}^*}{g_{it}^*\alpha}\right)^{1-\gamma}}{1-\alpha\beta\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{g_{it+1}^*}{g_{it}^*\alpha}\right)^{1-\gamma}} - \frac{\alpha^2\beta(1-\gamma)\frac{\mathcal{A}_1}{\mathcal{A}_2}\varphi_{it+2}\left(\frac{g_{it+2}^*}{g_{it+1}^*\alpha}\right)^{1-\gamma}}{1-\alpha\beta\frac{\mathcal{A}_1}{\mathcal{A}_2}\varphi_{it+2}\left(\frac{g_{it+2}^*}{g_{it+1}^*\alpha}\right)^{1-\gamma}}. \quad (5.1)$$

Equation (5.1) shows that the inverse of IES differs from what would be obtained with time-separable constant relative risk aversion CRRA preferences by the last two terms. These last two terms depend on all the parameters of the model, consumption growth rates, and taste shifters. Using the parameter estimates from specification (1), the sample mean of the IES is 0.80 with standard error 0.006, and the median is 0.95. The mean IES is larger than what is typically found using aggregated data (see Blundell, Browning, and Meghir, 1994; Atkeson and Ogaki, 1996; Attanasio and Weber, 1993; Attanasio and Browning, 1995; Vissing-Jorgensen, 2002) or over certain cohorts of individuals (Blundell, Browning, and Meghir, 1994; Attanasio and Weber, 1993; and Attanasio and Browning, 1995). However, the mean IES is consistent with the findings of Vissing-Jorgensen (2002) and Attanasio and Weber (1993).

Diaz, Pijoan-Mas, and Rios-Rull (2003) derive the IES implied by the multiplicative habits model along a balanced growth path to be $IES = 1/[\alpha + (1 - \alpha)\gamma]$. Using our parameter estimates, this works out to 0.95, which is the sample median of the IES obtained from our estimates. However, the implied IES from Diaz, Pijoan-Mas, and Rios-Rull (2003) does not change with age. The results Attanasio and Weber (1993) suggest a dome-shaped pattern of the IES over the life cycle in that it increases over the early stages of life and declines over the later years. A regression of the IES obtained from specification (1) on age and age squared obtains

$$IES_{it} = 0.5804 + 0.0114 * age_{it} - 0.0001 * age_{it}^2, \quad (5.2)$$

(0.1139) (0.0054) (0.00006)

which is consistent with the results of Attanasio and Weber (1993). The regression line is presented in Figure 1. The results from the analogous median regression exhibit a similar pattern.

Table 1: Estimation of the Euler equation with habit formation¹

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
γ	7.98 (0.884)	8.46 (0.616)	7.76 (0.878)	8.31 (0.872)	0.91 (0.039)	1.15 (0.011)	0.88 (0.039)	1.12 (0.091)
β	0.971 (0.354)	0.950 (0.224)	0.984 (0.357)	0.937 (0.313)	0.981 (0.048)	0.790 (0.015)	0.954 (0.039)	0.910 (0.138)
α	0.992 (0.061)	0.161 (0.034)	0.984 (0.065)	0.909 (0.055)	0.981 (0.075)	0.967 (0.050)	0.981 (0.061)	
σ^2	0.035 (0.001)	0.044 (0.002)	0.035 (0.001)	0.033 (0.004)				0.039 (0.120)
% of the noise	25	28	25	24				26
r_{it+1}	yes	yes		yes	yes	yes	yes	yes
T-bill			yes				yes	
Demographics	yes	no	yes	yes	yes	no	yes	no
Aggr. shocks				yes				

¹ Number of households in the sample 3,644. Number of time periods 8. Demographic characteristics include family size and age squared. Standard errors in parenthesis.

6 Conclusion

In this paper we investigate empirically the presence of habit formation in household food consumption. Habit formation in preferences over consumption is specified in the multiplicative form introduced by Abel (1990). We exploit the resulting structure of the Euler equation to develop a GMM estimator that allows for estimation of not only all structural parameters of interest (time-discount factor, utility curvature parameter and habit formation parameter), but also of the variance of (classical) measurement error in consumption.

The results, which are robust across different empirical specifications, support the existence of a habit in food consumption. The estimated strength of the habit is similar to that found using aggregate data, as in Fuhrer (2000) and Smith and Zhang (2007). In line with the existing body of literature that uses consumption data from the PSID, we find that measurement errors are substantial. Our results suggest that measurement errors account for approximately 25% of the variance of the observed log-consumption. Our results also show that ignoring measurement errors in consumption significantly affects the parameters of interest. Specifically, not accounting for measurement errors results in underestimation of the utility curvature parameter.

We also show that in our model framework we can analyze the implied household- and time-specific intertemporal elasticity of substitution. We estimate the mean of the IES at 0.80, which supports earlier findings for bondholders. We also find that the IES increases earlier in household life, peaks around age 40, and declines afterwards.

References

- [1] **Abel, Andrew B.** 1990. "Asset Prices Under Habit Formation and Catching Up With the Jones", *American Economic Review*, 80: 38-42.
- [2] **Alan, Sule and Martin Browning.** 2003. "Estimating Intertemporal Allocation Parameters Using Simulated Residual Estimation", University of Copenhagen CAM Working Paper 2003-03.
- [3] **Alan, Sule, Orazio Attanasio and Martin Browning.** Forthcoming. "Estimating Euler Equation with Noisy Data: Two Exact GMM Estimators", *Journal of Applied Econometrics*.
- [4] **Alonso-Carrera, Jaime, Jordi Caballe and Xavier Raurich.** 2005 "Growth, Habit Formation, and Catching-up with the Joneses". *European Economic Review*, 49(6): 1665-1691.
- [5] **Amemiya, Yasuo.** 1985. Instrumental Variable Estimator for the Nonlinear Errors-in-Variables Model, *Journal of Econometrics* 28: 273-289.
- [6] **Atkeson, Andrew and Ogaki, Masao.** 1996. Wealth-Varying Intertemporal Elasticities of Substitution: Evidence from Panel and Aggregate Data. *Journal of Monetary Economics*, 38: 507-534.
- [7] **Attanasio, Orazio, James Banks, and Sarah Tanner.** 2002. Asset Holding and Consumption Volatility. *Journal of Political Economy*, 110(4): 771-792.
- [8] **Attanasio, Orazio and Martin Browning.** 1995. Consumption over the Life Cycle and over the Business Cycle. *American Economic Review*, 85(5): 1118-1137.
- [9] **Attanasio, Orazio and Guglielmo Weber.** 1993. Consumption Growth, the Interest Rate and Aggregation. *The Review of Economic Studies*, 60(3): 631-649.
- [10] **Banks, James, Richard Blundell and Sarah Tanner.** 1998 Is There a Retirement Savings Puzzle?, *American Economic Review*, 88(4): 769-88.
- [11] **Barro, Robert J. and Robert G. King.** 1984. Time-separable Preferences and Intertemporal Substitution Models of Business Cycles", *Quarterly Journal of Economics*, 99: 817-839.
- [12] **Blundell, Richard W., Martin Browning and Costas Meghir.** 1994. Consumer Demand and the Life- Cycle Allocation of Household Expenditures, *Review of Economics Studies*, 61(1): 57-80.
- [13] **Browning Martin and M. Dolores Collado.** 2007. "Habits and Heterogeneity in Demands: A Panel Data Analysis", *Journal of Applied Econometrics*, 22: 625-640.

- [14] **Campbell, John.Y. and John H. Cochrane.** 1999. By Force of Habit. A Consumption-based Explanation of Aggregate Stock Market Behavior”, *Journal of Political Economy*, 107(2): 205-251.
- [15] **Carrasco, Raquel, Jos M. Labeaga and J. David Lopez-Salido.** 2005. Consumption and Habits: Evidence from Panel Data”, *The Economic Journal*, 115(January): 144-165.
- [16] **Carroll, Christopher D.** 2000. Solving Consumption Models with Multiplicative Habits” *Economics Letters*, 68(1): 67-77.
- [17] **Carroll, Christopher D., Jody R. Overland and David N.Weil.** 2000. Saving and Growth with Habit Formation”, *American Economic Review*, 90: 115.
- [18] **Carroll, Christopher D.** 2001. Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second-Order Approximation)”, *Advances in Macroeconomics*, 1(1): 6.
- [19] **Chen, Xiaohong and Sydney C.Ludvigson.** Forthcoming. Land of Addicts? An Empirical Investigation of Habit-Based Asset Pricing Behavior”, *Journal of Applied Econometrics*.
- [20] **Constantinides, George M.** 1991. Habit formation: A Resolution of the Equity Premium Puzzle”. *Journal of Political Economy*, 98(3): 519-543.
- [21] **Diaz, Antonia, Josep Pijoan-Mas and Jos-Vctor Rios-Rull.** 2003. Habit Formation: Implications for the Wealth Distribution”. *Journal of Monetary Economics*, 50(6): 1257-1291.
- [22] **Dynan Karen E.**2000. Habit Formation in Consumer Preferences. Evidence from Panel Data”. *American Economic Review*, 90: 391-406.
- [23] **Ferson, Wayne E. and George M. Constantinides.** 1991. Habit Persistence and Durability in Aggregate Consumption”, *Journal of Financial Economics*, 29: 199-240.
- [24] **Fuhrer, Jeffrey C.** 2000. Habit Formation in Consumption and Its Implications for Monetary-Policy Models”. *American Economic Review*, 90: 367-390.
- [25] **Hansen, Lars Peter, John C. Heaton and Amir Yaron.** 1996. Finite Sample Properties of some Alternative GMM Estimators, *Journal of Business and Economic Statistics*, 14: 262-280.
- [26] **Hayashi, Fumio.** 1985. The Permanent Income Hypothesis and Consumption Durability: Analysis Based on Japanese Panel Data”, *The Quarterly Journal of Economics*, 100(4): 1083-1113.
- [27] **Heaton, John C.** 1995. An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications”, *Econometrica*, 63: 681-717.

- [28] **Hiraguchi Ryoji.** 2008. Some foundations for multiplicative habits models”, *Journal of Macroeconomics*, 30: 873-884.
- [29] **Hong, Han and Elie Tamer.** 2003. A Simple Estimator for Nonlinear Error in Variable Models”, *Journal of Econometrics*, 117: 1-19.
- [30] **Kocherlakota, Narayana R.** 1996. The Equity Premium: Its Still a Puzzle, *Journal of Economic Literature* 24: 4271.
- [31] **Kydland, Finn E. and Edward C. Prescott.** 1982. Time to Build and Aggregate Fluctuations”, *Econometrica*, 50: 1345-1370.
- [32] **Lahiri, Kajal.** 1993. Panel Data Models with Rational Expectations”. In *Handbook of Statistics*, Vol.11, ed. G.S. Maddala, C.R. Rao, H.D. Vinod., Amsterdam: North-Holland.
- [33] **Meghir, Costas and Guglielmo Weber.** 1996. Intertemporal Nonseparability or Borrowing Restrictions? A Disaggregate Analysis Using a U.S. Consumption Panel,” *Econometrica*, 64(5): 1151-81.
- [34] **Mehra, Rajnish and Edward C. Prescott.** 1985. The Equity Premium: A Puzzle”, *Journal of Monetary Economics*, 15(1): 145-161.
- [35] **Muellbauer, John.** 1988. Habits, Rationality and Myopia in the Life Cycle Consumption Function, *Annales d' Economie et de Statistique*, 9: 4770.
- [36] **Naik, Narayan Y. and Michael J. Moore.** 1996. Habit Formation and Intertemporal Substitution in Individual Food Consumption. *Review of Economics and Statistics*, 78(2): 321-328.
- [37] **Runkle, David E.** 1991. Liquidity Constraints and the Permanent Income Hypothesis”, *Journal of Monetary Economics*, 27(1): 73-98.
- [38] **Shapiro, Matthew D.** 1984. The Permanent Income Hypothesis and the Real Interest Rate”. *Economics Letters*, 14: 93-100.
- [39] **Smith, William T. and Qiang Zhang.** 2007. Asset Pricing with Multiplicative Habit and Power-expo Preferences”. *Economics Letters*, 94 (3): 319-325.
- [40] **Ventura, Eva.** 1994. A Note on Measurement Error and Euler Equations”. *Economics Letters*, 45: 305-308.
- [41] **Vissing-Jorgensen, Annette.** 2002. Limited Asset Market Participation and the Elasticity of Intertemporal Substitution. *Journal of Political Economy*, 110(4): 825- 853.
- [42] **Zeldes, Stephen P.** 1989. Consumption and Liquidity Constraints: An Empirical Investigation”, *Journal of Political Economy*, 97(2): 305-346.

7 Appendix

Intertemporal Elasticity of Substitution

In this section we calculate individual-specific intertemporal elasticities of substitution. Individual-specific intertemporal elasticity of substitution can be found from:

$$\frac{1}{IES_{it}} = \left| \frac{\partial \ln \frac{MU_{it}}{MU_{it+1}}}{\partial \ln \frac{c_{it+1}}{c_{it}}} \right| \quad (7.1)$$

where

$$\begin{aligned} \frac{MU_{it}}{MU_{it+1}} &= \frac{\frac{\phi_{it}}{c_{it}} \left(\frac{c_{it}}{c_{it-1}^\alpha} \right)^{1-\gamma} - \alpha\beta \frac{\phi_{it+1}}{c_{it}} \left(\frac{c_{it+1}}{c_{it}^\alpha} \right)^{1-\gamma}}{\frac{\phi_{it+1}}{c_{it+1}} \left(\frac{c_{it+1}}{c_{it}^\alpha} \right)^{1-\gamma} - \alpha\beta \frac{\phi_{it+2}}{c_{it+1}} \left(\frac{c_{it+2}}{c_{it+1}^\alpha} \right)^{1-\gamma}} \\ &= \frac{\left(1 - \alpha\beta\phi_{it+1} \left(\frac{g_{it+1}}{g_{it}^\alpha} \right)^{1-\gamma} \right)}{\frac{\phi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it}^\alpha} \right)^{1-\gamma} \left(1 - \alpha\beta\phi_{it+2} \left(\frac{g_{it+2}}{g_{it+1}^\alpha} \right)^{1-\gamma} \right)} \end{aligned} \quad (7.2)$$

Taking logs of (7.2) and then taking partial derivative with respect to $\ln g_{it+1} = \ln \frac{c_{it+1}}{c_{it}}$ we obtain:

$$\frac{1}{IES_{it}} = \gamma - \frac{\alpha\beta(1-\gamma)\phi_{it+1} \left(\frac{g_{it+1}}{g_{it}^\alpha} \right)^{1-\gamma}}{1 - \alpha\beta\phi_{it+1} \left(\frac{g_{it+1}}{g_{it}^\alpha} \right)^{1-\gamma}} - \frac{\alpha^2\beta(1-\gamma)\phi_{it+2} \left(\frac{g_{it+2}}{g_{it+1}^\alpha} \right)^{1-\gamma}}{1 - \alpha\beta\phi_{it+2} \left(\frac{g_{it+2}}{g_{it+1}^\alpha} \right)^{1-\gamma}} \quad (7.3)$$

If we believe that the observed consumption c^* is contaminated with the measurement error, we must take it into account while calculating individual-specific IES. Consider the conditional expectation of the denominator and the numerator of (7.2). We start with the denominator:

$$\begin{aligned} \mathbb{E}_{\mathcal{C}_{it}|\mathcal{Z}_{it}} \left\{ \frac{\phi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it}^\alpha} \right)^{1-\gamma} \left(1 - \alpha\beta\phi_{it+2} \left(\frac{g_{it+2}}{g_{it+1}^\alpha} \right)^{1-\gamma} \right) \right\} &= \\ \mathbb{E}_{\mathcal{C}_{it}, \mathcal{S}_{it}|\mathcal{Z}_{it}^*} \left\{ \mathcal{A}_1^{-1} \frac{\phi_{it+1}}{g_{it+1}^*} \left(\frac{g_{it+1}^*}{g_{it}^\alpha} \right)^{1-\gamma} \left(1 - \alpha\beta \frac{\mathcal{A}_1}{\mathcal{A}_2} \phi_{it+2} \left(\frac{g_{it+2}^*}{g_{it+1}^\alpha} \right)^{1-\gamma} \right) \right\} \end{aligned} \quad (7.4)$$

If we drop expectation, we can write (7.4) as follows:

$$\frac{\phi_{it+1}}{g_{it+1}} \left(\frac{g_{it+1}}{g_{it}^\alpha} \right)^{1-\gamma} \left(1 - \alpha\beta\phi_{it+2} \left(\frac{g_{it+2}}{g_{it+1}^\alpha} \right)^{1-\gamma} \right) \varepsilon_{it+1} = \mathcal{A}_1^{-1} \frac{\phi_{it+1}}{g_{it+1}^*} \left(\frac{g_{it+1}^*}{g_{it}^\alpha} \right)^{1-\gamma} \left(1 - \alpha\beta \frac{\mathcal{A}_1}{\mathcal{A}_2} \phi_{it+2} \left(\frac{g_{it+2}^*}{g_{it+1}^\alpha} \right)^{1-\gamma} \right) \varepsilon_{it+1}^*$$

where

$$\varepsilon_{it+1} = \frac{\mathbb{E}_{C_{it}|Z_{it}} \left\{ \frac{\varphi_{it+1}}{s_{it+1}} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma} \left(1 - \alpha \beta \varphi_{it+2} \left(\frac{s_{it+2}}{s_{it+1}^* \alpha} \right)^{1-\gamma} \right) \right\}}{\frac{\varphi_{it+1}}{s_{it+1}} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma} \left(1 - \alpha \beta \varphi_{it+2} \left(\frac{s_{it+2}}{s_{it+1}^* \alpha} \right)^{1-\gamma} \right)} \quad (7.5)$$

and ε_{it+1}^* is defined analogously. Likewise, we can write for the numerator:

$$\left(1 - \alpha \beta \varphi_{it+1} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma} \right) e_{it} = \left(1 - \alpha \beta \mathcal{A}_3^{-1} \varphi_{it+1} \left(\frac{s_{it+1}^*}{s_{it}^* \alpha} \right)^{1-\gamma} \right) e_{it}^*$$

where

$$e_{it} = \frac{\mathbb{E}_{C_{it}|Z_{it}} \left\{ \left(1 - \alpha \beta \varphi_{it+1} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma} \right) \right\}}{\left(1 - \alpha \beta \varphi_{it+1} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma} \right)} \quad (7.6)$$

and e_{it}^* is defined analogously. Then (7.2) can be represented as:

$$\frac{MU_{it}}{MU_{it+1}} = \frac{MU_{it}^*}{MU_{it+1}^*} u_{it+1} \quad (7.7)$$

where $u_{it+1} = \frac{e_{it}^* \varepsilon_{it+1}}{e_{it} \varepsilon_{it+1}^*}$. Taking natural logs of (7.7) gives us:

$$\begin{aligned} \ln \frac{MU_{it}}{MU_{it+1}} &= \ln \frac{MU_{it}^*}{MU_{it+1}^*} + \ln u_{it+1} \\ &= \ln \frac{\left(1 - \alpha \beta \varphi_{it+1} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma} \right)}{\frac{\varphi_{it+1}}{s_{it+1}^*} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma} \left(1 - \alpha \beta \varphi_{it+2} \left(\frac{s_{it+2}}{s_{it+1}^* \alpha} \right)^{1-\gamma} \right)} + \ln u_{it+1} \end{aligned} \quad (7.8)$$

Thus we can write:

$$\begin{aligned} \frac{1}{IES_{it}} &= \left| \frac{\partial \ln \frac{MU_{it}}{MU_{it+1}}}{\partial \ln \frac{c_{it+1}}{c_{it}}} \right| \\ &= \left| \frac{\partial \ln \frac{MU_{it}^*}{MU_{it+1}^*}}{\partial \ln \frac{c_{it+1}^*}{c_{it}^*}} \right| \\ &= \gamma - \frac{\alpha \beta (1-\gamma) \mathcal{A}_3^{-1} \varphi_{it+1} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma}}{1 - \alpha \beta \mathcal{A}_3^{-1} \varphi_{it+1} \left(\frac{s_{it+1}}{s_{it}^* \alpha} \right)^{1-\gamma}} - \frac{\alpha^2 \beta (1-\gamma) \mathcal{A}_1 \mathcal{A}_2 \varphi_{it+2} \left(\frac{s_{it+2}}{s_{it+1}^* \alpha} \right)^{1-\gamma}}{1 - \alpha \beta \mathcal{A}_2 \varphi_{it+2} \left(\frac{s_{it+2}}{s_{it+1}^* \alpha} \right)^{1-\gamma}} \end{aligned} \quad (7.9)$$

Figure 1: Regression of the IES on age

