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# Statistical analysis of electric power production costs

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Electricity cannot be conviently stored. Thus there should be sufficient production at all times to meet the demand for electric power. If a low-cost generating unit fails this will lead to its substitution by a higher cost unit. The cost of producing electric power is a random variable because it depends upon two uncertain quantities, demand and the availability of the generating units. Analytical computation of the mean and the variance of the production costs can become quite cumbersome and time consuming for large systems, and therefore Monte Carlo simulation becomes an attractive alternative. A simulation study based on time series analysis of actual load data is described in which the primary objective was to determine the respective contributions of the demand and the generator availabilities to the variability of the estimates of the production cost. A secondary objective was to find out the extent to which an accurate temperature forecast reduces this variability. The results show that demand is a significant source of variation, and an accurate temperature forecast mitigates the effect of load uncertainty in the forecast of production costs.

## 1. Introduction

One of the characteristics of electric power production is that it cannot be conveniently stored. Thus at every instant of time there should be a sufficient amount of electricity production to meet the demand. If an electric power generating unit currently in operation fails or if the demand is unexpectedly very high, an electric utility company may have no other option but to import expensive power from another source or press into service one of its own inefficient generators. The cost of producing electric power is a random variable because it is dependent upon the uncertain mix of available generators and the uncertain demand. It is the purpose of this paper to describe a statistical model for the electric power production cost insofar as it is affected by these two factors and report on a statistical analysis based on this model. The cost of fuel for the electric power generators which is a major component of the production cost can be considered to be non-random when the study horizon spans only a limited time period.

In the current regulated climate, production costing models are widely used in the electric power industry by the individual utilities for the purpose of forecasting the cost of electricity production. This use covers short, medium and long-term electricity operations and planning. For example, in the very short-term, which may range from the next hour to the next few days, the models are used to forecast electricity spot prices and guide decisions regarding whether the utility's own generators should be used to produce power or purchase from outside independent producers. In the moderate term horizon which may last anywhere from one month to one year, these models are used to write supply contracts with major users and vendors. For the long-term spanning a period of one year and beyond, they are used to guide decisions regarding acquisition and disposal of generation assets. These models account for the expected variation of load (i.e., demand for power) over time and the different degrees of utilization of the generating units. In the standard models, the production cost of a power generating system over a given time interval is obtained by adding the amounts of energy produced by each generator in MegaWatt-Hours (MWH) multiplied by its operating cost ($/MWH). The amount of energy produced by a given unit of course depends upon the magnitude of the load, its capacity, cost and availability that dictate the extent to which it will be called upon to deliver power. In the coming deregulated climate of the future, it is expected that these models will also continue to play an important role. One of their anticipated uses will be to help buyers and sellers of electricity to devise hedging strategies for transactions in the open market.

For the most part, in the current versions of the probabilistic production costing models that are routinely used by the industry, load is considered to be deterministic. The only source of uncertainty that is accounted for by these models is generator availability. In this paper we specifically account for load uncertainty, and show that

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for short-range forecasts, this source of uncertainty is no less important than that associated with generator availability. In this connection, we also investigate to what extent an accurate temperature forecast reduces the magnitude of this source of variation.

Traditionally, the electric power industry has made its decisions based on the anticipated expected values of production costs only, but it is being increasingly recognized that a measure of variance is equally necessary. The reasons for this as they pertain to a regulated environment are outlined in Lee *et al.* [1] and Ryan and Mazumdar [2,3]. In the deregulated marketplace, in order to carry out hedging strategies for risk management, measures of volatility (i.e., the variance) for both the price and volume of the electricity demanded will be required [4]. An approximate estimate of the market-clearing prices for electric power is given by the short-term marginal costs, and the production costing model that is being described in this paper can be used to extract information about these costs as well [5]. We consider the effect of the variability of load and generator availability on the variation of production cost of a generation system consisting of a typical assortment of generating units using representative data. The results show that load variability plays an important role in the variation of production costs which points out the need for developing and using models that explicitly include this source of variability.

We first describe in Section 2 two models for evaluating production costs. Section 3 reviews the analytical formulas for the computation of the expected value and the variance of the production costs of a generating system. Since the analytical formulas are seen to be quite complex, Monte Carlo techniques remain an effective alternative. Section 4 gives the results of statistical analysis of a Monte Carlo simulation output in order to determine the relative importance of the two stochastic components of production costs, load and generator availability.

## 2. An overview of production costing models

Electric power consumption varies with time reflecting the predictably cyclical nature of human activities. The demand is higher during the day and early evening, weekdays and the summer or winter seasons as compared to late night and early morning, weekends and fall or spring seasons. In order to run the electric power generation system economically so as to reliably meet the demand it is thus necessary to turn on and off the generating machines at appropriate times. The process of turning on a machine is known as *unit commitment* in electric power parlance. But the units cannot be turned on and off in a haphazard manner. Besides the start-up costs, one also needs to consider certain operating constraints that dictate how frequently and in what manner the units can be turned on or off. They are, for example: *minimum up time*, *minimum downtime*, *minimum capacity*, *maximum capacity*, *ramping rate*, etc. The minimum up time (downtime) constraint refers to the fact that a unit once turned on (off) needs to be running for at least a certain amount of time before it can be turned off (on). The minimum and maximum capacity constraints of a generating unit specify the levels between which a generator needs to operate. The ramping rate gives the maximum rate at which a generator can attain full power from an "off" state. Given an hourly load profile for a specified time horizon, the optimization problem whose solution gives the sequence in which the different units are turned on and off so as to meet the demand without violating these constraints is called the *unit commitment* problem. It is solved in conjunction with the *economic dispatch* problem that determines how much power each *committed* generating unit produces so as to minimize the production cost.

The unit commitment problem gives rise to a combinatorial optimization problem that is not particularly easy to solve for large systems. Solution methods for the unit commitment and economic dispatch problems in a purely deterministic situation where neither the load nor the unit operating status is considered to be random are given in Wood and Wollenberg [6]. The solution of this pair of problems for a generation system gives the optimal production costs for the time horizon to which the generation and load data refer. This can then be used to forecast the production costs over a given time period when the load is regarded as deterministic and the generators are assumed to be available with certainty. To the best of our knowledge counterparts of these optimization problems have not been considered for the situation in which the load or the generator operating states are considered as random variables. Monte Carlo simulation [7] is often done using this model to obtain estimates of production costs when these quantities are considered random, but they need more extensive development.

A second set of models into which it is relatively easy to incorporate the stochastic features of load and generator operating status is often used to obtain approximate estimates of production costs. In these models the unit commitment constraints are completely ignored and it is postulated that a *strict predetermined merit order of loading* prevails according to which the generating units will be run in order to meet the demand. The units will be used to their full capacity if available and if needed to produce power. The particular set of units that will be used to supply the demand at any given time thus depends on the magnitude of the load, the availability of the generating units and the postulated loading order among generating units. Unlike the unit commitment problem, this model does not account for the history of the operation of the generating units. This model is often used in practice under the assumption that it provides over the long run a close approximation to the first model that more closely mimics utility operations. In this paper we will consider the second model only because the probabilistic aspects of production costing have been developed almost exclusively for this case. This model was first proposed by Ryan and Mazumdar [2] and has been used in several other recent articles [1,3,8].

It is assumed that the costs are being calculated for a power generation system consisting of $N$ generating units over a time interval [0, $T$]. The following additional assumptions are made:

(i) The generators are dispatched at each instant in a fixed, predetermined loading order, and the actual set of units being used at any given time depends upon the instantaneous system load and the availability of the generating units. This assumption ignores the operating constraints associated with unit commitment.

(ii) The $j$th unit in the loading order has a capacity $c_j$ (MW), variable energy cost $d_j$ (\$/MWH), and the unavailability index (also known in the power system parlance as the forced outage rate), $p_j$. The cost of unserved energy (i.e., the unmet demand) is $d_{N+1}$ (\$/MWH). The sequence of values $\{d_j, j = 1, \ldots, N\}$ is increasing. The operating state of each unit $j$ follows an alternating renewal process $Y_j(t)$, which is in the steady state:

$$Y_j(t) = \begin{cases} 1 & \text{if unit } j \text{ is up at time } t, \\ 0 & \text{if unit } j \text{ is down at time } t. \end{cases}$$

The relation $E[Y_j(t)] = 1 - p_j$ holds for all values of $t$ in the interval [0, $T$]. (In Section 4, we will assume $Y_j(t)$ to be a continuous-time Markov chain).

(iii) The load at time $t$, which is denoted by $u(t)$, is also a stochastic process in the steady state. A special case is when $u(t)$ is a deterministic time-varying function. The load has a certain amount of predictable variation depending upon the day of the week, the time of the day, season, etc., but on top of this, there is random variation.

(iv) For all $i \neq j$, $Y_i(t)$ and $Y_j(s)$ are statistically independent for all values of $t$ and $s$. Also, each $Y_j(t)$ is independent of $u(t)$ for all values of $t$.

(v) The up and down process of a generating unit continues whether or not it is in use.

It is necessary to make assumption (v) in order to guarantee that the steady-state condition for the generating units holds. The steady-state condition is used to make the analytical formulas tractable. When Monte Carlo simulation is performed, this assumption is no longer essential. In the simulation reported in Section 4, we have however retained this assumption.

Let the index $i$ denote the $i$th position in the loading order for the generating units. Let $e_i(t)\mathrm{d}t$ and $Z_i(T)$ denote respectively the energy produced by unit $i$ during the time intervals [$t$, $t + \mathrm{d}t$], and [0, $T$], respectively. From the above assumptions it follows that

$$e_1(t) = \min[u(t), c_1]Y_1(t),$$

$$e_i(t) = \min\left[\max\left(u(t) - \sum_{k=1}^{i-1} c_k Y_k(t), 0\right), c_i\right] Y_i(t),$$

$$i = 2, 3, \ldots, N.$$

$$Z_i(T) = \int_0^T e_i(t)\mathrm{d}t. \qquad (1)$$

## 3. Analytical formulation for the expected value and variance of production costs

We briefly review the formula for the expected value of production costs. First, consider unit 1. From the above, we see that

$$E[Z_1(T)] = E\left[\int_0^T e_1(t)\mathrm{d}t\right] = \int_0^T E[e_1(t)]\mathrm{d}t$$

$$= \int_0^T E[\min(u(t), c_1)Y_1(t)]\mathrm{d}t.$$

Now define

$$I(t;x) = \begin{cases} 1 & \text{if } u(t) \geq x, \\ 0 & \text{if } u(t) < x. \end{cases}$$

Thus we can express

$$\min(u(t), c_1) = \int_0^{c_1} I(t;x)\mathrm{d}x.$$

Because of the assumed independence of $Y_1(t)$ and $u(t)$,

$$E[Z_1(t)] = \int_0^T \int_0^{c_1} E[I(t;x)Y_1(t)]\mathrm{d}x\ \mathrm{d}t,$$

$$= (1 - p_1) \int_0^{c_1} \int_0^T P\{u(t) \geq x\}\mathrm{d}t\ \mathrm{d}x,$$

$$= T(1 - p_1) \int_0^{c_1} G_T(x)\mathrm{d}x, \qquad (2)$$

where $G_T(x)$ is the average probability that the load is greater than $x$, the average being taken over the interval [0, $T$]. That is,

$$G_T(x) = \frac{1}{T}\int_0^T P\{u(t) \geq x\}\mathrm{d}t.$$

When the load $u(t)$ is a deterministic time-varying function, $P\{u(t) \geq x\} = I(t; x)$. That is, $G_T(x)$ now measures the proportion of the time that the load exceeds or equals $x$ in the time interval [0, $T$]. This quantity is known as the load–duration curve in the power systems literature, Proceeding in the same fashion, it can be shown [9] that

$$E[Z_i(T)] = T(1 - p_i) \int_0^{c_i} E\left[G_T\left(x + \sum_{j=1}^{i-1} c_j Y_j\right)\right] \mathrm{d}x. \quad (3)$$

Here, the $Y_j$'s are independently distributed Bernoulli random variables with $P\{Y_j = 0\} = p_j$, and $P\{Y_j = c_j\} = 1 - p_j$, and the integrand is equivalent to:

$$\frac{1}{T} \int_0^T P\left\{u(t) - \sum_{j=1}^{i-1} c_j Y_j \geq x\right\} \mathrm{d}t.$$

This reduces to the well-known Baleriaux formula [9–12] when the load is deterministic. Although it is not particularly easy to compute (3), several effective approximation procedures [12,13] have been developed for routine use.

In a similar way, it can be shown that the expected amount of unserved energy (i.e., the unmet demand by the generating system), denoted here by $V(T)$ is given by

$$E[V(T)] = \int_0^T \int_0^\infty P\left\{u(t) - \sum_{i=1}^{N} c_i Y_i \geq x\right\} \mathrm{d}x \, \mathrm{d}t, \\ = T \int_0^\infty E\left[G_T\left(x + \sum_{i=1}^{N} c_i Y_i\right)\right] \mathrm{d}x.$$

This integral also can be evaluated using the same approximations as those referred to above. The expected production cost $K(T)$ for the system during the interval [0, $T$] is obtained as follows:

$$E[K(T)] = \sum_{i=1}^{N} d_i E[Z_i(T)] + d_{N+1} E[V(T)],$$

The variance of the production cost is given by the following formula:

$$\mathrm{Var}[K(T)] = \sum_{i=1}^{N} \sum_{j=1}^{N} d_i d_j \,\mathrm{cov}(Z_i(T), Z_j(T)) \\ + d_{N+1}^2 \mathrm{Var}[V(T)] + 2 \sum_{i=1}^{N} d_i \,\mathrm{cov}(Z_i(T), V(T)).$$

However, the computation of the above covariance terms is not an easy proposition. To appreciate the reasons why, let us consider the first covariance term. From the definition of $Z_i(T)$ in (1), it follows that

$$\mathrm{cov}(Z_i(T), Z_j(T)) = \int_0^T \int_0^T \mathrm{cov}(e_i(t), e_j(s)) \mathrm{d}s \, \mathrm{d}t. \quad (4)$$

Now,

$$e_i(t) = \min\left[\max\left(u(t) - \sum_{k=1}^{i-1} c_k Y_k(t), 0\right), c_i\right] Y_i(t),$$

$$e_j(s) = \min\left[\max\left(u(s) - \sum_{k=1}^{j-1} c_k Y_k(s), 0\right), c_j\right] Y_j(s).$$

Although $Y_k(t)$ and $Y_j(s)$ are independent for $k \neq j$, $Y_k(t)$ and $Y_k(s)$ are correlated for each $k$. For a two-state continuous-time Markov chain in the steady-state, if the transition rate from the up state to the down state is denoted by $\lambda_k$ and the rate from the down state to the up state is $\mu_k$, then it is well-known [14] that

$$\mathrm{cov}(Y_k(t), Y_k(s)) = p_k q_k \exp(-(\lambda_k + \mu_k)|s - t|), \quad (5)$$

where $p_k = \lambda_k/(\lambda_k + \mu_k) = 1 - q_k$, and is the unavailability index of unit $k$. The approach of Ryan and Mazumdar [2] to compute the variance assumes that the load $u(t)$ is a deterministic function of $t$ and the stochastic process $Y_i(t)$ is a continuous-time Markov chain. Their procedure can be illustrated by considering the term $\mathrm{Var}(Z_2(T))$. From the definition of $e_j(t)$ it follows that

$$e_2(t) = \min(\max(u(t) - c_1, 0), c_2) Y_1(t) Y_2(t) \\ + \min(u(t), c_2)(1 - Y_1(t)) Y_2(t),$$

$$e_2(s) = \min(\max(u(s) - c_1, 0), c_2) Y_1(s) Y_2(s) \\ + \min(u(t), c_2)(1 - Y_1(s)) Y_2(s).$$

Thus in order to compute the variance term one needs to compute the covariance of $e_2(t)$ and $e_2(s)$ using (5) and then integrate the covariance function $\mathrm{cov}(e_2(t), e_2(s))$ in (4). This process needs to be repeated for each pair of the indices $i$ and $j$ ($i, j = 1, 2, \ldots, N$). Thus the extent of the required computations becomes enormous for large systems. In this procedure, the difficulty arises from the need to account for the $2^N$ possible availability states of a system of $N$ generating units. Several recent papers have suggested procedures for simplifying the computation of the variance of the production costs but they still remain quite complex and cumbersome [1,3,15,16]. Also, in none of these papers is the randomness of the load taken into account.

## 4. Statistical analysis of a Monte Carlo output

The complexities of the analytical formulae should not be a deterrent for the use of the production costing models because many of the answers to questions involving them can be obtained with the aid of Monte Carlo simulation. Properly designing the simulation remains an important issue and we have addressed previously several ways in which variance reduction can be achieved [17]. We illustrate an application of Monte Carlo simulation in attempting to answer two important questions relating to the magnitude of uncertainty associated with the load in estimating the electric power production costs. Many of the production costing models that are currently used by the industry assume that the load is a deterministic quantity with no inherent randomness. But it is clear that such an assumption is hardly justifiable. Thus it is natural to ask how important the variability associated with the load is as compared to the variability resulting from the generator outages insofar as the estimate of production cost is concerned. One of the important factors that affect the magnitude of the short-term variations in the load is the ambient temperature. Thus a question that is very relevant is as follows: if an accurate forecast of the temperature is available for the study horizon, can it be utilized to reduce the variance of the estimated production cost? In an attempt to answer these two questions using Monte Carlo simulation we analyzed a data set that gave the actual ambient temperature and the corresponding load for each hour in a region covering the north-eastern United States during the calendar years 1995 and 1996.

The data set was divided into two subsets: spring–summer and fall–winter. The load data pertaining to holidays such as Labor Day, Christmas, and Independence Day were deleted because they did not represent typical load profiles for the weekdays. In order to answer the two questions raised in the preceding paragraph, we present an analysis considering the spring-summer weekdays only. The purpose of this analysis is to predict the production costs for six selected weekdays during the spring and summer seasons, and estimate the variance components due to load and generator availabilities for these days. The six chosen days for which the predictions are being made were also deleted from the data set. Thus, the reduced data set consists of 246 weekdays. Figure 1 gives a plot of the last 900 hourly load data points from this set.

We next considered a generation system patterned after the generation mix of an actual electric utility company. The relevant characteristics of the units, in their loading order, are given in Table 1.

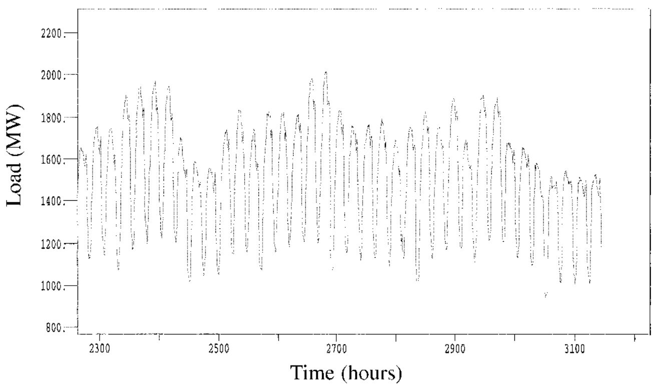


**Fig. 1.** Hourly load data for a summer season.

### 4.1. *Load models*

We consider two separate models for the load. In Model I, the load is characterized as a discrete time series, and the temperature data is not used. In Model II, the hourly load is represented as the sum of two components: a regression equation with temperature as an independent variable and a discrete time series for the residual values.

#### 4.1.1. *Model I: time series for hourly load*

The data sequence here consists of the hourly load for the weekdays starting at 12 midnight of March 21 1996 and ending at 11 pm of September 20 1996. To maintain an unbroken sequence in the data, the average hourly load values for the respective weekdays were inserted for the holidays. Missing information and the hourly data for the six test days were also replaced with the corresponding average values. The result is an unbroken sequence of 132 weekdays. After the weekly cycle (difference = 120 hours for five weekdays) is removed from the data, an AR(3) model is found to provide a good fit to the time series. Plots of the Auto-Correlation Function (ACF) and the Partial Auto-Correlation Function (PACF) of the hourly load data, the theoretical AR(3) model, and the residuals are given in Fig. 2(a–c). The ACF and PACF of Fig. 2(a) closely match those of Fig. 2(b), and those of Fig. 2(c) match the characteristics of a white noise process. The fitted model, an ARIMA $(3,0,0)\times(0,1,0)_{120}$, is:

$$\begin{aligned} u(t) = & -2.49 + 1.103[u(t-1) - u(t-121)] \\ & + 0.000\,070\,24[u(t-2) - u(t-122)] \\ & - 0.1412[u(t-3) - u(t-123)] + u(t-120) + z(t). \end{aligned}$$

Here $z(t)$ is a Gaussian white noise process with mean 0 and estimated variance $\hat{\sigma}_z^2 = 990.04$.

**Table 1.** A generation system

| *Units* | *Capacity (MW)* | *Mean time to failure (1/λ)* | *Mean time to repair (1/μ)* | *Energy cost ($/MWH)* |
|---|---|---|---|---|
| 1–2 | 400 | 1100 | 150 | 6.00 |
| 3 | 350 | 1150 | 100 | 11.40 |
| 4–7 | 150 | 960 | 40 | 11.40 |
| 8–9 | 150 | 1960 | 40 | 14.40 |
| 10–12 | 200 | 950 | 50 | 22.08 |
| 13–15 | 100 | 1200 | 50 | 23.00 |
| 16 | 50 | 2940 | 60 | 27.60 |
| 17 | 100 | 450 | 50 | 43.50 |

cost of unserved energy $d_{18}$: $135/MWH

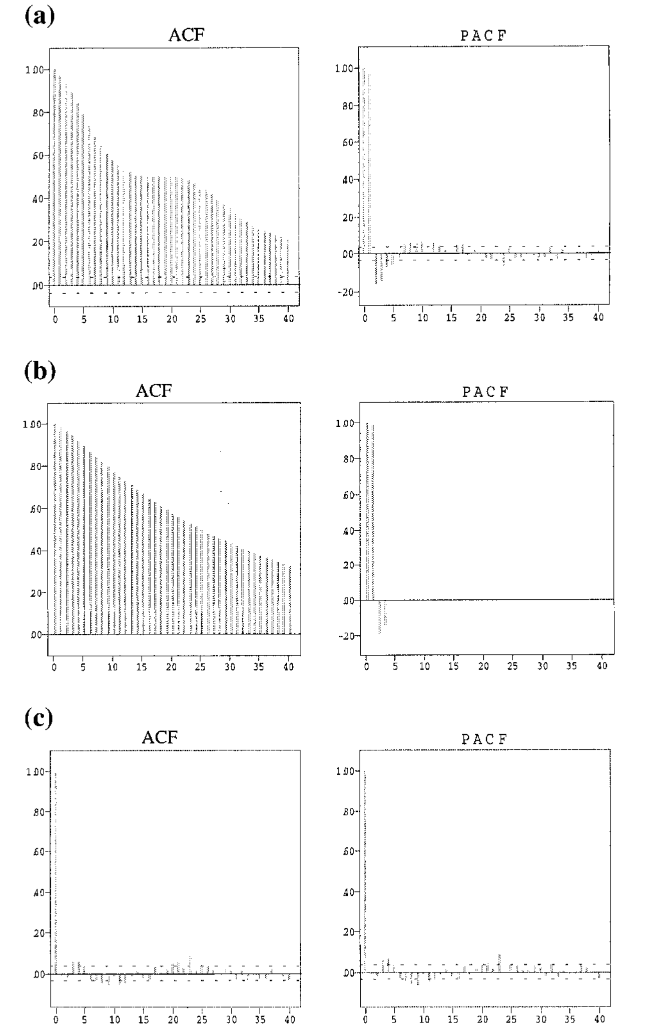


**Fig. 2.** (a) ACF and PACF of $x(t) = u(t) - u(t-120)$; (b) ACF and PACF of an AR(3) process; and (c) ACF and PACF of the residuals.

### 4.1.2. *Model II: time series for hourly load with temperature as an independent variable*

In this model, individual regression equations are first fitted for each hour of a 24-hour period in which the load u(t) is the response and the hourly temperature $\tau_t(^\circ\mathrm{F})$ is the independent variable. A plot of the load versus temperature at each hour, shown in Fig. 3 for the hour beginning at 12 noon, suggested the following regression equation:

$$u(t) = \beta_{0,t} + \beta_{1,t}\tau_t + \beta_{2,t}(\tau_t - 65)\delta(\tau_t) + x(t),$$

where $\delta(\tau_t)$ is defined as:

$$\delta(\tau_t) = \begin{cases} 0 & \text{if } \tau_t \le 65, \\ 1 & \text{if } \tau_t > 65, \end{cases}$$

and $x(t)$ is a time series to be suitably determined.

We used a reduced data set of 246 weekdays (spring–summer seasons of 1995 and 1996) to estimate the least-square regression coefficients, $\hat{\beta}_t$, which are given in Table 2. A $t$-test of the regression coefficients concluded that all regression coefficients at each hour differ significantly from zero (the $p$-values were 0.000+). The parameters of the time series $x(t)$ were next estimated based on the residuals. For this purpose, the same data sequence as used in Model I (March 21 1996 to September 20 1996) was used. The plots of the ACF and the PACF of $x(t) - x(t-120)$, when compared to those of an AR(1) process and those of the residuals with the corresponding functions of a white noise process, suggest that an AR(1) model is suitable for representing the data. (see Fig. 4(a–c)). For Model II, an ARIMA $(1,0,0) \times (0,1,0)_{120}$, is obtained as follows:

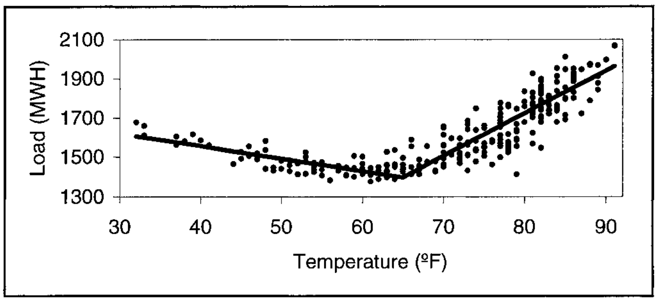


**Fig. 3.** Load versus temperature at 12 noon.

$$u(t) = \hat{\beta}_{0,t} + \hat{\beta}_{1,t}\tau_t + \hat{\beta}_{2,t}(\tau_t - 65)\,\delta(\tau_t) + x(t),$$
$$x(t) = x(t-120) + 0.879[x(t-1) - x(t-121)] + z(t),$$

**Table 2.** Least-square estimates of regression coefficients

| *Hour* | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|---|---|---|---|
| 0 | 1266.488 | −3.043 | 24.923 |
| 1 | 1228.883 | −3.098 | 23.214 |
| 2 | 1200.436 | −2.784 | 21.825 |
| 3 | 1210.148 | −3.110 | 23.743 |
| 4 | 1252.080 | −3.378 | 25.518 |
| 5 | 1396.442 | −4.181 | 26.683 |
| 6 | 1615.254 | −5.040 | 24.353 |
| 7 | 1730.754 | −4.980 | 25.635 |
| 8 | 1722.957 | −4.150 | 25.587 |
| 9 | 1725.510 | −4.286 | 25.880 |
| 10 | 1779.210 | −5.160 | 26.604 |
| 11 | 1810.045 | −5.940 | 27.871 |
| 12 | 1801.092 | −6.194 | 27.849 |
| 13 | 1828.385 | −6.812 | 28.760 |
| 14 | 1837.352 | −7.484 | 29.619 |
| 15 | 1860.203 | −8.327 | 30.908 |
| 16 | 1890.890 | −8.823 | 30.870 |
| 17 | 1973.484 | −10.534 | 33.116 |
| 18 | 2046.137 | −11.728 | 33.575 |
| 19 | 2055.195 | −11.765 | 33.010 |
| 20 | 2003.196 | −10.099 | 32.082 |
| 21 | 1699.140 | −4.486 | 28.454 |
| 22 | 1524.169 | −3.820 | 28.715 |
| 23 | 1392.461 | −3.821 | 28.062 |

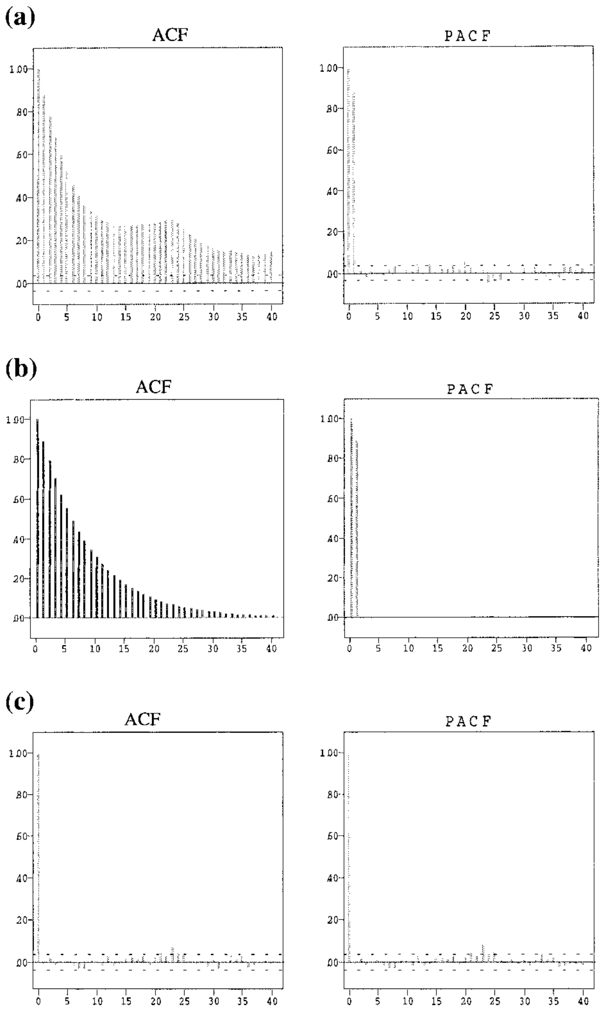


**Fig. 4.** (a) ACF and PACF of $x(t) - x(t - 120)$; (b) ACF and PACF of an AR(1) process; and (c) ACF and PACF of the residuals.

where $z(t)$ is a Gaussian white noise with mean 0 and estimated variance $\hat{\sigma}_z^2 = 2032.55$.

### 4.2. *Monte Carlo procedure*

In the Monte Carlo simulation, the stochastic inputs for each run are the hourly load and the operating state for each generating unit at each instant during the six chosen 24-hour periods. The outputs are the total energy produced by each unit and the production cost of each unit and the system during the same period. We use the following notation to describe the simulation:

$N$ = Number of generating units.
$T$ = Number of hours.
$\lambda_i$ = Failure rate of generating unit $i$ $(i = 1, \ldots, N)$.
$\mu_i$ = Repair rate of generating unit $i$ $(i = 1, \ldots, N)$.
$\mathbf{U}_l$ = $[u_l(1), u_l(2), \ldots, u_l(T)]$ = sample $l$ $(l = 1, \ldots, L)$ of an hourly load profile during hours 1 to $T$.
$L$ = Number of hourly load profiles to be sampled.
$\mathbf{S}_q$ = $\{[t_1, t_2, \ldots, t_j..]^1,\ [t_1, t_2, \ldots, t_j..]^2, \ldots.\ [t_1, t_2, \ldots, t_j..]^N\}$ = sample $q(q = 1, \ldots, Q)$ of a generator up and down time sequence covering the period [0, $T$].
$[t_1, t_2, \ldots, t_j..]^i$ = Up and down time subsequence of generator $i$. If $j$ is odd, $t_j$ is the time at which the generator $i$ fails, otherwise $t_j$ denotes the time that the generator $i$ is repaired and made available $(t_j < T)$.
$Q$ = Number of sequences $\mathbf{S}_q$ to be sampled.
$C_{l,q}$ = Total production cost when the sampled hourly load profile is $\mathbf{U}_l$ and the sampled generator up and down time sequence is $\mathbf{S}_q$.

#### 4.2.1. *Steps of the Monte Carlo procedure*

*Step 1.* Read the parameters pertaining to capacity, costs, failure and repair rates for each generating unit as well as the estimated parameters for the two load models.

*Step 2.* Repeat for $l = 1$ to $L$. (*sampling of load vectors*)

*2.1* Obtain a load vector $\mathbf{U}_l$ by sampling each hours load for 24 hours based on the known values of the earlier hours and the load model.

*2.2* Repeat for $q = 1$ to $Q$. (*sampling of generator up and down states*)

*2.2.1* Set the state of generators at time $t = 0$ to all units being up.

*2.2.2* Obtain $\mathbf{S}_q$ by generating successive up and down times until time $T$. For each generating unit $i$ draw random samples of uptime from an exponential distribution with parameter $\lambda_i$ and draw samples of downtime from an exponential distribution with parameter $\mu_i$ $(i = 1, 2, \ldots, N.)$

*2.2.3* Using the predetermined loading order among the available units, compute the energy produced by each unit during the time interval [0, $T$] using the model of Section 3.

*2.2.4* Calculate the production cost $C_{l,q}$ for each unit and the system for the time interval [0, $T$].

*2.2.5* Add the cost of unserved energy (if any) to $C_{l,q}$.

### 4.3. *Statistical analysis of Monte Carlo output*

The simulation output yielded a total of $L \times Q$ values of production costs for the period under consideration. This set of output values has two identifiable sources of variation: load and generator up and down times. A random-effect one-factor model [18] was used to represent this data. In this model, the cost $C_{ij}$ is given as follows:

$$C_{ij} = \mu + l_i + g_{ij} \quad (i = 1, 2, \ldots, L;\ j = 1, 2, \ldots, Q), \qquad (6)$$

where $\mu$. is a constant, $l_i$ are iid normal random variables with expectation zero and variances $\sigma_l^2$, $g_{ij}$ are iid normal random variables with expectation zero and variances $\sigma_g^2$, and $l_i$ and $g_{ij}$ are independent.

The variation of $C_{ij}$ within each load value $l_i$ results from generator outages during the study period. The quantities $l_i$ and $g_{ij}$ are the load and generator effects, and the quantities $\sigma_l^2$ and $\sigma_g^2$ respectively give the contributions of load and generator to the variance of production costs $\sigma^2$ which equals $\sigma_l^2 + \sigma_g^2$.

Performing a standard analysis of variance on the simulation output, we obtained the mean square between loads (MSL) and the mean square within loads (MSG). It is well known [18] that the expected values of these two quantities are given by

$$E[MSL] = \sigma_g^2 + Q\sigma_l^2, \quad E[MSG] = \sigma_g^2$$

Thus, the estimates of $\sigma_g^2$ and $\sigma_l^2$ are

$$S_g^2 = MSG, \quad S_l^2 = \frac{MSL - MSG}{Q}, \quad S^2 = S_g^2 + S_l^2. \qquad (7)$$

Here, $S^2$ is an estimate of the variance of the production cost $\sigma_2$. Therefore, the estimate of the variance of the production cost is broken into two components, $S_g^2$ and $S_l^2$. The former measures the variability of the production cost due to the generator effect and the latter due to the load effect.

### 4.4. *Estimation of variance components*

The Monte Carlo simulation was run, with the initial state all units up at hour 0, for both load models for a period covering 24 hours of six different days, April 10, May 3, June 4, July 30, August 23, and September 20, 1996. It was assumed that the actual loads for these six particular days were unknown but that the hourly loads of their predecessor days and 1 week earlier were known. Moreover, perfectly accurate forecasts of the hourly temperatures were assumed to be available for all 6 days. A total of 90 000 runs were made for each 24 hour period with $L = 300$ and $Q = 300$. For each Model I and II, the variance components of the production costs were estimated from Equation (7). Table 3 gives the results. They show that quite different estimates for the expected production costs were obtained when the forecast temperature was considered. Moreover, the overall variance $S^2$ as well as the contribution of the variance component $S_l^2$ decreased when temperature was included in Model II. The explanation is that knowing the temperature reduces the residual load variability and therefore its effect on the variance of the production cost is decreased. The Monte Carlo simulation was also run using the actual values of

**Table 3.** Estimated variance components of the production cost

| *Season/date* | *Load model* | $\hat{E}[C]$ | $S_l^2$ (× 1000) | $\hat{Var}[C]$ (× 1000) | $S_l^2/\hat{Var}[C]$ |
|---|---|---|---|---|---|
| Spring | | | | | |
| April 10 1996 | Model 0 | 309 675 | – | 64 785 | – |
| | Model I | 283 497 | 568 239 | 619 104 | 0.915 |
| | Model II | 307 619 | 273 060 | 342 642 | 0.795 |
| May 3 1996 | Model 0 | 249 875 | – | 40 277 | – |
| | Model I | 235 565 | 554 193 | 592 845 | 0.932 |
| | Model II | 240 352 | 257 369 | 296 682 | 0.865 |
| June 4 1996 | Model 0 | 261 903 | – | 46 388 | – |
| | Model I | 233 398 | 547 599 | 588 399 | 0.928 |
| | Model II | 252 268 | 256 485 | 308 485 | 0.829 |
| Summer | | | | | |
| July 30 1996 | Model 0 | 287 461 | – | 57 195 | – |
| | Model I | 286 627 | 580 042 | 640 446 | 0.903 |
| | Model II | 272 122 | 261 141 | 311 337 | 0.836 |
| Aug 23 1996 | Model 0 | 363 814 | – | 187 657 | – |
| | Model I | 310 348 | 623 696 | 703 818 | 0.884 |
| | Model II | 356 323 | 380 384 | 545 140 | 0.696 |
| Sept 20 1996 | Model 0 | 259 321 | – | 44 140 | – |
| | Model I | 254 843 | 559 610 | 604 238 | 0.923 |
| | Model II | 281 138 | 265 961 | 324 651 | 0.817 |

the hourly load for each of these 6 days. We call this case "Model 0." Thus this model includes only one source of uncertainty namely that resulting from generator availability. Estimated 95% prediction intervals of the production cost corresponding to Models 0, I, and II were obtained based on the Monte Carlo estimates of the mean and the variance of the production costs. These intervals, illustrated in Figs 5 and 6, give the approximate range within which the production cost is expected to lie with probability 0.95. We observe that the intervals predicted by Model II contain those of Model 0 for all the six test days, which is not the case for Model I, and the intervals for Model II are much shorter compared to those for Model I. We also notice that the effect of temperature is more evident in the results obtained from the summer test days compared to the spring test days in that the intervals predicted by Model I contain all those of Model 0 for the spring season.

From the Monte Carlo results we observe that the load effect is a major component of variation of production costs, and the variation in costs is large. Therefore, load should not be treated as deterministic in production costing models. The variability associated with it should be explicitly modeled. We also see that the estimates of the short-term production costs may be made more precise by using the temperature forecasts and time series models that govern the load distribution. Efforts toward more accurate load prediction using covariates such as temperature, humidity, wind speed, etc. should pay a dividend by reducing the error of production cost forecasts.

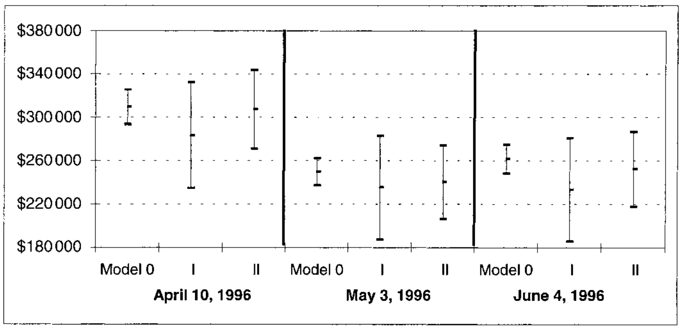


**Fig. 5.** Prediction intervals of the production cost (spring).

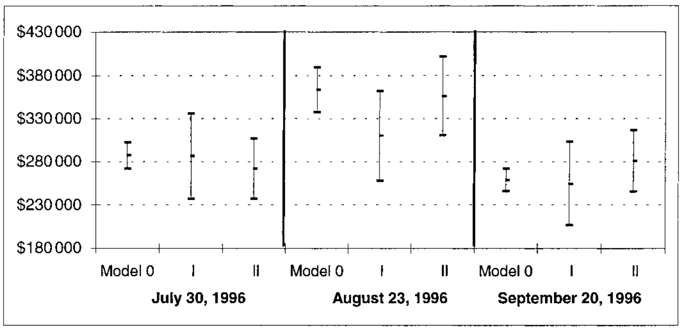


**Fig. 6.** Prediction intervals of the production cost (summer).

## 5. Concluding remarks

Cost of electricity production is a topic of general public interest. In this paper we have attempted to illustrate the importance of probability models and statistical analysis for assessing this cost. Probability models are important because the production cost is affected by two stochastic variables – demand and the availability of the generating units. The reliability of the generators plays a crucial role in determining costs because electricity cannot be conveniently stored and the failure of a low-cost unit will lead to a higher-cost unit being substituted in its place. The state space associated with a realistic power generation system can be quite large resulting from a combination of the operating states of the individual generating units and the load states. This makes the analytical computation of the mean and the variance of the production costs quite difficult. However, Monte Carlo simulation can always be used to obtain answers to practical questions of interest. In this paper we gave an example of a Monte Carlo simulation from which we can conclude that in order to obtain accurate forecasts of electric power production costs the hourly load should not be considered deterministic but regarded as a stochastic process. Better load prediction by considering predictor variables such as temperature will improve the accuracy of prediction of production costs. The findings of this paper suggest that analytical models, which explicitly consider load as a stochastic process, need to be developed.

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